Generalized dynamic observers for LPV systems

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Abstract: In this paper, an observer for linear parameter varying (LPV) systems, based on the generalized dynamic observer (GDO) is proposed. The design of the GDO is derived from the solution of linear matrix inequalities (LMIs) and the solution of the algebraic constraints obtained from the estimation error analysis. The efficiency of the proposed approach is illustrated through the estimation of the outlet temperatures of a double pipe heat exchanger. In this paper, a new observer structure named generalized dynamic observer (GDO) for linear parameter varying (LPV) systems is proposed. The design of the GDO is derived from the solution of linear matrix inequalities (LMIs) and the solution of the algebraic constraints obtained from the estimation error analysis. The efficiency of the proposed approach is illustrated through the estimation of the outlet temperatures of a double pipe heat exchanger.

Keywords: Generalized dynamic observer, LPV systems, heat exchanger.

1. INTRODUCTION

A linear parameter varying (LPV) system is a set of linear state space models that are interpolated trough an activation function. Commonly, a certain number of points in the scheduling space are selected forming a regular grid. At each point, a LTI system is assigned to describe the dynamic behavior of the local neighborhood. The study of LPV modeling theory was introduced by Shamma (1988). It has been increasing in the last decades and it has other advantages in comparison with a nonlinear model. For instance, it is possible to represent the nonlinear behavior of a nonlinear system through many linear submodels that describe different operation points. These submodels are interpolated through a mechanism depending of the scheduling variables. If an state variable is used as scheduling variable, the system is called quasi-LPV and represents a large class of nonlinear systems. There are works that have focused on the design and implementation of algorithms for estimation and control for LPV systems.

Advanced observer techniques for LPV systems have been reported in the literature such as proportional and integral observers by Hamdi et al. (2011), sliding mode observers by Pakki et al. (2014), descriptor observers by López-Estrada et al. (2015), generalized dynamic observers by Osorio-Gordillo et al. (2015) and adaptive observers by Nguyen et al. (2015) used for fault estimation to determine the type, size and shape of the fault interested, which is a useful information for fault tolerant control.

In control and estimation schemes it is assumed that the state vector is measured. However in a practical case, this assumption is not always fulfilled. Therefore, there exists the necessity of estimate these variables with an observer.

In the design of observers for LPV systems there are a lot of works that solve the estimation issue. In Sename et al. (2013); Bara et al. (2000); Daafouz et al. (2000); Stilwell and Rugh (1999) a proportional observer (PO) for LPV system is proposed. On the other hand the estimation task is boarded by proportional integral observers (PIO), as is made in Youssef et al. (2014); Hamdi et al. (2011); Ichalal et al. (2009).

The objective of the estimation is to get the estimation error equals to zero in steady state. In a PO there always exists a static estimation error in presence of constant perturbations, this consideration is solved with an integral term provided by the PIO to deal with the effect of the perturbation in the estimation error. The integral term is conformed by the difference between the estimated output and the system output and an integral gain.

In Osorio-Gordillo et al. (2016, 2015) and Gao et al. (2016) a new structure of observer called generalized dynamic observer (GDO) was developed. This structure is inspired by Goodwin and Middleton (1989); Park et al. (2002) which add additional dynamics in the observer introducing a new alternative for state estimation. The PIO and GDO have the same characteristic of cancel the effect of the disturbance in the estimation error in the steady state. The main difference between these structures is the additional dynamics in the observer and the degrees of freedom added to the structure. In the PIO it is possible manipulate just one matrix, while the GDO has two available matrices that allows of achieving steady state accuracy and improve robustness in estimation error against disturbances and parametric uncertainties. The practical contribution of this paper focuses in the application of the proposed method to a realistic model of a heat exchanger.

2. PRELIMINARIES

Throughout this paper, $\|.\|$ denotes the Euclidean norm. Let W a subset of vector space V. Then, we define the left orthogonal complement $W^\perp$ by $W^\perp = \{x \in V : x^T y = 0 \text{ for all } y \in W\}$.

