Oversampled modal analysis of a turbocharger rotor from the experimental lateral vibrations

L. A. Montoya* E. E. Rodríguez**
H. J. Zúñiga* I. Mejía******

**Centro de Ingeniería y Desarrollo Industrial (CIDESI), MEX (Tel: 442-177-0869; e-mail: lmontoya@posgrado.cidesi.edu.mx).*
*** Centro de Ingeniería y Desarrollo Industrial (CIDESI), MEX (e-mail: eloy.rodriguez-vazquez.mx@ieee.org)*
**** Centro de Ingeniería y Desarrollo Industrial (CIDESI), MEX, (e-mail: hzuniga@posgrado.cidesi.edu.mx)*
***** Centro de Ingeniería y Desarrollo Industrial (CIDESI), MEX, (e-mail: imejia@posgrado.cidesi.edu.mx)*

Abstract: The dynamic responses of turbomachinery is affected by some factors such as rotor unbalance, coupling defects and support bearings, as well as the demand for higher operating speeds, which can cause vibrations that affect the behavior and reliability of the rotating system. Therefore, it is required to study a dynamic model to predict some vibration characteristics, in this case, the natural frequencies and mode shapes (both of free vibration) of a turbocharger rotor by using modal analysis. The natural frequencies and mode shapes of the rotor were measured with modal testing by using an impact hammer. Several degrees of freedom were found for the experimental model, while for estimating the modal analysis the order was reduced to only three degrees of freedom. Then, the rotor was sectioned into three parts to make an oversample. The model of three degrees of freedom for each section was solved and the sections were coupled to generate a model 9 degrees of freedom equivalent to the experimental system. The dynamic model is based on frequencies and displacements obtained experimentally, so it is automatically validated by comparing the output data. The results predicted by the modal model are in good agreement with the experimental test. The model is also flexible with other geometries and has a great time and computing performance, which can be evaluated with respect to other commercial software's in the future.

Keywords: transfer functions, dynamic systems, eigenvalues, eigenvectors, vibration measurement.

1. INTRODUCTION

The vibrations in turbomachinery are responsible for over 40% of the problems (Meher-Homji, (1995), Chang, (1996), Szász et al., (2000), Morrison et al., (2005)), where rotating parts are the most important source (Shahgholi, (2014)). These problems affect principally the bearings supports because these are the physical support of the rotor system, which play an important role in the vibration modes and critical frequencies (Kozanecka et al., (2008)). Therefore, many methods have been developed to predict failures by analyzing the dynamic response of rotating systems.

The natural frequencies of a gas turbine rotor were obtained experimentally and were found that each order natural frequency increases with contact stresses. It was showed that using these frequencies and the finite element analysis software SAMCEF Field, the effect of the contact stiffness can be obtained (Zhang and Du., (2009)). The natural frequencies of a rotor vary with many factors such as the structure, the supports and the accuracy of the model, so it becomes complex to get a good prediction (Xu et al., (2014)).

The natural frequencies and mode shapes of a rotor were estimated using finite element models in ANSYS considering the bearings stiffness and damping. It was noted that the mode shapes define where the mass unbalance forces have to be positioned, and that using different bearings, the natural frequencies and critical speeds can be controlled (Xu et al., (2014)).

According to (Lu et al., (2010)) each order of natural frequency of a gas turbine rotor increases with rod preloads, because the rigidity changes. These preloads have a significant effect on the deflections and stresses, which modifies the modal analysis.

A dynamical analysis of a gas turbine rotor using the Dynrot program with code based on the finite element method was made by (Taplak and Parlak, (2012)), they found the system has an unstable behaviour when it is near to the critical speeds, and that small imbalance values do not affect the behaviour but decrease the critical speeds.

The natural frequencies and mode shapes of a rotor shaft were obtained using ANSYS, and the results were validated with an experimental test using FFT (Fast Fourier Transform) analyzer. If the system is modified with mountings and accessories, then stiffness increases and therefore the natural frequency. The results were in good agreement, although there was no feedback to the model with experimental data (Pagar and Gawande, (2014)).

