

Dissipative observers for time-delay nonlinear systems

Jesús D. Avilés^{* *} Jaime A. Moreno[‡]
Fernanda Alvarez-Mendoza[‡]

**Instituto Politécnico Nacional, Sección de Estudios de Posgrado e Investigación, ESIME-UPT, C.P. 07430, Ciudad de México, México
(e-mail: jesus.david.aviles@gmail.com).*

*‡Instituto de Ingeniería-UNAM, Coyoacán DF, 04510 México D.F.,
(e-mail: jmorenoP@iingen.unam.mx)*

Abstract:

In this paper, we investigate the problem for designing dissipative nonlinear observer for a family of time-delay systems. The methodology guarantees the (practical) asymptotic convergence of the observer depending on the (presence) absence of disturbances and/or uncertainties in the system dynamics. This dissipative design method is applied to the observer error dynamics, which are represented as a closed-loop system between a Linear Time-invariant (LTI) subsystem with multiple time delays and a static nonlinearity. A wide variety of nonlinearities can be included by means of dissipative design. In addition, the sufficient design conditions are studied in order to be treated in the ambient of the Linear Matrix Inequalities (LMI's).

Keywords: Nonlinear observer, Dissipative observer, Time delay systems.

1. INTRODUCTION

Many applications in communications, teleoperation, chemical, physiological and biological processes, electrical, mechanical and hydraulic systems, can be characterized by means of nonlinear time-delay systems. The analysis of these systems is more complex compared to the classic systems without any delay (Naifar et al., 2015; Ghanes et al., 2013). The stability and control of systems with delayed state present several interesting problems due to that the presence of a delay in the system dynamics can state instability, or bad behaviors for the closed-loop schemes (Niculescu and Lozano, 2001; Hale and Lunel, 2013).

Recently, the state observers for time-delay systems appear as an increasing area for the researchers. This class of systems has some open problems (Naifar et al., 2015), where observation methodologies have been proposed in order to solve it, for instance, there exist approaches based on asymptotic method (Bhat and Koivo, 1976; Darouach, 2001; Mazenc et al., 2012), interval methods (Li and Lam, 2012; Efimov et al., 2014, 2015), sliding mode methods (Niu et al., 2004; Iskander et al., 2013), H_∞ approach (Fattouh et al., 1998; Wang et al., 2001).

In this work we present a new method for designing state observers for a family of nonlinear time-delay systems

subject to multiple time delays, using the dissipative approach proposed in (Moreno, 2004). This method considers the absence/presence of additive disturbances. The main idea consists in decomposing the observer error dynamics in (i) a linear time invariant (LTI) subsystem with constant multiple delays and (ii) a nonlinear time-varying feedback, when the nonlinear system is disturbance-free. If the nonlinearity is dissipative with respect to (w.r.t.) a quadratic supply rate, it is required that the LTI subsystem to be dissipative w.r.t a related supply rate, in order to assure exponential stability of the closed-loop system. This notion can easily be extended for working with nonlinear time-delay systems affected by additive perturbations. The observation methodology permits including a wide variety of the nonlinearities, which are used in some observer approaches in the literature.

The paper is organized as follows. The problem formulation is considered in Section 2. Some preliminaries are given in Section 3. In Section 4 the dissipative observer design for nominal nonlinear systems subject to multiple delays; In the same section the observation method is extended deals time-delay systems with disturbances. Some computational aspects are discussed in Section 5.