The following lemma will be used in the sequel of this paper.

Lemma 1: Skelton et al. (1997) Let matrices $\mathcal{B}$ and $\mathcal{Q} = \mathcal{Q}^T$ be given. Then the following statements are equivalent:

i) There exists a matrix $\mathcal{X}$ satisfying.
$$\mathcal{B}\mathcal{X} + (\mathcal{B}\mathcal{X})^T + \mathcal{Q} < 0$$
ii) The following conditions holds.
$$\mathcal{B}^\perp \mathcal{Q} \mathcal{B}^{\perp T} < 0 \quad \text{or} \quad \mathcal{B}\mathcal{B}^T > 0$$

suppose the above statements hold and further assume that $\mathcal{B}^T\mathcal{B} > 0$. Then all matrices $\mathcal{X}$ in statement i) are given by

$$\mathcal{X} = -\sigma \mathcal{B}^T + \sqrt{\sigma}\mathcal{L}\vartheta^{1/2}$$

where $\mathcal{L}$ is any matrix such that $\|\mathcal{L}\| < 1$ and $\sigma > 0$ is any scalar such that

$$\vartheta \triangleq \sigma \mathcal{B}\mathcal{B}^T - \mathcal{Q} > 0$$

3. PROBLEM FORMULATION

Let us consider the following LPV system

$$\dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) \tag{1a}$$
$$y(t) = Cx(t) \tag{1b}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ the input vector, $y(t) \in \mathbb{R}^p$ represents the measured output vector and $\rho(t) \in \mathbb{R}^j$ is a varying parameter vector. It is assumed that all parameters $\rho = \{\rho_1 \dots \rho_j\}$ are bounded, measurable and their values remain in a convex polytope of M vertices as in Rodrigues et al. (2007). The LPV system (1) can be rewritten as the following polytopic representation:

$$\dot{x}(t) = \sum_{i=1}^{M} \mu_i(\rho(t))(A_i x(t) + B_i u(t)) \tag{2a}$$
$$y(t) = Cx(t) \tag{2b}$$

where
$$\sum_{i=1}^{M} \mu_i(\rho(t)) = 1, \quad 0 \le \mu_i(\rho(t)) \le 1 \tag{3}$$

$\forall i \in [1, \dots, M]$ where $M = 2^j$. $\mu_i(\rho(t)) = \mu(\overline{\rho}_i, \underline{\rho}_i, \rho_i(t), t)$ ($\overline{\rho}_i$) and $\underline{\rho}_i$) represent the maximum and minimum value of ρ_i respectively).

Now, let us consider the following GDO for system (2)

$$\dot{\zeta}(t) = \sum_{i=1}^{M} \mu_i(\rho(t))(N_i\zeta(t) + H_i v(t) + F_i y(t) + J_i u(t)) \tag{4a}$$
$$\dot{v}(t) = \sum_{i=1}^{M} \mu_i(\rho(t))(S_i\zeta(t) + L_i v(t) + M_i y(t)) \tag{4b}$$
$$\hat{x}(t) = P\zeta(t) + Qy(t) \tag{4c}$$

where $\zeta(t) \in \mathbb{R}^{q_0}$ represents the state vector of the observer, $v(t) \in \mathbb{R}^{q_1}$ is an auxiliary vector and $\hat{x}(t) \in \mathbb{R}^n$ is the estimate of $x(t)$. Matrices N_i, H_i, F_i, S_i, L_i, M_i, P and Q are unknown matrices of appropriate dimensions which must be determined such that $\hat{x}(t)$ converges asymptotically to $x(t)$.