A scale-down model to predict the free or forced lateral vibrations of a rotor-bearing system was presented by (Wu, (2007)). It was shown that scaling laws and scaling factors approach the prediction of the full-size rotor from its scale model.

In the study by (Fegade et al., (2014)) was presented a harmonic analysis to identify frequency of a system with different configuration of bearings using ANSYS. They used the unbalance of the rotor as excitation to perform the analysis, and mentioned that rotating critical speeds are associated with high vibration amplitude. In this case, a system with symmetric orthotropic bearings gave the less critical speed, so this kind of analysis is important to know the vibration amplitude response for minimizing the noise of the rotor.

A finite element model of a Jeffcott rotor in ANSYS, approaching the stiffness and damping coefficients of hydrodynamic bearings, was used by (Ramírez Vargas et al. (2013)). It was performed a harmonic analysis to determine the vibrational response in steady and transient state. The transient analysis showed the required excitation of the rotor to go through a natural frequency. The modal analysis reveals if any natural frequencies will be near the operating speed, and it is necessary to carry out a harmonic analysis. The natural mode shapes help to identify if the motion is near the bearing supports, which could affect the rotor critical speeds (Moore, (2010)).

As described in different investigations, there are few studies that link the experimental side on estimates of theoretical models. Hence, the natural frequencies and mode shapes calculated experimentally were used as input values in the modal analysis approach to get estimated modal parameters such as natural frequencies and complex deflections shapes of the turbocharger rotor.

2. EXPERIMENTAL METHOD

A turbocharger rotor of 1.249 kg was used (Fig. 1). The rotor was hanged by two bungees to simulate the free boundary conditions. Thereafter, it was impacted with an impact hammer (model PCB 086C03) to measure the response of the acceleration using two miniature triaxial piezoelectric CCLD accelerometers (model Brüel & Kjaer type 4506). An accelerometer was moved on a total of 9 points along the rotor to oversample the mode shapes while the other was static as reference.

When the shaft is excited the frequency response functions (FRFs) are obtained and processed with an acquisition and processing code developed for this purpose, which makes it easy to adjust the parameters like sample frequency, record length, profits, etc., as well as the calculation of frequency response functions and averages. Only the first three mode shapes obtained experimentally were used in the following modal analysis approach.

Fig. 1. Turbocharger rotor.

The average acceleration response (Fig. 2), was used for the identification of the natural frequencies. The FRFs are developed by dividing the output by the input.

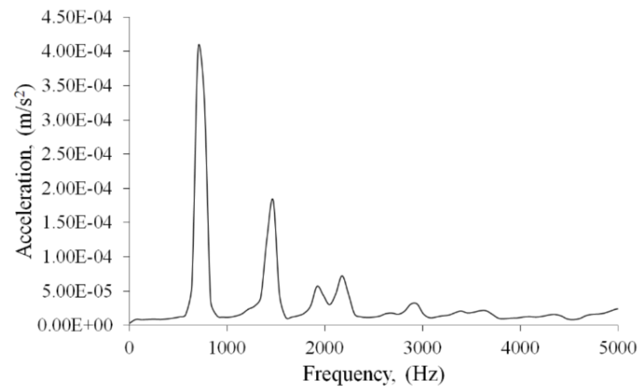

Fig. 2. Average acceleration response.

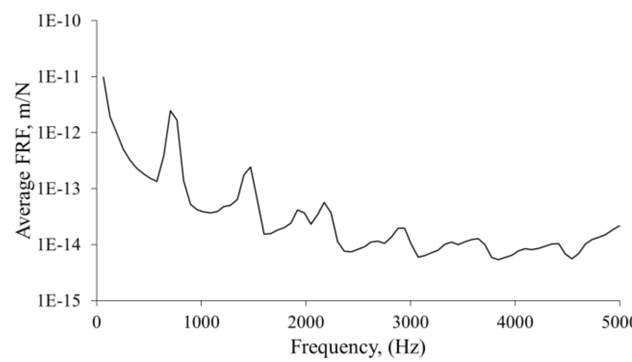

Fig. 3. Average FRF from the modal test.