2. PROBLEM FORMULATION

We consider a class of time-delayed nonlinear systems described by

^{*} J.D. Avilés is on leave from Autonomous University of Baja California. Faculty of Engineering and Business. C.P. 21460, Tecate, B.C, México.

$$\Sigma_{\text{pf}} : \begin{cases} \dot{x}(t) = A_0 x(t) + A_1 x(t - h_1) + G f_n(\sigma; t, y, u) \\ \quad + \varphi(t, y, u) + b(t), \\ y(t) = C x(t), \\ \sigma(t) = H x(t), \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $y(t) \in \mathbb{R}^p$ is the output vector. $h_1 > 0$ is a constant time delay. $b(t)$ is an unknown input for Σ_{pf} . $\sigma(t) \in \mathbb{R}^r$ is a (not necessarily measured) linear function of the state, $f_n(\sigma; t, y, u) \in \mathbb{R}^m$ is a nonlinear function locally Lipschitz in σ , and uniformly in (t, u, y) , $\varphi(t, y, u)$ is a nonlinear function locally Lipschitz in (u, y) and piecewise continuous in t .

The aim of this paper is to design a state observer for time-delayed nonlinear system, in absence or presence of perturbations, using the (Q, S, R) dissipative approach. Moreover, the observer design can be generalized for nonlinear systems subject to constant multiple delays.

3. PRELIMINARIES

We introduce the characterization of dissipativity tailored for specific case of systems with a point-wise delay. For further details see (Haddad et al., 2004; Rocha-Cózatl and Moreno, 2010; Avilés and Moreno, 2014).

3.1 Dissipativity property

We consider a time-delay system described by a functional differential equation

$$\Gamma_{\text{NL}} : \begin{cases} \dot{x}(t) = F(x(t), x(t-h), u(t)), \\ x(\theta_h) = \phi(\theta_h), \quad -h \leq \theta_h \leq 0, \quad t \geq 0, \\ y(t) = H(x(t), x(t-h), u(t)) \end{cases} \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^p$ is the output, $u(t) \in \mathbb{R}^m$ is the input. F and H are locally Lipschitz in its arguments. Note that the state at the time t , in the second expression, of Γ_{NL} in (2) is the *piece of trajectories* x between $x-h$ and t , or equivalently, the element x_t in the space of continuous functions defined on the interval $[-h, 0]$, taking values in \mathbb{R}^n ; this denoted by $x_t \in \mathcal{C}([-h, 0], \mathbb{R}^n)$. Thus, we have $x_t(\theta_h) = x(t+\theta_h)$, $\theta_h \in [-h, 0]$. Furthermore, the initial state is indicated as $\phi(\cdot) \in \mathcal{C}([-h, 0], \mathbb{R}^n)$ for any $h \geq 0$, which represents a continuous vector valued function.

Definition 1. Σ_{NL} is a *time-delay state strictly dissipative system with respect to*, the so called *supply rate* $w(u, y)$, which is locally integrable with $w(0, 0) = 0$, if there exists a positive-definite *storage rate* with $V(0) = 0$, such that the following the *dissipation inequality*

$$\dot{V}(x(t), x(t-h)) \leq -\epsilon V(x(t)) + w(y(t), u(t)) \quad (3)$$

is fulfilled for all $t \geq 0$.

We will consider, without loss of generality, quadratic forms for both,

- A quadratic *supply rate*, which can be expressed by

$$w(y, u) = \begin{bmatrix} y \\ u \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} \quad (4)$$

where $Q = Q^T \in \mathbb{R}^{p \times p}$, $S \in \mathbb{R}^{p \times m}$, $R = R^T \in \mathbb{R}^{m \times m}$.

- A storage rate, represented by the Lyapunov – Krasovskii functional candidate,

$$V = x^T(t) P x(t) + \int_{t-h}^t x^T(s) Q x(s) ds, \quad (5)$$

where $P, Q \in \mathbb{R}^{n \times n}$ are positive definite matrices.

We introduce the following definitions, in according to Moreno (2004), for the time-delay systems using the previous functions.

