Supervisory Control for a Gas Turbine Continuous Timed Petri Net Model

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Abstract
In this work a Continuous Timed Petri Net model for a Gas Turbine is presented. In order to guarantee a safety operation, a supervisory controller is designed to ensure the combustion chamber temperature is lower than 1000 Celsius degrees. The model is based in the approach of the Brayton cycle, which rules the functioning of a gas turbine, and it is composed by four states: Compression, Combustion, Expansion and Cooling. In every state (excepting the cooling phase) temperature and pressure are modeled as first order linear systems, therefore, every system is translated into a continuous timed Petri Net. The complete model can be used in further work in order to design supervisory controllers to ensure an efficient performance of a gas turbine. Although the model presented is extremely simplified, it will be used as a starting point to developed more complex models.

Keywords: Supervisory Control Theory, Petri Nets, Gas Turbine

1. INTRODUCTION

The gas turbine thermodynamic and mechanic nature are considered dynamic but the control algorithms are referred to specific punctual measurements, making the control algorithm work as a supervisor instead of a dynamic regulator. In this scenario, the need of modeling the gas turbine system as a Discrete Event System (DES) has become greater in the latest years. The design of supervisory controllers (SCs) with Petri Nets (PNs) using the Invariant Based Control Design (IBCD) method (Moody and Antsaklis (1998), Iordache and Antsaklis (2006)) has been previously considered for the avoidance of forbidden states (Giua et al. (1992)) and for constraining the system behavior using linear inequalities induced by Behavioral Constraints (BCs) (Yamalidou and Kantor (1991)). The application of Supervisory control theory (SCT) has been studied in Núñez and Sánchez (2015) for manufacturing uses. Plenty of works are dedicated to analyze the behavior of the so-called hybrid systems, using as main approach continuous and fluid PNs. In Gribaudoa et al. (2003), a non-deterministic model of a temperature control system is developed. In this work the assumption that the gas turbine is in fact a deterministic system is strongly considered, in order to propose simplified first order models. Gas turbines are used in transport systems as aircraft and in power generating systems. The principle of the gas turbine is developed by the Brayton cycle, a thermodynamic process which intervenes in the gas turbine components. The steady-state behavior of the gas turbine has been widely investigated in engineering area. Moreover, the dynamic behavior has been studied using non linear models of its components, leading to complicated mathematical representations. However, it is impossible to ignore the dynamic behavior of the system during the Brayton cycle, so a standard modeling of the gas turbine as discrete state system is not enough to capture its behavior. The full analysis of the Continuos timed Petri net (CTPN) model is developed in Núñez et al. (2016). In Section 2, the basic concepts of gas turbines, continuous Petri Nets and supervisory controller (SC) are introduced. The mathematical reduction and the CTPN model of the gas turbine dynamics, developed in Núñez et al. (2016) is presented in Section 3. In Section 4 a SC is designed using the IBCD to ensure a combustion chamber temperature bounded by 1000 Celsuis degrees and a simulation is obtained in Matlab. Finally, the conclusions and further work are presented in Section 5.

2. FUNDAMENTALS

Gas turbine, Brayton cycle and SCT is briefly introduced in this section.

2.1 Gas turbine

A generic gas turbine is composed by three elements, each one is associated to a state of the Brayton cycle. The elements are shown in Fig. 1. For further reading, review Valera (2014) and Thirunavukarasu (2013).

2.2 Brayton cicle

The Brayton cycle is used to define the behavior of a gas turbine. It is composed by four states:

(1) Compression. Ideally is an adiabatic process (no heat transfer with the environment). However, the temperature does increase, as has been shown in several experiments.

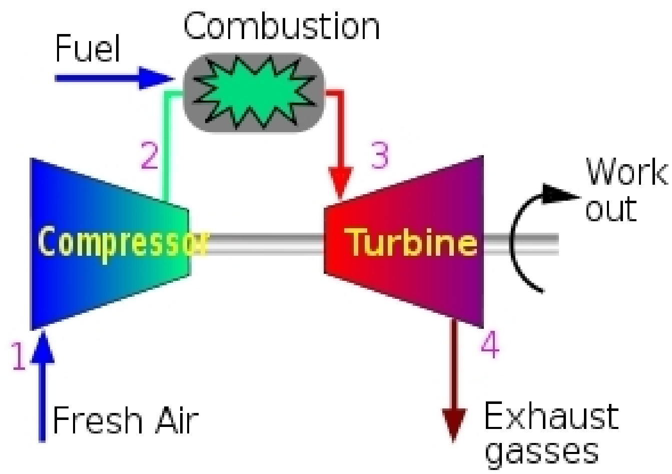

Figure 1. Gas turbine

(2) Heat addition (combustion). An isobaric process (no change on the fluid volume).
(3) Expansion. Again, an ideally adiabatic process.
(4) Heat rejection. An isobaric process.

For simplicity of the measurements, entropy is not included in the present analysis.

