Modal Model for a Combustion Chamber; Synthesized from Vibrations

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Abstract: This paper deals with the perturbation the pressure alongside the combustion chamber. This perturbation is modeled by vibrations modes on surface tube. These vibration modes are obtained experimentally using accelerometers along the prototype. The analytic model of the combustion chamber is based on a modal model where their degrees of freedom are in accordance with its flexible modes and their corresponding modal natural frequencies that joined from impact tests conducted throughout the axial shaft. The main contribution of this work, at the moment, it is a modal characterization method to get a mass-spring model system which is solving using algorithm recursive to find the closest parameters values.

Keywords: Identification system, Modal model, combustor.

NOMENCLATURE

M	Mass matrix
K	Stiffness matrix
D	Damping matrix
B	Input matrix
x	Nodal displacement vector
Ω	Matrix of natural frequencies
Φ	Matrix of mode shapes
ϕ	ith displacement of the ith mode
A_d	Matrix displacement

1. INTRODUCTION

Most of the time, the aero derivative gas turbines are installed always at wellhead because they excite extraction machinery. The fuel extracted is used into the gas turbine just after it has been slightly filtered then its quality highly depends of its natural deposition and composition, so when this fuel burns into the gas turbine combustor, it generates perturbations which sometimes induce the phenomenon known as acoustics.

The main indicator for the acoustics phenomena is the excessive heat released that generates pressure oscillations in the combustion chamber. These discontinuities can cause several problems in the fuel system as well as: its efficiency degradation, premature wear of its components and even its catastrophic failure (Lieuwen et al., 2006). This phenomenon and its combination with high pressures and temperatures (including the incorrect fuel-air mixing process) produce highly polluting particles such as NOx (Vigueras-Zuñiga et al., 2014).

Nowadays the art state about the thermoacoustic phenomena effects in gas turbines performance, has been mainly focused some techniques as well as the swirling flows (Yadav et al., 2010) which provides aerodynamic stability to the combustion process by producing regions of recirculating flows that reduce the flame length and increase the residence time of the reactants in the flame zone. Into the experimental way, some analysis from combustion test rig using different kind of injectors, constrictors, air-fuel mixes have been performed in order to find the best technique for combustion system (Valera-Medina et al., 2012). Also other analysis has included premixed fluid and also Helmholtz resonator (Kai-Uwe et al., 2006).

In the same way the gas turbine combustion modelling and simulations with the acoustic phenomenon is needed to implement any certain kind of control algorithm. The basic gas turbine model equations (Rowen, 1983) are important for analysis, design and simulation of control system especially for Combined Cycle Power Plants (CCPP) (Rai et al., 2013; Shalan et al., 2010). So by focusing on acoustics, new passive and active control techniques for such instabilities have been studied and developed (Lieuwen et al., 2006; Rai et al.,2013; Samad et al., 2011) as well as techniques of Adaptive Sliding Phasor Averaged Control (ASPAC: adapted the phase of the valve-commanded fuel flow variations) and Multiscale extended Kalman (MSEK: predict the time-delayed states) are promising techniques for reduce the power in the pressure oscillations (Doing et al.,2009) but more research is required.

The acoustic model proposed here has been planned to be the first part of a synthetization of a control algorithm to diminish the effects of the thermoacoustic phenomena into the gas turbine performance. Considering that new electronic controllers can allocate and compute more complex algorithms which take in account more accurate and also complex models.

1.1 Gas turbine

It starts with the hypothesis that you can have a better understanding of the system through the formulation of a mathematical model derived from experimental evidence is that considers all those measurable aspects and perhaps they are not considered when realizer the analytical modelling.

The purpose of this research is to approach the problem seen as a vibration system. During the combustion process burning air-fuel potential energy converted to kinetic energy by raising the pressure and temperature. This process apart to release energy, so intrinsic produces shock waves that impact the housing of the combustion chamber.

This is an area of opportunity that has not yet been exploited it is to take the problem with a finite analysis of a vibrational system to know the phenomenon of combustion element.

It was considering geometry as the combustion Chamber as shown in figure 1.