Remark 1: The GDO (4) is in a generalized form. In fact:

- For $H_i = 0$, $S_i = 0$, $M_i = 0$ and $L_i = 0$ the observer reduces to the PO for LPV systems.
$$\dot{\zeta}(t) = \sum_{i=1}^{M} \mu_i(\rho(t))(N_i\zeta(t) + F_i y(t) + J_i u(t))$$
$$\hat{x} = P\zeta(t) + Qy(t)$$
- For $L_i = 0$, $S_i = -CP$ and $M_i = -CQ + I$ then the following PIO for LPV systems is obtained
$$\dot{\zeta}(t) = \sum_{i=1}^{M} \mu_i(\rho(t))(N_i\zeta(t) + H_i v(t) + F_i y(t) + J_i u(t))$$
$$\dot{v}(t) = y(t) - C\hat{x}(t)$$
$$\hat{x}(t) = P\zeta(t) + Qy(t)$$

The following lemma gives existence conditions of the observer (4).

Lemma 2: There exists an observer of the form (4) for the system (2) if the following two statements hold

1. There exists a matrix T of appropriate dimension such that the following conditions are satisfied
   (a) $N_i T + F_i C - TA_i = 0$
   (b) $J_i = TB_i$
   (c) $S_i T + M_i C = 0$
   (d) $PT + QC = I_n$
2. The matrices $\sum_{i=1}^{M} \mu_i(\rho(t)) \begin{bmatrix} N_i & H_i \\ S_i & L_i \end{bmatrix}$ are Hurtwitz $\forall i \in [1, \dots, M]$.

Proof. Let $T \in \mathbb{R}^{q_o \times n}$ be a parameter matrix and consider the transformed error $\varepsilon(t) = \zeta(t) - Tx(t)$, then its derivative is given by:

$$\begin{aligned}\dot{\varepsilon}(t) = \sum_{i=1}^{M} \mu_i(\rho(t))(N_i\varepsilon(t) + (N_i T + F_i C - TA_i)x(t) \\ + H_i v(t) + (J_i - TB_i)u(t))\end{aligned} \tag{7}$$

by using the definition of $\varepsilon(t)$, equations (4b) and (4c) can be written as:

$$\dot{v}(t) = \sum_{i=1}^{M} \mu_i(\rho(t))(S_i\varepsilon(t) + (S_iT + M_iC)x(t) + L_iv(t)) \tag{8}$$

$$\hat{x}(t) = P\varepsilon(t) + (PT + QC)x(t) \tag{9}$$

If conditions a-d of Lemma 2 are satisfied, the following observer error dynamics is obtained from (7) and (8)

$$\underbrace{\begin{bmatrix} \dot{\varepsilon}(t) \\ \dot{v}(t) \end{bmatrix}}_{\dot{\varphi}(t)} = \sum_{i=1}^{M} \mu_i(\rho(t)) \underbrace{\begin{bmatrix} N_i & H_i \\ S_i & L_i \end{bmatrix}}_{\mathbb{A}_i} \underbrace{\begin{bmatrix} \varepsilon(t) \\ v(t) \end{bmatrix}}_{\varphi(t)} \tag{10}$$

From (9), we have

$$\hat{x}(t) - x(t) = e(t) = P\varepsilon(t) \tag{11}$$

in this case if matrices $\mathbb{A}_i$ are Hurtwitz then $\lim_{t\to\infty} e(t) = 0$.

4. GDO DESIGN

4.1 Parameterization of the observer matrices

In this section, we shall give the parameterization of the algebraic constraint equations of Lemma 2. Let $E \in \mathbb{R}^{q_0 \times n}$ be a full row rank matrix such that the matrix $\Sigma = \begin{bmatrix} E \\ C \end{bmatrix}$ is of full column rank and let $\Omega = \begin{bmatrix} I_n \\ C \end{bmatrix}$. Conditions c and d can be written as:

$$\begin{bmatrix} S_i & M_i \\ P & Q \end{bmatrix} \begin{bmatrix} T \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \tag{12}$$

the necessary and sufficient condition for equation (12) to have a solution is:

$$\text{rank} \begin{bmatrix} T \\ C \end{bmatrix} = \text{rank} \begin{bmatrix} T \\ C \\ 0 \\ I_n \end{bmatrix} = n \tag{13}$$