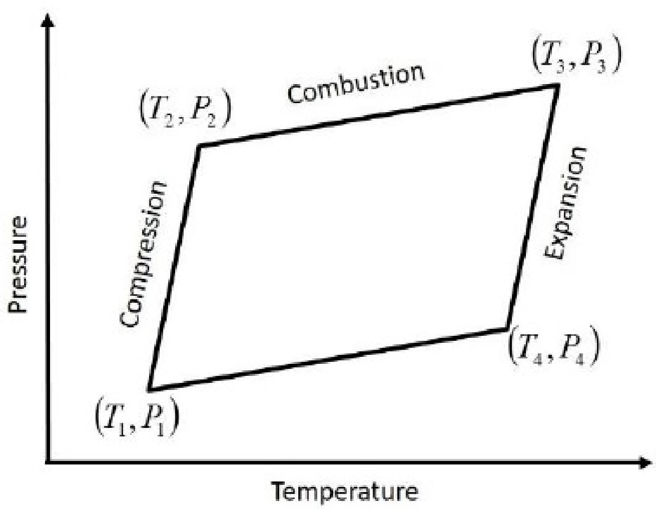

Figure 2. Brayton cycle

In this description is remarkable that each state occurs in a component of the gas turbine.

2.3 Continuos timed Petri Nets

The following subsection has been taken directly form Tovany et al. (2013). The reader is refered to Alla and David (1998) and Recalde et al. (2005) for related subjects in Continuos timed Petri Nets.

A continuous timed Petri net is a 3-tuple $ContPN = (N, \lambda, m_0)$, where (N, m_0) is a continous Petri Net and $\lambda : T \to \{R^{+^{|T|}}\}$ is a function associating a firing rate with each transition.

The state equation of a ContPN is shown in Eq. 1

$$\dot{m}(\tau) = C * f(\tau) \quad (1)$$

where τ is the time variable. C is the net incidence matrix, $C = Post - Pre$.

A ContPN is called infinite server semantic if the flow of a transition t_i is

$$f_i = \lambda_i * min_{p \in pre(t_i)}\{m(p)/Pre[p, t_i]\} \quad (2)$$

Eq. 1 can be expressed as a piecewise linear system given by

$$\dot{m} = C\Lambda\Pi(m)m \quad (3)$$

The firing rate matrix is denoted by $\Lambda = diag(\lambda_1, ..., \lambda_{|T|})$. A configuration of a ContPN at m is a set of (p, t) arcs describing the effective flow of all transitions:

$$\Pi(m)[i, j] = \begin{cases} \dfrac{1}{Pre[i, j]} \text{ if } p_i \text{ is constraining } t_j \\ 0 \text{ otherwise} \end{cases} \quad (4)$$

2.4 Supervisory Control Theory (SCT)

The automata version of SCT is developed in Wonham (2013). In this subsection, the fundamentals SCT for discrete event system modeled as Petri Net (PN) are introduced.

Definition 1. (Control pattern). Let N be a PN and T be its set of transitions.

The control pattern Γ is defined as the set of transitions enabled in a marking M of (N,M).

Definition 2. (Transition sequence). Let (N, M) be a PN system and T be its set of transitions.

$\sigma = t_1 t_2 \cdots t_n$ is a transition sequence of transitions such that

- $M_{i-1} \xrightarrow{t_i} Mi$
- t_i is enabled in M_{i-1}

with $t_i \in T$, $\forall i = 1, 2, \cdots, n$.

Definition 3. (Petri Net Supervisor). Consider a constraint $L * M \leq b$ for the marking vector of a PN system (N,M) with incidende matrix D and L a matrix corresponding the set of inequalities in the constraint. $S : M \to \Gamma$ is a supervisor for PN system (N,M). Let C be a PN with marking M_c and set of transition T. C is the implementation of S as a PN such that

- Marking vector $M_c = b - L * M_0$.
- Incidence matrix $Dc = -L * D$
- Γ is the control pattern for (C, M_c).

Definition 4. (Admissible marking). Let marking M' be reachable from M_0. M' is an admissible marking for the constraint $L * M \leq b$ if

- $L * M' \leq b$
- For all reachable markings M_r from $M\prime$ trough the occurrence of uncontrollable transitions in (N,M) $L * M_r \leq b$

Definition 5. (Admissible constraint). Let (N,M) a PN system with initial marking M_0. An admissible constraint satisfies

- $L * M_0 \leq b$
- All reachable markings from M_0 are admissible markings.

Corollary 6. Given a plant with uncontrollable transitions represented by its incidence matrix D_p, partitioned in the following form

$D_p = [D\ D_{uc}]$

with D representing the columns of controllable transitions and D_{uc} uncontrollable transitions.