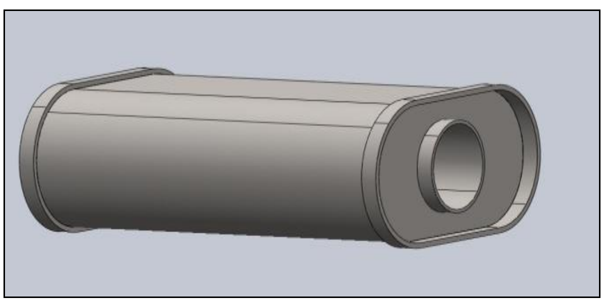

Fig. 1. Combustion chamber.

1.2 Modal Testing

Accelerometers were used for data acquisition. Those are miniature triaxial piezoelectric CCLD accelerometer, model 4506B Brüel &Kjær and their acquisition data module including Workstation PC (Fig. 2).

Fig. 2. Measurements devices

The Combustion chamber housing was hanging out using the axial shaft as horizontal axis. Accelerometers were place on different points (named: X1 to X7) of the housing of combustion chamber as it can appreciate on figure 3. It start determinate a modal model of a combustion chamber through impulse response test using an instrumented hammer.

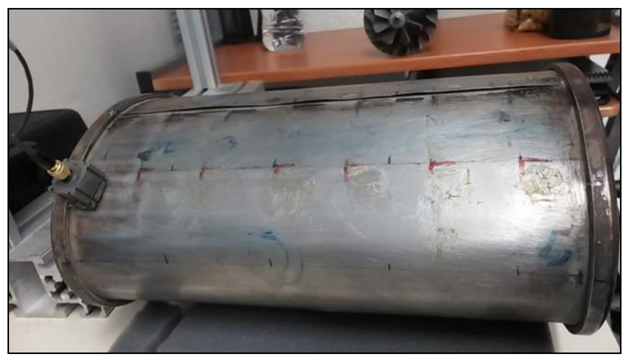

Fig. 3. Accelerometer placement

Five measurements were made on each point. This is to contemplate the experimental error. With the acceleration measurements from the impact tests, all natural frequencies and mode shapes were shown. The figure 4 shows the frequency spectrum of the filtered impact response which is represented by accelerations (m/s^2) against Frequency (Hz) in X1 placement.

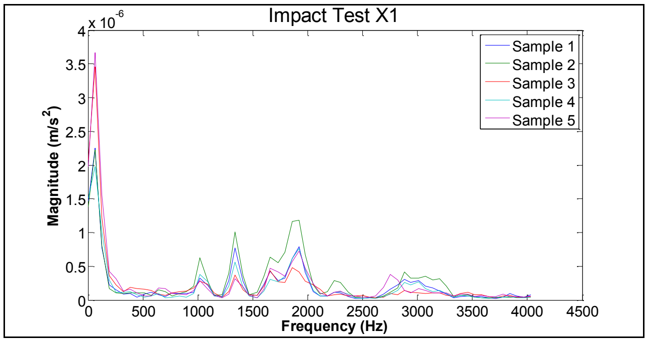

Fig. 4. Fourier spectrum

It is clear that the experimental values at each point differ somewhat from each other mainly in the magnitude values, but the natural frequencies are always present in each one. The reason is because each section responds more to a given stimulus (natural frequency) but the sum of each section, and thus all the answers, leads the construction of the mode shapes. This is the way that is believed that commercial software using for the structural analysis of materials operate, only that they do it in a simulated environment and our approach is through experimental measurements in situ.

1.3 Vibration System

The nodal models are derived in nodal coordinates, in terms of nodal displacements, velocities, and accelerations. The model is characterized by the mass, stiffness, and damping matrices.