Now, since rank $\begin{bmatrix} T \\ C \end{bmatrix} = n$, there always exist matrices $T \in \mathbb{R}^{q_0 \times n}$ and $K \in \mathbb{R}^{q_0 \times p}$ such that:

$$T + KC = E \tag{14}$$

which can be written as:

$$[\, T \;\; K \,]\, \Omega = E \tag{15}$$

and since rank(Ω)=rank$\begin{bmatrix} \Omega \\ E \end{bmatrix}$. The particular solution of equation (15) is given by:

$$[\, T \;\; K \,] = E\Omega^+ \tag{16}$$

Equation (16) is equivalent to:

$$T = T_1 \tag{17}$$

$$K = K_1 \tag{18}$$

where $T_1 = E\Omega^+ \begin{bmatrix} I_n \\ 0 \end{bmatrix}$ and $K_1 = E\Omega^+ \begin{bmatrix} 0 \\ I_p \end{bmatrix}$. Now, inserting the equivalence T from (14) into condition (a) it leads to:

$$N_iE + \tilde{K}_iC = TA_i \tag{19}$$

where $\tilde{K}_i = F_i - N_iK$ and equation (19) can be written as:

$$\begin{bmatrix} N_i & \tilde{K}_i \end{bmatrix} \Sigma = TA_i \tag{20}$$

The general solution of (20) is given by:

$$\begin{bmatrix} N_i & \tilde{K}_i \end{bmatrix} = TA_i\Sigma^+ - Z_i(I_{n+p} - \Sigma\Sigma^+) \tag{21}$$

by replacing matrix T from equation (17) into equation (21) it gives:

$$N_i = N_{1,i} - Z_iN_3 \tag{22}$$

$$\tilde{K}_i = \tilde{K}_{1,i} - Z_i\tilde{K}_3 \tag{23}$$

where $N_{1,i} = T_1A_i\Sigma^+ \begin{bmatrix} I_{qo} \\ 0 \end{bmatrix}$, $N_3 = (I_{qo+p} - \Sigma\Sigma^+) \begin{bmatrix} I_{qo} \\ 0 \end{bmatrix}$, $\tilde{K}_{1,i} = T_1A_i\Sigma^+ \begin{bmatrix} 0 \\ I_p \end{bmatrix}$, $\tilde{K}_3 = (I_{qo+p} - \Sigma\Sigma^+) \begin{bmatrix} 0 \\ I_p \end{bmatrix}$ and Z_i are arbitrary matrices of appropriate dimension. As matrices N_i, T, K, $\tilde{K}_i$ are known, we can deduce the matrix F_i as:

$$F_i = F_{1,i} - Z_iF_3 \tag{24}$$

where $F_{1,i} = T_1A_i\Sigma^+ \begin{bmatrix} K \\ I_p \end{bmatrix}$, $F_3 = (I_{n+p} - \Sigma\Sigma^+) \begin{bmatrix} K \\ I_p \end{bmatrix}$.

On the other hand from equation (14) we obtain:

$$\begin{bmatrix} T \\ C \end{bmatrix} = \begin{bmatrix} I_{qo} & -K \\ 0 & I_p \end{bmatrix} \Sigma \tag{25}$$

inserting equation (25) into the equation (12) we get:

$$\begin{bmatrix} S_i & M_i \\ P & Q \end{bmatrix} \begin{bmatrix} I_{qo} & -K \\ 0 & I_p \end{bmatrix} \Sigma = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \tag{26}$$