- The linear time invariant (LTI) system with delay, given by,

$$\Gamma_{\text{L}} : \begin{cases} \dot{x}(t) = A_0 x(t) + A_1(x-h_1) + Bu(t), \\ x(\theta_h) = \phi(\theta_h), \quad -h \leq \theta_h \leq 0, \quad t \geq 0, \\ y(t) = Cx(t) \end{cases} \quad (6)$$

with the initial state $x(\theta_h) = \phi(\theta_h)$, $\forall \theta_h \in [-h, 0]$, where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^p$ is the measurement output, $u(t) \in \mathbb{R}^m$ is the control input. h is a known time delay of the system satisfying $h > 0$. Γ_{L} is *Time-Delay State Strictly Dissipative* (SSD) w.r.t. $w(y, u)$ in (4), denoted as $\text{SSD}\{Q, S, R\}$, if and only if there exist the matrices $P = P^T > 0$, $Q = Q^T > 0$ and scalar constant $\epsilon > 0$ such that

$$\begin{bmatrix} PA_0 + A_0^T P + \epsilon P + Q - C^T Q C & \star & \star \\ A_1^T P & -Q & 0 \\ B^T P - S^T C & 0 & -R \end{bmatrix} \leq 0 \quad (7)$$

where \star is the symmetric matrix block.

- A varying-time *static function* given by

$$y = f(t, u) \quad (8)$$

$f : [0, \infty) \times \mathbb{R}^m \rightarrow \mathbb{R}^p$, piecewise continuous in t and locally Lipschitz in u , such that $f(t, 0) = 0$, is *dissipative* w.r.t. $w(y, u)$ in (4), which is denoted in short-form $\text{D}\{Q_N, S_N, R_N\}$, if

$$w(y, u) = w(f(t, u), u) \geq 0. \quad (9)$$

for any $t \geq 0$ and $u \in \mathbb{R}^m$,

The sector conditions for quadratic nonlinearities ($m = p$) can be represented in terms of dissipativity. For instance,

- If $f \in [K_1, K_2]$, that is, $(y - K_1 u)^T (K_2 u - y) \geq 0$, then f is $\text{D}\{Q_N, S_N, R_N\}$ with:

$$\text{D}\{-I, \frac{1}{2}(K_1 + K_2), -\frac{1}{2}(K_1^T K_2 + K_2^T K_1)\}$$

- If $f \in [K_1, \infty]$, that is, $(y - K_1 u)^T u \geq 0$, then f is $\text{D}\{Q_N, S_N, R_N\}$ with:

$$(Q_N, S_N, R_N) = \left(0, \frac{1}{2}I, -\frac{1}{2}(K_1 + K_1^T) \right)$$

Remark 2. The above conditions includes H_∞ and passivity as special cases.

- (1) When $Q = -I$, $S = 0$, and $R = \gamma^2 I$, the *dissipative* nonlinearity is reduced to an H_∞ performance requirement.
- (2) When $Q = 0$, $S = I$, and $R = 0$, the *dissipative* nonlinearity corresponds to the strict passive case.

Remark 3. A multivariable nonlinearity f can be $D\{Q_i, S_i, R_i\}$ for several triple (Q_i, S_i, R_i) , i.e., $\omega_i(f(t, u), u) = f^T Q_i f + 2f^T S_i u + u^T R_i u \geq 0$, for $i = 1, 2, \dots, \mu$ Moreno (2004). In this case, it is easy to see that f is $\sum_{i=1}^\mu \theta_i(Q_i, S_i, R_i)$ -D for every $\theta_i \geq 0$, i.e., f is dissipative with respect to the supply rate $\omega_\theta(f, u) = \sum_{i=1}^\mu \theta_i \omega_i(f, u)$.

We now apply the foregoing notions for the following closed-loop system given by a linear time-delay subsystem with the negative feedback connection of a static nonlinearity. The use of the Lyapunov–Krasovskii functional candidate allows deriving delay-independent stability results for the closed-loop system.