A structural model is represented by the second-order linear differential equations.

$$M x + D x + K x = Bu \qquad (1)$$

Where dimensions matrix are define as *m*-by-*n*. Consider free vibrations of a structure without damping, structure without external excitation (u=0) and with the damping matrix D=0. The equation of motion in this case turns into the following equation:

$$M \ddot{x} + K x = 0 \qquad (2)$$

Now we proceed by assuming the form of solution (just as with differential equations). In this case, since there is no damping, we choose a purely oscillatory solution.

$$(K - \omega^2 M)\, \phi\, e^{j\omega t} = 0 \qquad (3)$$

Where K resulting matrix corresponding to 4 masses system is:

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & -k_4 & -k_4 \end{bmatrix} \qquad (4)$$

This is a set of homogeneous equation that became just an eigenvalue problem. As result of this problem get ith values of frequency ω. These frequencies do not exceed the number of degrees of freedom. Eigenvector problem gives a vector ϕ_i corresponding to the ith natural frequency is called the ith natural mode, or ith mode shape.

Although it can be considered a set n of mass and stiffness and a small number of ω to excite the system, it is known that a set of mass can respond to one group at a frequency and the other not, but being linked, these answers as a chain direction. Therefore it is proposed to take for each natural frequency, representing a mass described above. By having four exciting frequencies are considered four masses linked by stiffness.

The case studied was considering only 4 masses which are the most strong on each measurement as shown in figure 4 (Gutierrez et al., 2012).

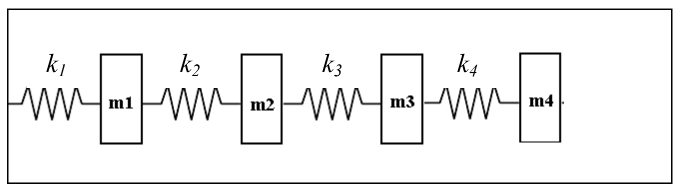

Fig. 5. Mass-spring system

The mass-stiffness systems using for now the natural frequencies and modal shapes is show in figure 5, where $m_1=m_2=m_3=m_4$ for this case.

It’s very important to make emphasis the analysis gotten by software always presents results on free vibration. On our case, the experimental results don’t put away the damping factor. For this first step only try to get a good approach only with mass and stiffness values. The accuracy adjustment considering damping is object to other paper.

The test bed result gives the matrix of natural frequencies which are showing in the fig 4.

$$\Omega = \begin{bmatrix} 64 & 0 & 0 & 0 \\ 0 & 1024 & 0 & 0 \\ 0 & 0 & 1344 & 0 \\ 0 & 0 & 0 & 1920 \end{bmatrix}$$

Through this experimental data it was easy to determinate the matrix of modes shapes, or modal matrix Φ.

$$\Phi = \begin{bmatrix} 5.494 & 3.057 & 4.27 & 2.145 \\ 5.424 & 5.989 & 5.586 & 3.210 \\ 8.457 & 2.701 & 2.044 & 1.136 \\ 4.837 & 3.655 & 3.099 & 2.065 \end{bmatrix}$$

2. METHOD

The methodology used to, from experimental values, a reconstruction of matrices that satisfy the same based on the solution of system of equations. On this basis an algorithm that consists of 3 sections are performed.

2.1 First section

The first section of the algorithm uses as input values the ω (eigenvalues or natural frequencies) and Φ (eigenvectors or mode shapes) who were finding on test bed. A_d matrix was reconfigured from equation 3.

$$Ax = B$$

Where A is equal to K-ωM, and x is a modal shapes. Now, A_d matrix was made in terms of the values obtained in the mode shapes and M matrix with unit values. A_d matrix was named displacement matrix.

$$A_d = \begin{bmatrix} \Phi_{1,n} & \Phi_{1,n} - \Phi_{2,n} & 0 & 0 & 0 \\ 0 & \Phi_{2,n} - \Phi_{1,n} & \Phi_{2,n} - \Phi_{3,n} & 0 & 0 \\ 0 & 0 & \Phi_{3,n} - \Phi_{2,n} & \Phi_{3,n} - \Phi_{4,n} & 0 \\ 0 & 0 & 0 & \Phi_{4,n} - \Phi_{3,n} & \Phi_{4,n} \end{bmatrix}$$

Where now, S_{fn} contains the value of the stiffness to find:

$$s_{fn} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix}$$

The main idea is to solve the lineal system equations is by the form:

$$S_{fn} = J^{-1} * B \tag{5}$$

Where B vector corresponding to input force signal.

$$J = (A_d - I\omega_n) \tag{6}$$

The displacement matrix must be considering each natural frequency (ω) to solve the system. A_d is not a square Matrix. The pseudoinverse of an m-by-n matrix is using:

$$S_{fn} = (J^T * J)^{-1} * J) * B \tag{7}$$

At the end of this algorithm results in three vectors (S_{fn1}, S_{fn2}, S_{fn3}) for each mode shape. Each vector is initial approximations to start looking for suitable values of the final stiffness value that fit the given ω and Φ matrix.