Since matrix Σ is of full column rank and

$$\begin{bmatrix} I_n & -K \\ 0 & I_{n_y} \end{bmatrix}^{-1} = \begin{bmatrix} I_{qo} & -K \\ 0 & I_p \end{bmatrix}$$

the general solution to equation (26) is given by:

$$\begin{bmatrix} S_i & M_i \\ P & Q \end{bmatrix} = \left(\begin{bmatrix} 0 \\ I_n \end{bmatrix} \Sigma^+ - \begin{bmatrix} U_{1,i} \\ U_2 \end{bmatrix} (I_{qo+p} - \Sigma\Sigma^+) \right) \times \begin{bmatrix} I_{qo} & K \\ 0 & I_p \end{bmatrix} \tag{27}$$

where $U_{1,i}$ and U_2 are arbitrary matrices of appropriate dimensions. Then matrices S_i, M_i, P and Q can be determined as:

$$S_i = -U_{1,i}N_3 \tag{28}$$
$$M_i = -U_{1,i}F_3 \tag{29}$$
$$P = \Sigma^+ \begin{bmatrix} I_{qo} \\ 0 \end{bmatrix} - U_2 N_3 \tag{30}$$
$$Q = \Sigma^+ \begin{bmatrix} K \\ I_p \end{bmatrix} - U_2 F_3 \tag{31}$$

Now, by using (22) and (28) the observer error dynamics (10) can be rewritten as:

$$\dot{\varphi}(t) = \sum_{i=1}^{M} \mu_i(\rho(t))((\mathbb{A}_i - \mathbb{Y}_i\mathbb{A}_2)\varphi(t)) \tag{32}$$

where

$$\mathbb{A}_i = \begin{bmatrix} N_{1,i} & 0 \\ 0 & 0 \end{bmatrix}, \ \mathbb{A}_2 = \begin{bmatrix} N_3 & 0 \\ 0 & -I_{qo} \end{bmatrix}, \ \mathbb{Y}_i = \begin{bmatrix} Z_i & H_i \\ U_{1,i} & L_i \end{bmatrix}.$$

4.2 Stability analysis

In this section, a method to design a GDO from (4) is presented. This method is obtained from the determination of matrix $\mathbb{Y}_i$, such that system (32) is stable. The GDO matrices can be obtained by using the following theorem.

Theorem 1: There exists a parameter matrix $\mathbb{Y}_i$ such that the system (32) is asymptotically stable if and only if there exists a matrix $X = \begin{bmatrix} X_1 & X_1 \\ X_1 & X_2 \end{bmatrix} > 0$ where $X_1 = X_1^T$ such that the following LMI's are satisfied.

$$N_3^{T\perp} \left[N_{1,i}^T X_1 + X_1 N_{1,i}\right] N_3^{T\perp T} < 0 \tag{33}$$

Then, by using the elimination lemma matrix $\mathbb{Y}_i$ is parameterized as

$$\mathbb{Y}_i = X^{-1}(-\sigma\mathcal{B}^T + \sqrt{\sigma}\mathcal{L}\vartheta_i^{1/2})^T \tag{34}$$

where $\mathcal{L}$ is any matrix such that $\|\mathcal{L}\| < 1$ and $\sigma > 0$ is any scalar such that

$$\vartheta_i \triangleq \sigma\mathcal{B}\mathcal{B}^T - \mathcal{Q}_i > 0 \tag{35}$$

with

$$\mathcal{Q}_i = \begin{bmatrix} X_1 N_{1,i} + N_{1,i}^T X_1 & N_{1,i}^T X_1 \\ X_1 N_{1,i} & 0 \end{bmatrix},$$
$$\mathcal{B} = \begin{bmatrix} -N_3^T & 0 \\ 0 & I_q \end{bmatrix}.$$

Proof Consider the following Lyapunov function candidate

$$V(\varphi(t)) = \varphi(t)^T X \varphi(t) > 0 \tag{36}$$

with $X = \begin{bmatrix} X_1 & X_1 \\ X_1 & X_2 \end{bmatrix} > 0$ and $X_1 = X_1^T$. Its derivative along the trajectory of (32) is given by

$$\dot{V}(\varphi(t)) = \sum_{i=1}^{M} \mu_i(\rho(t))\varphi(t)^T((\mathbb{A}_i - \mathbb{Y}_i\mathbb{A}_2)^T X + X(\mathbb{A}_i - \mathbb{Y}_i\mathbb{A}_2))\varphi(t) < 0 \tag{37}$$