Lemma 4. Let the interconnected time-delayed system

$$\Gamma_I : \begin{cases} \dot{x}(t) = A_0 x(t) + A_1 x(t - h_1) + Bu(t), \\ y(t) = Cx(t), \\ u(t) = -\xi(t, y), \end{cases} \quad (10)$$

if there exists a triplet of dissipative function, that is $D(Q_N, S_N, R_N)$, such that the LTI time-delay subsystem is $SDD\{-R_N, S_N^T, -Q_N\}$, then the origin of Γ_I is *globally exponentially stable*.

Proof. Considering the Lyapunov-Krasovskii functional candidate

$$V(x(t), x(t - h)) = x^T(t)Px(t) + \int_{t-h}^t x(s)Qx(s)ds$$

which is, clearly, positive definite and radially unbounded. Its derivate along the trajectories of Σ_E is given by $\dot{V}(x) = x^T(t)(A_0^T P + PA_0 + Q)x(t) + 2x^T(t)B^T P v(t) - x(t - h)Qx(t - h) + 2x^T(t - h)A_1^T P x(t)$

which is rewritten and bounded by

$$\begin{aligned} & \begin{bmatrix} x(t) \\ x(t - h) \\ v(t) \end{bmatrix}^T \begin{bmatrix} A_0^T P + PA_0 + Q & \star & \star \\ A_1^T P & -Q & 0 \\ B^T P & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - h) \\ v(t) \end{bmatrix} \\ & \leq \begin{bmatrix} x(t) \\ -\xi(t, y) \end{bmatrix}^T \begin{bmatrix} -C^T R_N C & S_N^T C^T \\ CS_N & -Q_N \end{bmatrix} \begin{bmatrix} x(t) \\ -\xi(t, y) \end{bmatrix} \\ & -\epsilon V(x) \\ & \leq -\epsilon V(x) \end{aligned}$$

selecting $SSD\{Q, S, R\} = \{-R_N, S_N^T, -Q_N\}$. Then the origin of Σ_I is Globally Exponentially Stable equilibrium point. ■

Now, additive disturbances/uncertainties have been considered for the interconnected system Γ_I . This can be expressed as

$$\Gamma_P : \begin{cases} \dot{x}(t) = Ax(t) + A_h x(t - h) + Bu(t) + b(t), \\ y(t) = Cx(t), \\ u(t) = -\xi(t, y), \end{cases} \quad (11)$$

where $b(t)$ represents the exogenous term for Γ_P . In the following Lemma, we present the practical stability characteristics for Γ_P .

Lemma 5. The nonlinear time-delay system Γ_P is Input-to-State Stable (ISS) w.r.t $b(t)$, if the requirements of Lemma 3 are satisfied.

Proof. Considering the same Lyapunov-Krasovskii functional candidate for Γ_P , we have its bounded derivative as follows When b is different from zero the inequality $\dot{V} \leq -\epsilon V + 2x^T P b$ can be rewritten, for any $\theta \in (0, 1)$, as

$$\begin{aligned} \dot{V}(x) & \leq -(1 - \theta)\epsilon V - \theta \epsilon x^T P x + 2x^T P b \\ & \leq -(1 - \theta)\epsilon V - \theta \epsilon \lambda_{\max}(P) \|x\|_2^2 + 2\lambda_{\max}(P) \|b\|_2 \|x\|_2 \\ & \leq -(1 - \theta)\epsilon V + \lambda_{\max}(P) \|x\|_2 (2\|b\|_2 - \theta \epsilon \|x\|_2) \\ & \leq -(1 - \theta)\epsilon V, \quad \forall \|x\|_2 \geq \frac{2}{\theta \epsilon} \|b\|_2. \end{aligned}$$

Applying (Khalil, 2002, Theorem 4.19) it follows that Ξ_P is ISS with respect to $b(t)$.

■

The Lemma 4 states the exponential stability property for Γ_P , and represents a special case of the Lemma 5 when the input is vanished, that is $b(t) = 0$.

4. DISSIPATIVE OBSERVER DESIGN

In this section, we present a method for designing dissipative observers for nominal/disturbed nonlinear time-delay systems.