Due the theoretical modal analysis (following section) is based on a 3 order model, only the first three natural frequencies was selected, which correspond to the first three resonance peaks in Fig. 2. The corresponding natural frequencies are 704 Hz, 1470 Hz and 1920 Hz.

It was decided to position the accelerometer in 9 points along the rotor with the aim of achieving a greater number of displacements, this in order to use a method of oversampling proposed in this paper, shown in section 3, where the same three natural frequencies, but with a greater number of displacements, it can be achieved a better approximation of the model. The response was measured 10 times for each point to reduce the lack of trust, then the average FRFs are shown in (Fig. 3).

The average of averages FRFs corresponding to the 9 measurement points are shown in (Fig. 3), where the displacement between the applied load is plotted, this according to the frequency. The FRFs information is used to determine the mode shapes. The mode shapes are calculated with FRF values for each point, in this case 9, for each natural frequency.

3. OVERSAMPLING MODAL ANALISIS APPROACH

The mathematical morphology of the model proposed here is a set of finite variational equations, which comes from the modal model of the shaft which was obtained from an experimental modal analysis. Once natural frequencies and modal shapes are known experimentally, the degrees of freedom (DOFs) for the modal model are the same as the number of natural frequencies of the frequency spectrum at the working range. With these DOFs the modal shapes are described spatially, but most of the time at highest frequencies the mode shapes have complex forms, that cannot be described precisely with the DOFs and more DOFs are needed. Then, the mathematical model proposed here converts the modal model with limited DOFs to a set of finite variational equations with enough number of DOFs to describe more accurate the natural frequencies and mode shapes. This set of finite variational equations are defined as several modal models which have the same natural frequencies that the ones obtained experimentally, but their modal shapes emulate just a segment of the experimental shapes. So after it computes the coefficients of each modal model (in this case, stiffness and masses), the experimental mode shapes are achieved by coupling the mode shapes of each segment. The model approach is to explain how the general model is formulated to then explain how the model is oversampled.

The equation of motion (1) for the complete system is represented with the following model

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{U(t)\} \tag{1}$$

where, M, C and K are the mass, damping and stiffness matrix, respectively. As the particular interest of this research is the free and undamped vibration, so (2) is given by

$$[M]\{\ddot{X}\} + [K]\{X\} = 0 \tag{2}$$

The turbocharger rotor showed several experimental natural frequencies, which means that several degrees of freedom are required. The number of the equation of motions is given by de number of natural frequencies obtained experimentally. The proposed general model in (Fig. 4) is the kind of spring-mass model proposed in (Yu et al., (2012), to get the modal parameters. The model in (Fig. 4) is based on the first three estimated natural frequencies and mode shapes because they have higher amplitudes, so the system equations can be represented in (3) as follows

$$\begin{aligned} u_1(t) &= m_1\ddot{x}_1 + k_2(x_1 - x_2) + k_1x_1 \\ u_2(t) &= m_2\ddot{x}_2 + k_2(x_2 - x_1) + k_3(x_2 - x_3) \\ u_3(t) &= m_3\ddot{x}_3 + k_3(x_3 - x_2) + k_4x_3 \end{aligned} \tag{3}$$

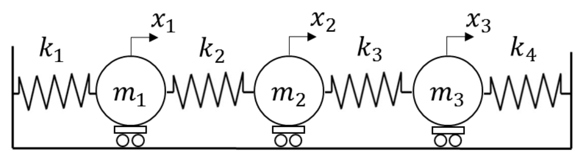

Fig. 4. Proposed general model.