the inequality $\dot{V}(\varphi(t)) < 0$ is valid for all $\varphi(t) \neq 0$ if and only if

$$(\mathbb{A}_i - \mathbb{Y}_i\mathbb{A}_2)^T X + X(\mathbb{A}_i - \mathbb{Y}_i\mathbb{A}_2) < 0 \tag{38}$$

which can be written as

$$\sum_{i=1}^{M} \mu_i(\rho(t))(\mathcal{B}\mathcal{X}_i + (\mathcal{B}\mathcal{X}_i)^T + \mathcal{Q}_i) < 0 \tag{39}$$

where $\mathcal{B} = -\mathbb{A}_2^T$ and $\mathcal{Q}_i = \mathbb{A}_{1,i}^T X + X\mathbb{A}_{1,i}$ and $\mathcal{X}_i = X\mathbb{Y}_i$. According to elimination lemma there exists a matrix $\mathcal{X}_i$ satisfying (39) if and only if the following condition holds:

$$\mathcal{B}^\perp \mathcal{Q}_i \mathcal{B}^{\perp T} \tag{40}$$

with $\mathcal{B}^\perp = \begin{bmatrix} -N_3^{T\perp} & 0 \end{bmatrix}$. By using the definition of matrix $\mathcal{Q}_i$ we obtain (33). If (40) is satisfied, the parameter $\mathbb{Y}_i$ is obtained as in (34).

5. APPLICATION TO DOUBLE PIPE HEAT EXCHANGER

In order to illustrate our results, let us consider a double pipe heat exchanger. It is used for energy exchange between at least two fluid streams, a hot and a cold stream. In this case, the hot water flows through the inner pipe and the cooling water flows through the annular section (outside of the inner pipe) López-Zapata et al. (2016).

To obtain a simple model of the heat transfer, the following modeling assumptions are used:

A1. Constant volume and mass in the heat exchanger pipes.
A2. Physico-chemical properties of the fluid are constant.
A3. Global heat transfer coefficient (U) and area (A) are constant.
A4. There is not heat transfer with the environment.
A5. Inlet temperatures are measured.

The continuous time state equations that represent the energy balance are given in Eq. (41)

$$\dot{T}_{co}(t) = \frac{v_c}{V_c}(T_{ci}(t) - T_{co}(t)) + \frac{UAr}{c_{pc}\rho_c V_c}(T_{ho}(t) - T_{co}(t)) \tag{41a}$$

$$\dot{T}_{ho}(t) = \frac{v_h}{V_h}(T_{hi}(t) - T_{ho}(t)) + \frac{UAr}{c_{ph}\rho_h V_h}(T_{co}(t) - T_{ho}(t)) \tag{41b}$$

where V_c is the volume in external side, V_h is the volume in the inner side, v_c is the flow in the cold stream, v_h is the flow in the hot stream, c_{pc} is the specific heat of cold water, c_{ph} is the specific heat of hot water, ρ_c is the density of cold water, ρ_h is the density of hot water, Ar is the heat transfer surface area and U is the global heat transfer coefficient. $T_{ci}(t)$ and $T_{hi}(t)$ are the inlet temperatures in the cold and hot streams respectively. $T_{co}(t)$ and $T_{ho}(t)$ are the outlet temperatures in the cold and hot streams respectively. Consider the following LPV system described by (1) where there exist one scheduling parameters $\rho(t) \in [0.5 \times 10^{-5}, 3 \times 10^{-5}]$ which represents the variation of the flow in the hot stream v_h. Therefore, the LPV system is

$$A(\rho(t)) = \begin{bmatrix} -\dfrac{UAr}{c_{pc}\rho_c V_c} - \dfrac{v_c}{V_c} & \dfrac{UAr}{c_{pc}\rho_c V_c} \\ \dfrac{UAr}{c_{ph}\rho_h V_h} & -\dfrac{UAr}{c_{ph}\rho_h V_h} - \dfrac{\rho(t)}{V_h} \end{bmatrix},$$

$$B(\rho(t)) = \begin{bmatrix} \dfrac{v_c}{V_c} & 0 \\ 0 & \dfrac{\rho(t)}{V_h} \end{bmatrix}, \quad \text{and} \quad C = [\,1\ 0\,].$$