4.1 Nominal case

Consider the nonlinear system with multiple time delays, which are not commensurable,

$$\Sigma_S : \begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{nd} A_i x(t - h_i) + Gf(\sigma; t, y, u) \\ \quad + \varphi(t, y, u), \\ y(t) = Cx(t), \\ \sigma(t) = Hx(t), \end{cases} \quad (12)$$

with the initial state $x(\theta_h) = \phi(\theta_h)$, $\forall \theta_h \in [-h, 0]$, where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^p$ is the measurement output, $u(t) \in \mathbb{R}^m$ is the control input. $\sigma(t) \in \mathbb{R}^r$ is a (not necessarily measured) linear function of the state, $f_n(\sigma; t, y, u) \in \mathbb{R}^m$ is a nonlinear function locally Lipschitz in σ , and uniformly in (t, u, y) , $\varphi(t, y, u)$ is a nonlinear function locally Lipschitz in (u, y) and piecewise continuous in t . h_i , $i = 1, \dots, nd$, are a known time delays of the system satisfying $0 < h_1 < h_2 < \dots < h_{nd}$. Furthermore, A, A_i, G, C, H are constant matrices with appropriate dimensions.

We introduce an state observer for Σ_S , which takes the form

$$\Sigma_O : \begin{cases} \dot{\hat{x}}(t) = A_0\hat{x}(t) + \sum_{i=1}^{nd} A_i\hat{x}(t-h_i) + \varphi(t, y, u) \\ \quad + Gf(\hat{\sigma}(t) + N(\hat{y}(t) - y(t)); t, y, u) \\ \quad + L_0(\hat{y} - y) + \sum_{i=1}^{nd} L_i(\hat{y}(t-h_i) - y(t-h_i)), \\ \hat{\sigma}(t) = H\hat{x}(t), \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \quad (13)$$

where \hat{x} is the estimated state of Σ_O . The observer gains are $L \in \mathbb{R}^{n \times p}$, $N \in \mathbb{R}^{r \times p}$.

The difference $e = \hat{x} - x$ is the so called error estimation. Thus, the dynamics of observer error are given by

$$\begin{aligned} \dot{e}(t) &= A_{L0}e(t) + \sum_{i=1}^{nd} A_{Li}e(t-h) + \\ &\quad G[f(\hat{\sigma}(t) + NCe(t); t, y, u) - f(\sigma(t); t, y, u)], \\ \tilde{\sigma}(t) &= He(t), \\ \tilde{y}(t) &= Ce(t), \end{aligned}$$

where $e(t-h) = \hat{x}(t-h) - x(t-h)$ is the same estimation error that depend on the delay h . It is worth to note that the estimation error systems can be rewritten as

$$\Sigma_E : \begin{cases} \dot{e}(t) = A_{L0}e(t) + \sum_{i=1}^{nd} A_{Li}e(t-h) + Gv(t) \\ z(t) = H_N e(t), \\ v(t) = -\xi(\sigma(t), z(t)), \end{cases} \quad (14)$$

taking into account $\hat{\sigma} + N(\hat{y} - y) = Hx + He + NCe = \sigma + (H + NC)e$. Defining $z \triangleq (H + NC)e = \tilde{\sigma} + N\tilde{y}$. Moreover, the matrices $A_{L0} = A_0 + L_0C$, $A_{Li} = A_i + L_iC$. The incremental nonlinearities can be expressed by

$$\xi(z, \sigma; t, u, y) = f(\sigma; t, u, y) - f(\sigma + z; t, u, y) \quad (15)$$

Now, we consider the characteristics on the dissipative observers for the family of the nonlinear time-delay systems.