Then expressing the model (3) in the frequency domain and ordering in matrix form

$$[\mathrm{U}]_{3\times1} = [\mathrm{A}]_{3\times3}[\mathrm{X}]_{3\times1} \tag{4}$$

where A consists as follows

$$\mathrm{A} = \begin{bmatrix} k_1 + k_2 - \omega_1{}^2 m & -k_2 & 0 \\ -k_2 & k_2 + k_3 - \omega_1{}^2 m & -k_3 \\ 0 & -k_3 & k_3 + k_4 - \omega_1{}^2 m \end{bmatrix} \tag{5}$$

The matrix U is the force vector, which is considered unitary because the impacts used as experimental excitation were modeled as unitary pulses. The matrix X is the displacement vector, filled with experimental data. The matrix A consists of the values of mass, stiffness and system frequency, where the mass and stiffness values are unknown. As the experimental displacements and frequencies are known, one can express the model in the following way

$$[\mathrm{MK}]_{7\times1} = [\mathrm{D}]^{-1}_{7\times3}[\mathrm{U}]_{3\times1} \tag{6}$$

where D is a 3x7 matrix and the pseudoinverse D^{-1} as in (6). The system shown below in (7), is repeated but changing ω to the next frequency and the displacements for the next mode shape.

$$\mathrm{D} = \begin{bmatrix} x_1 & x_1 - x_2 & 0 & 0 & -\omega^2 x_1 & 0 & 0 \\ 0 & x_2 - x_1 & x_2 - x_3 & 0 & 0 & -\omega^2 x_2 & 0 \\ 0 & 0 & x_3 - x_2 & x_3 & 0 & 0 & -\omega^2 x_3 \end{bmatrix} \tag{7}$$

The MK matrix contains the stiffnesses and masses of the first section of the rotor. In general, what is being made is to obtain the distribution of stiffnesses and masses by using the experimental data of frequency and displacement.

The method shown above is considering the complete rotor, however, the oversampled method consists, in general, to use the proposed general model for each uncoupled section, as in (Fig. 5), using the corresponding frequencies and displacements of the 1st, 2nd and 3rd mode shape of each section, as well as the respective mass of the section, data which are known. For example, for the first section the displacements values of the proposed general model x_1, x_2

and x_3, now would be $x_{1,1}$, $x_{1,2}$ and $x_{1,3}$, to obtain a MK matrix for the first section as in (Fig. 5).

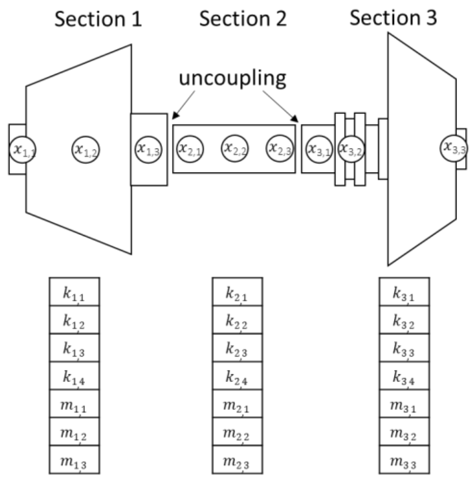

Fig. 5. Rotor sectioning.

The resulted MK matrix is normalized. The analysis has to be repeated for the following sections of the rotor, to calculate the stiffnesses and masses corresponding to each section.

$$R = \begin{bmatrix} k_{1,1}+k_{1,2}-\omega^2 m_{1,1} & -k_{1,2} & 0 & \cdots & 0 \\ -k_{1,2} & k_{1,2}+k_{1,3}-\omega^2 m_{1,2} & -k_{1,3} & \cdots & 0 \\ 0 & -k_{1,3} & k_{1,3}+\left\{\dfrac{1}{\dfrac{1}{k_{1,4}}+\dfrac{1}{k_{2,1}}}\right\}-\omega^2 m_{1,3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & -k_{3,3} \\ 0 & 0 & 0 & -k_{3,3} & k_{3,3}+k_{3,4}-\omega^2 m_{3,3} \end{bmatrix} \tag{8}$$

After obtaining all values of stiffness and mass of the sections, the resulting matrix R, which follows the model in (5), is formed by making the coupling of the sections adding serially the last stiffness of the first section with the first stiffness of the adjacent section, as shown in (8), and so on with the following sections until get a total of 10 coupled stiffnesses for this case of study. Therefore, the eigenvalues and eigenvectors are calculated from (8) and consequently the estimated natural frequencies and mode shapes.