Such that the scheduling functions $\mu_i(\rho(t))$ are

$$\mu_1(\rho(t)) = \frac{\overline{\rho} - \rho(t)}{\overline{\rho} - \underline{\rho}} \tag{42}$$

$$\mu_2(\rho(t)) = \frac{\rho(t) - \underline{\rho}}{\overline{\rho} - \underline{\rho}} \tag{43}$$

The problem is to estimate the states $[\,T_{co}\ T_{ho}\,]^T$ by using the GDO. By solving the LMI's of Theorem 1 and choosing the matrix $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathcal{L} = \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}$ and $\sigma = 10$ we obtain the following results:

$$X = \begin{bmatrix} 13.7138 & -0.0542 & 13.7138 & -0.0542 \\ -0.0542 & 11.0021 & -0.0542 & 11.0021 \\ 13.7138 & -0.0542 & 27.4277 & 0 \\ -0.0542 & 11.0021 & 0 & 27.4277 \end{bmatrix}$$

$$\mathbb{Y}_1 = \begin{bmatrix} 0.7677 & 0.0385 & -0.6907 & 0.7677 & 0.0361 \\ 0.1973 & 0.1967 & 0.1961 & 0.1943 & 0.8055 \\ -0.3100 & 0.0546 & 0.4192 & -0.6746 & 0.0570 \\ 0.0072 & 0.0060 & 0.0048 & 0.0084 & -0.6028 \end{bmatrix}$$

$$\mathbb{Y}_2 = \begin{bmatrix} 0.7782 & 0.0490 & -0.6802 & 0.7782 & 0.0466 \\ 0.5534 & 0.5528 & 0.5522 & 0.5504 & 1.1616 \\ -0.3141 & 0.0505 & 0.4151 & -0.6787 & 0.0529 \\ -0.1215 & -0.1227 & -0.1239 & -0.1203 & -0.7315 \end{bmatrix}$$

Finally, we can get all the matrices of the observer as:

$$N_1 = \begin{bmatrix} -0.7596 & 0.0263 \\ 0.1144 & -0.4539 \end{bmatrix}, \ N_2 = \begin{bmatrix} -0.7596 & 0.0263 \\ 0.1144 & -1.5731 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 0.3646 & 0 \\ -0.0012 & 0 \end{bmatrix}, \ S_2 = \begin{bmatrix} 0.3646 & 0 \\ -0.0012 & 0 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.7677 & 0.0361 \\ 0.1943 & 0.8055 \end{bmatrix}, \ H_2 = \begin{bmatrix} 0.7782 & 0.0466 \\ 0.5504 & 1.1616 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -0.6746 & 0.0570 \\ 0.0084 & -0.6028 \end{bmatrix}, \ L_2 = \begin{bmatrix} -0.6787 & 0.0529 \\ -0.1203 & -0.7315 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.3191 \\ 0.1728 \end{bmatrix}, \ F_2 = \begin{bmatrix} 0.3191 \\ 0.1728 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} -0.1823 \\ 0.0006 \end{bmatrix}, \ M_2 = \begin{bmatrix} -0.1823 \\ 0.0006 \end{bmatrix},$$

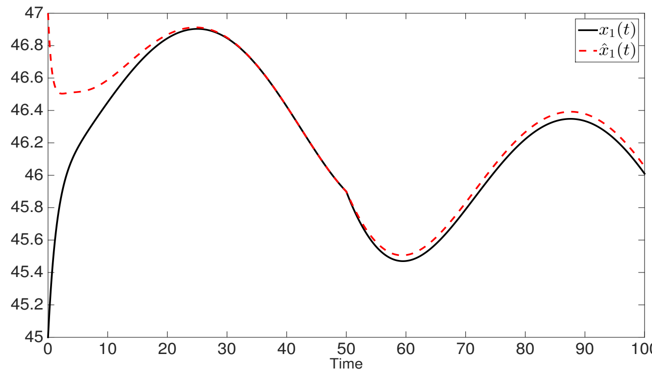

Fig. 1. Estimation of x_1.