2.2 Second section

The second part corresponds to the search of vector S_{fn} is best suited to consider. This is through using Eq. (4) for each S_{fn} and gets the new eigenvalues (v) and eigenvectors (u). Values which are checked them has minor mistake from the Euclidean norm:

$$\|Err\|_2 = (\sum_{i=1}^{n} |dV|^2)^{1/2} \tag{8}$$

Where dV is the delta v, its corresponding the different between the experimental value and the calculated value:

$$dV = [\omega] - [v] \tag{9}$$

2.3 Third section

The third part is to adjust the values of stiffness such that these approximate the experimental natural frequencies.

It is noteworthy that each of the S_{fn} vectors obtained fully meets the mode shapes as they were with these that gave rise S_{fn}. Now, multiply the S_{fn} vector by a scalar, regardless of the change in the mode shapes is almost negligible.

To find the values of S_{fn} was necessary to use the bisection method of Bolzano (Mathews et al., 2004). Was established as a focal point the value of the natural frequency searched and through the algorithm iterations is ends until the difference between the real and the imaginary value is less or equal to epsilon value established.

3. TESTING ALGORITHM

The way to test this algorithm was first starting from a known system, in which stiffness and masses known and of course the natural frequency and mode shapes.

The next example (Gawronsky, 2004) considers unit values. The system use masses $m_1=m_2=m_3=1$, stiffness $k_1=k_2=k_3=1$ $k_4=0$ and damping matrix proportional to the stiffness matrix, D=0.01K. This is the values of the system. And these are their eigenvalues and eigenvectors.

$$\Omega_{real} = \begin{bmatrix} 0.594168 & 0 & 0 \\ 0 & 4.664844 & 0 \\ 0 & 0 & 9.7409388 \end{bmatrix}$$

$$\Phi_{real} = \begin{bmatrix} -0.3279853 & -0.7369762 & -0.5910090 \\ -0.5910090 & 0.3279853 & -0.7369762 \\ -0.7369762 & -0.5910090 & -0.3279853 \end{bmatrix}$$

Running each of the functions described in the methodology which incorporating Ω_{real} and Φ_{real}, give as a result vector S_{fn}:

$$S_{fn} = \begin{bmatrix} 3.1327479 \\ 3.0466369 \\ 3.0010702 \end{bmatrix}$$

Using these values of stiffness and using equation 4 gets the natural frequencies and mode shapes:

$$\Omega_{calculated} = \begin{bmatrix} 0.601658 & 0 & 0 \\ 0 & 4.667707 & 0 \\ 0 & 0 & 9.7097657 \end{bmatrix}$$

$$\Phi_{calculated} = \begin{bmatrix} -0.3225243 & -0.7293917 & -0.6032959 \\ -0.5895643 & 0.3438351 & -0.7308839 \\ -0.7405350 & -0.5914095 & -0.3191281 \end{bmatrix}$$

Using the Euclidean norm to calculate the error is obtained:

$$\left\| Err_{\Omega} \right\|_2 = 0.0311731$$

$$\left\| Err_{\Phi} \right\|_2 = 0.0226590$$

4. CONCLUSIONS

The development of an algorithm for obtaining the values of stiffness from experimental data was performed. The implementation thereof and the incorporation of the other axes are being developed. The mode shapes and their natural frequencies were very useful to determine an accurate approach of mode shapes at low frequencies.

The future work to improve the accuracy to estimate of the mode shapes for the higher frequencies will start by measuring the impact test accelerations using lower uncertainty sensors. Then, a study of error transference along the recursive algorithm is needed to increase its accuracy.

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