4. RESULTS

Table 1 shows a summary comparison of the natural frequencies of the turbocharger rotor corresponding to the three vibration modes measured experimentally and those predicted by the oversampled modal analysis approach for the lateral vibration modes. It can be observed that the estimation of the theoretical model closely approximates the experimental, which showed an average percentage of relative error, from the three errors in Table 1, acceptable low of 8.82%. It can be said that the results of the proposed dynamic model are in good agreement with the experiments. It is to emphasize that the estimated frequencies correspond to primary or crude model, without using recursive algorithms, so small differences could be reduced by subsequent adjustments in future works. There are several ways to determine the modal frequencies, however, most authors use simulation by FEM, which requires more computing performance and have the disadvantage of not be able to feedback control systems in operation.

Table 1. Comparison of natural frequencies.

No.	Experimental Natural Frequency (Hz)	Natural Frequency by Modal Analysis (Hz)	Error (%)
1	704	604.28	14.17
2	1472	1487.73	1.07
3	1920	1704.4	11.23

The measured and estimated mode shapes are shown below in Fig. 6-8.

The natural frequencies approach proposed with this method are in the order of magnitude of the analysis shown by (Jalali et al., (2014)). In this research (Jalali et al., (2014)), it was used a 3D FEM and 1D beam model, also analyzing a high speed rotor, however, errors were up to 20% between the frequencies estimated, and there was not enough comparison against experimental test. The mode shapes also have some similarity, but the difference may be due to the geometry of the rotors.

Although this research is based on the non-rotating vibration modes, it has been said that non-rotating modes can be used to describe the dynamic behavior of a rotor without losing much accuracy (Wei Meng et al., (2015)).

In the study by (Gutiérrez Wing et al., (2011)) was cited that it is important to know the modal parameters to relate spatial parameters and to determine the response of the system and unbalance forces, this in a model they proposed for balancing rigid rotors. The modal parameters estimated in this study has the advantage that are based on the actual system response. An experimental rig to get the modal parameters was used by (Yu et al., (2012), Yu et al. (2016)), where proposed a damping identification system to identify the natural frequencies and mode shapes of the lumped masses, getting good approach with experimental results and FEM, but the procedure is evaluated in a turbomachinery foundation system.

The stiffnesses calculated for the first section of the rotor, where the turbine side is, were higher than the other two sections, which may be because this first section contains greater mass. This differs from what mentioned in (Meng et al., (2013)), where the stiffness of the turbine side is lower than the compressor side and the vibration amplitude is greater on the turbine side.

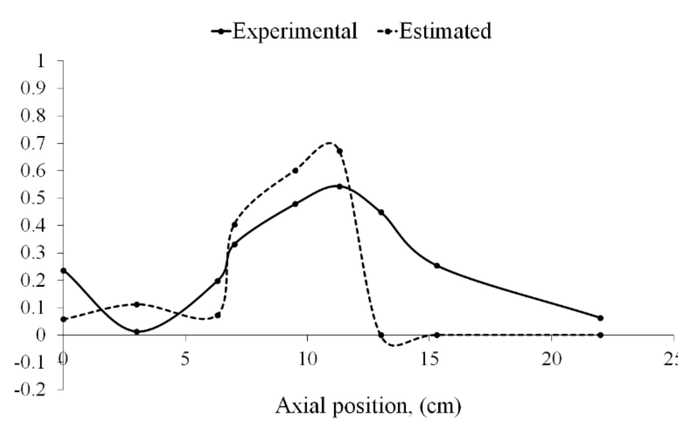

Fig. 6. 1st mode shape.