Definition 6. Σ_O is an *dissipative observer* for the system Σ_S , if the error estimation error is SSD $\{-R_N, S_N^T, -Q_N\}$, that is, the estimation asymptotically converges to the real state trajectory,

$$\|e(t)\| \rightarrow 0 \quad \text{as } t \rightarrow \infty.$$

The following Theorem represents the main result of this paper, which provide the sufficient conditions for designing dissipative observers for a family of nonlinear systems with delay Σ_S .

Theorem 7. Assume that ϕ is D $\{Q_N^j, S_N^j, R_N^j\}$ for any quadratic form

$$\omega_N^j(\phi, \vartheta) = \phi^T Q_N^j \phi + 2\phi^T S_N^j \vartheta + \vartheta^T R_N^j \vartheta \geq 0, \forall \sigma, j = 1, \dots, \mu \quad (16)$$

Suppose that the matrices $P = P^T > 0$, $Q_i = Q_i^T > 0$, L_0 , L_i , N , an vector $\theta = (\theta_1, \dots, \theta_\mu) \succeq 0$ and a scalar $\epsilon > 0$, $i = 1, 2, \dots, nd$, such that :

$$\begin{bmatrix} \Omega & \star & \star & \star & \star \\ A_{L1}^T P & -Q_1 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{Lnd}^T P & 0 & 0 & -Q_{nd} & 0 \\ G^T P - S_N H_N & 0 & 0 & 0 & Q_N \end{bmatrix} \leq 0 \quad (17)$$

where $\Omega = A_{L0}^T P + P A_{L0} + H_N^T R_N H_N + \epsilon P + \sum_{i=1}^{nd} Q_i$. Then, Σ_O is an globally exponentially convergent dissipative observer for Σ_S . \diamond

Proof. We consider the scalar case for the nonlinearity $\xi(\sigma, z)$, that is, $\mu = 1$. Given the Krasovskii-Lyapunov candidate functional

$$\begin{aligned} V(e) &= e^T(t) P e(t) + \int_{t-h_1}^t e^T(s) Q_1 e(s) ds \\ &\quad + \int_{t-h_2}^t e^T(s) Q_2 e(s) ds + \dots + \int_{t-h_{nd}}^t e^T(s) Q_{nd} e(s) ds \end{aligned}$$

its derivate along the trajectories of Σ_E is given by $\dot{V}(e) = e^T(t)(A_{L0}^T P + P A_{L0} + Q_1 + Q_2 + \dots + Q_{nd})e(t) + 2e^T(t)G^T P v(t) - e(t-h)(Q_1 + Q_2 + \dots + Q_{nd})e(t-h) + 2e^T(t-h)(A_{L1}^T P + A_{L2}^T P + \dots + A_{Lnd}^T P)e(t)$ which is rewritten and bounded by

$$\begin{aligned} &[r(t)]^T \begin{bmatrix} \Omega_0 & \star & \star & \star & \star \\ A_{L1}^T P & -Q_1 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{Lnd}^T P & 0 & 0 & -Q_{nd} & 0 \\ G^T P - S_N H_N & 0 & 0 & 0 & Q_N \end{bmatrix} [r(t)] \\ &\leq \begin{bmatrix} z(t) \\ -\xi(\sigma, z) \end{bmatrix}^T \begin{bmatrix} -H_N^T R_N H_N & S_N^T H_N^T \\ H_N S_N & -Q_N \end{bmatrix} \begin{bmatrix} z(t) \\ -\xi(\sigma, z) \end{bmatrix} \\ &\quad - \epsilon V(e) \end{aligned}$$

with $r(t) = [e(t), e(t-h_1), \dots, e(t-h_{nd}), v(t)]^T$, $\Omega_0 = A_{L0}^T P + P A_{L0} + \epsilon P + Q_i$, and selecting SSD $\{Q, S, R\} = \{-R_N, S_N^T, -Q_N\}$. Then the origin is Globally Exponentially Stable equilibrium point. \blacksquare

Remark 8. The Dissipative observer design, in the preceding Theorem 7, is very flexible in the kind of nonlinearities considered for the system Σ_S . It is clear to see that the convergence to zero depends on the observer gains L_i associated to the multiple delays.