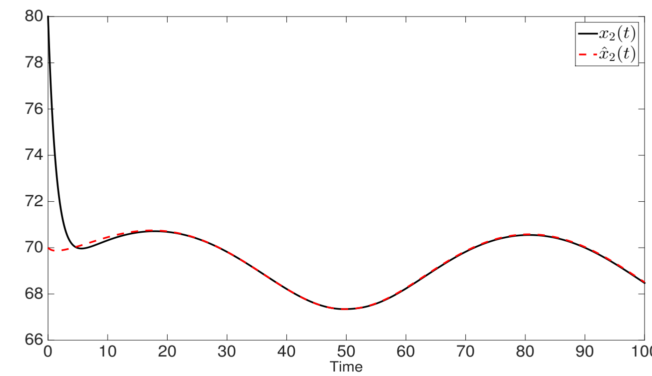

Fig. 2. Estimation of x_2.

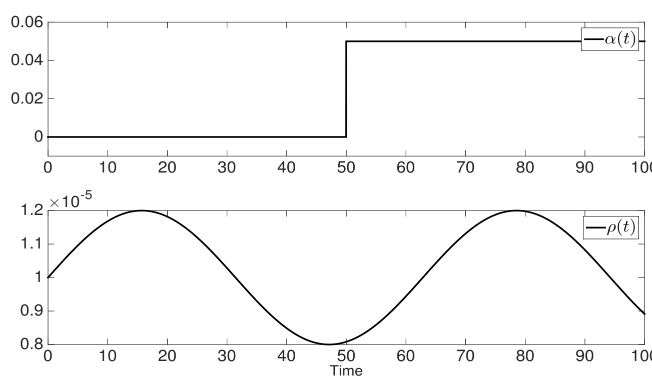

Fig. 3. Uncertainty $\alpha(t)$ and parameter variant $\rho(t)$.

$P = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 0.75 \\ 0 \end{bmatrix}$.

The initial condition for the system are $x(0) = [45, 80]^T$ for the GDO are $\zeta(0) = [26.5, 70]^T$, $v(0) = [0, 0]^T$ and $u(t) = [29, 81]^T$. To evaluate the performance of the observers an uncertainty is added in the flow of the cold stream $\Delta v_c = v_c + \alpha(t) v_c$. The results of the simulation are depicted the following Figures

6. CONCLUSION

In this paper a generalized dynamic observer for LPV systems is presented. The conditions for the existence of the GDO are provided and its stability is proved in form

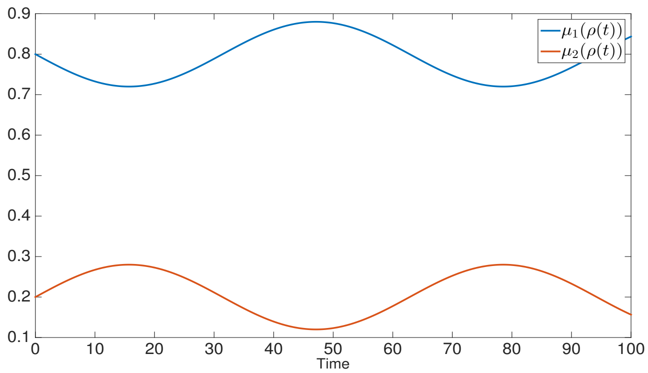

Fig. 4. Weighting functions $\mu_1(\rho(t))$ and $\mu_2(\rho(t))$

of LMIs. In order to illustrate the observer performances, a double pipe heat exchanger is used. From the simulation results, it can be seen that the GDO has characteristics of robustness to certain class of uncertainties in the steady state regime.

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