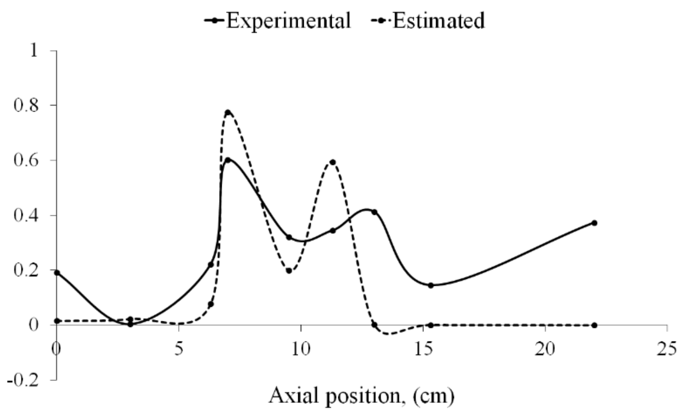

Fig. 7. 2nd mode shape.

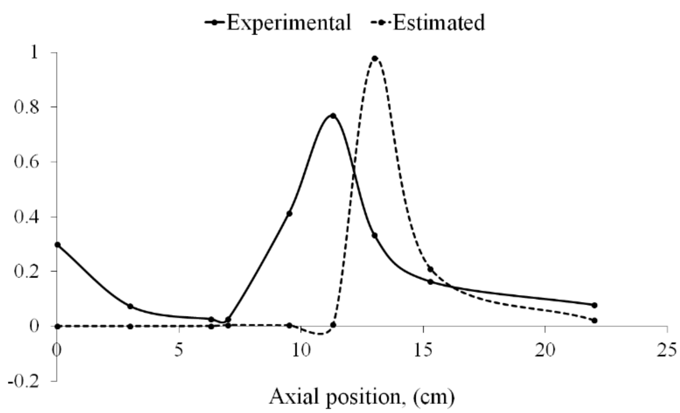

Fig. 8. 3rd mode shape.

5. CONCLUSIONS

The proposed model showed very good agreement with experimental results and is validated because is based on experimental input data. The model is also flexible with other geometries and has a great time and computing performance, which can be evaluated with respect to other commercial software's in the future, as well as can be used to synthesize more accurate control algorithms. It is intended to estimate the natural frequencies and mode shapes with the effect of bearing supports in operating conditions, as well as the imbalance and other dynamic conditions in future works. Depth knowledge in rotordynamics can contribute to the development of more reliable and predictable systems and support design tasks, monitoring and maintenance.

ACKNOWLEDGMENTS

Authors thank CONACYT for the support through its programs of doctoral scholarships (492895, 437556, 256316). Also thank to the LaNITeF and the Postgraduate Department of CIDESI.

REFERENCES

Chang, J. C. I. (1996). Integrated research approach to attack engine HFC problem. *Proceedings of the ASME Aerospace Division American Society of Mechanical Engineers*, *ASME*, Vol. 52, pages 313-320.

Fegade, R., Patel, V., Nehete, R. S. and Bhandarkar, B. M. (2014). Unbalanced response of rotor using ansys parametric design for different bearings. *International Journal of Engineering Sciences & Emerging Technologies,* Vol. 7, No. 1, pages 506-515.

Gutiérrez Wing, E. S., Aguirre Romano, J. E., Colín Ocampo, J. and Cortés García, C. (2011). Balanceo de rotores rígidos sin emplear rodados de prueba. *Ingeniería Mecánica. Tecnología y Desarrollo, SOMIM,* Vol. 3, No. 6, pages 195-205.

Jalali, M. H., Ghayour, M., Ziaei-Rad, S. and Shahriari, B. (2014). Dynamic analysis of a high speed rotor-bearing system. *Measurement,* Vol. 53, pages 1-9.

Kozanecka, D., Kozanecki, Z. and Lech, T. (2008). Experimental identification of dynamic parameters for active magnetic bearing. *Journal of Theoretical and Applied Mechanics*, Vol 46, No. 1, pages 41-50.