4.2 Disturbed system

Consider the following nonlinear system

$$\Psi_S : \begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{nd} A_i x(t-h_i) + Gf(\sigma; t, y, u) \\ \quad + \varphi(t, y, u) + d(t, x), \\ y(t) = Cx(t), \\ \sigma(t) = Hx(t), \end{cases} \quad (18)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $y(t) \in \mathbb{R}^p$ is the output vector. $d(t)$ represents the term of perturbation or uncertainty.

We use the state observer Σ_O to estimate the state vector for the disturbed system Ψ_S . Considering, observer error $e = \hat{x} - x$, its dynamics can be rewritten as

$$\Psi_E : \begin{cases} \dot{e}(t) = A_{L0}e(t) + \sum_{i=1}^M A_{Li}e(t-h) + Gv(t) + b(t) \\ z(t) = H_N e(t), \\ v(t) = -\xi(\sigma(t), z(t)), \end{cases} \quad (19)$$

where $b(t) = -d(t, x)$ is represented as external signal for the disturbed system Ψ_S .

Definition 9. Σ_O is an *dissipative observer* for the family of disturbed systems Ψ_S , if the estimation error is $SSD\{Q, S, R\}$, such that, the estimation asymptotically converges to a vicinity of the real state trajectory,

$$\|e(t)\| \rightarrow \beta_b \quad \text{as } t \rightarrow \infty.$$

The following Theorem provide the sufficient conditions for designing dissipative observers for a family of disturbed nonlinear systems with delay Σ_S .

Theorem 10. Assume that ϕ is $D\{Q_N^j, S_N^j, R_N^j\}$ for any quadratic form as in (16)

Suppose that the matrices $P = P^T > 0$, $Q_i = Q_i^T > 0$, L_0, L_i, N , a vector $\theta = (\theta_1, \dots, \theta_\mu) \succeq 0$ and a scalar $\epsilon > 0$, such that the dissipation matrix inequality: is fulfilled. Then, Σ_O is an globally ISS dissipative observer for Σ_S . \diamond

Remark 11. The matrices $P = P^T > 0$, $Q_i = Q_i^T > 0$, L_0, L_i, N , an vector $\theta = (\theta_1, \dots, \theta_\mu) \succeq 0$ and a scalar $\epsilon > 0$, represent the design parameters of the dissipative observers.

5. COMPUTATIONAL IMPLEMENTATION

The dissipativity property ensures the convergence of the nonlinear observer Σ_O for the family of nonlinear time-delay systems, in absence/presence of disturbances. In order to build an dissipative observer, we require to find the variables $(P, Q_i, L_i, N, \theta_j, \epsilon)$ such that the dissipation matrix inequality in (17) is satisfied.

The dissipation matrix inequality (MI) in (17) represents a feasibility nonlinear problem. This MI can be converted to a problem of bilinear matrix inequality in the variables $(P, Q_i, PL_i, \theta_j, \epsilon)$, taking into account $N = 0$. Moreover, If ξ is scalar, and when $R = 0$, then the dissipation matrix inequality in (17) is an LMI in $(P, Q_i, PL_i, N, \epsilon)$. There are other cases to convert the LMI's, using the theory of the Schur's complement. It is important to mention that the inequality in (17) can be leaded to the bilinear matrix inequalities, for example, (17) is a BMI in $(P, Q_i, L_i, \theta_j, \epsilon)$ when $N = 0$.

6. CONCLUSIONS

A new method to design dissipative observers for a family of nonlinear systems subject to multiple time delays, in presence/absence of disturbances, has been presented. The method includes some nonlinearities of H_∞ and passivity as special cases. The sufficient condition can be expressed in terms of Linear Matrix Inequalities (LMI's) or Bilinear Matrix Inequalities (BMI's).

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