Lu, M., Geng, H., Yang, B. and Yu, L. (2010). Finite element method for disc-rotor dynamic characteristics analysis of gas turbine rotor considering contact effects and rod preload. *International Conference on Mechatronics and Automation (ICMA), IEEE*, pages 1179-1183.

Meher-Homji, C. B. (1995). Blading vibration and failures in gas turbines Part A: Blading dynamics & the operating environment. *ASME*, pages 1-11.

Meng, C., Su, M. and Wang, S. (2013). An investigation on dynamic characteristics of a gas turbine rotor using an improved transfer matrix method. *Journal of Engineering for Gas Turbines and Power, ASME,* Vol. 135, pages 122505:1-8.

Moore, J. J., Vannini, G., Camatti, M. and Bianchi, P. (2010). Rotordynamic analysis of a large industrial turbocompressor including finite element substructure modeling. *Journal of Engineering for Gas Turbines and Power, ASME,* Vol. 132, No. 8, pages 082401-082401-9.

Morrison, C. R., Provenza, A., Kurkov, A., Montague, G., Duffy, K., Mehmed, O., Johnson, D. and Jansen, R. (2005). Fully suspended, five-axis, three-magnetic-bearing dynamic spin ring with forced excitation. *Society for Experimental Mechanics*, Vol. 45, No. 3, pages 226-237.

Pagar, N. D. and Gawande, S. H. (2014). Investigations of dynamic characteristics of eccentric rotary shaft of wankelengine. *Journal of Mechanical Design and Vibration,* Vol. 2, No. 2, pages 53-59.

Ramírez Vargas, I., Palacios Pineda, L. M. and Corro Hernández, H. (2013). Respuesta vibratoria de un rotor apoyado en chumaceras hidrodinámicas cortas. *Memorias del XIX Congreso Internacional Anual de la SOMIM,* pages 1085-1093.

Shahgholi, M., Khadem, S. E. and Bad, S. (2014). Free vibration analysis of a nonlinear slender rotating shaft with simply support conditions. *Mechanism and Machine Theory*, Vol. 82, pages 128-140.

Szász, G., Flowers, G. T. and Hartfield, R. J. (2000). Hub-based vibration control of multiple rotating airfoils. *Journal of Propulsion and Power*, Vol. 16, No. 6, pages 1155-1163.

Taplak, H. and Parlak, M. (2012). Evaluation of gas turbine rotor dynamic analysis using the finite element method. *Measurement,* Vol. 45, No. 5, pages 1089-1097.

Wei Meng, M., Jian Jun, W. and Zhi, W. (2015). Frequency and stability analysis method of asymmetric anisotropic rotor-bearing system based on three-dimensional solid finite element method. *Journal of Engineering for Gas Turbines and Power, ASME,* Vol. 137, pages 102502:1-9.

Wu, J. J. (2007). Prediction of lateral vibration characteristics of a full-size rotor-bearing system by using those of its scale models. *Finite Elements in Analysis and Design,* Vol. 43, pages 803-816.

Xu, G., Zhou, J., Geng, H., Lu, M. and Cheng, W. (2014). Unbalance response analysis of the circumferential tie-rod combined rotor considering different support. *International Conference on Mechatronics and Automation (ICMA), IEEE*, pages 1323-1328.

Yu, M., Feng, N. and Hahn, E. J. (2012). On the identification of the modal parameters for a flexible turbomachinery foundation. *Proceedings of ASME Turbo Expo 2012, Copenhagen,* pages 1-9.

Yu, M., Feng, N. and Hahn, E. J. (2016). Experimental evaluation of a modal parameter based system identification procedure. *Mechanical Systems and Signal Processing,* Vol. 68-69, pages 302-315.

Zhang, Y. and Du, Z. (2009). Dynamic characteristics calculation study of a gas turbine rod fastening rotor. *Power Engineering Conference, APPEEC 2009. Asia-Pacific*, *IEEE*, pages 1-4.