Let $L * M_p \leq b$ be a marking constraint. If Eq. 5 holds then constraint is admissible.

$$L * D_{uc} \leq 0 \tag{5}$$

3. CONTINUOUS TIMED PERTRI NET MODEL

In this section the model of the Brayton cycle as ah hybrid system is developed. A high-level discrete event system is constructed using the four states of the Brayton cycle, explained in section 2.2. In each state, the dynamic equations of temperature and pressure are obtained, based on the thermodynamical relationships introduced in Rodriguez (2015).

3.1 High-level discrete state model

Brayton cycle is composed by 4 states

(1) Compression
(2) Combustion
(3) Expansion
(4) Cooling

In add to this states, an initial state representing no operation of the gas turbine is included in the model. The corresponding Finite State Machine of the Brayton cycle is presented in Fig. 3

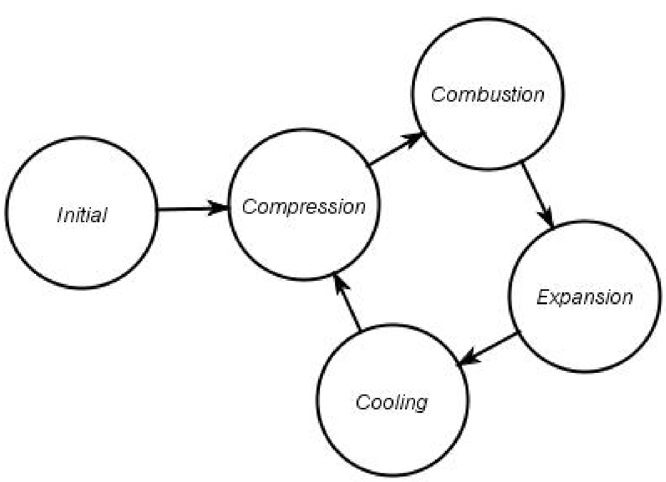

Figure 3. Brayton cycle modeled as a Finite State Machine

3.2 Mathematical representation of temperature and pressure

In each state of the system, the temperature and pressure evolve according to dynamical non linear equations. For this work, the relationships have been reduced into first order linear dynamical equations in the form of equation 6.

$$\dot{m} = -\alpha m + \kappa u \tag{6}$$

$\alpha > 0$ and $\kappa > 0$.

With u an input variable which increases the value of f. The coefficients α and κ are determined based in experimental results of the temperature and pressure behavior in each state.

Compression dynamic equations In the state of compression, the input variable is the angular speed ω of the compressor and it increases the value of the pressure in the output of the compressor P_2, giving as result Eq. 7

$$\dot{P}_2 = -\alpha_{P_2} P_2 + \kappa_{P_2} \omega \tag{7}$$

With initial condition the atmospheric pressure P_1. The thermodynamic relationship between pressure and temperature into the compressor is developed in Rodriguez (2015) and is presented in Eqs. 8-9.

$$K_c(t) = \frac{P_2}{P_1} \tag{8}$$

$$T_2 = K_c T_1 \tag{9}$$

Being T_2 the temperature in the output of compressor and T_1 the environment temperature. From 8 and 9

$$T_2 = \frac{P_2}{P_1} T_1 \tag{10}$$

Considering P_1 and T_1 constants (this assumption is valid because of the slow rate change of environmental conditions) and differentiating 10

$$\dot{T}_2 = \frac{\dot{P}_2}{P_1} T_1 \tag{11}$$

$$\dot{T}_2 = \frac{T_1}{P_1}(-\alpha_{P_2} P_2 + \kappa_{P_2} \omega) \tag{12}$$

$$\dot{T}_2 = -\alpha_{P_2} T_2 + \kappa_{P_2} \frac{T_1}{P_1} \omega \tag{13}$$

Combustion dynamic equations Temperature in the output of combustion chamber T_3 evolves with the applied heat Q generated by the ignition. Q is proportional to the amount of air mass in the combustion chamber, which is proportional to the open percentage of angle of the stator position θ_{Air}. Eq. 14 represents the evolution of T_3 with a control input θ_{Air}.

$$\dot{T}_3 = -\alpha_{T_3}[T_3 - T_1] + \kappa_{T_3} \theta_{Air} \tag{14}$$

Pressure in the output of combustion chamber P_3 evolves according to the heat transference, such as T_3, leading to Eq. 15

$$\dot{P}_3 = -\alpha_{P_3}[P_3 - P_2] + \kappa_{P_3}\theta_{Air} \tag{15}$$

Expansion dynamic equations Similarity to compression, pressure P_4 in the output of the turbine is represented as a first order linear system with a control input ω, as seen in Eq. 16. Making an analysis of the temperature exchange in this process similar to compression process, Eq. 17 represents the temperature behavior in the output of the turbine, considering $T3$ and $P3$ as constants during the expansion process.

$$\dot{P}_4 = -\alpha_{P_4}P_4 + \kappa_{P_4}\omega \tag{16}$$

$$\dot{T}_4 = -\alpha_{P_4}T_4 + \kappa_{P_4}\frac{T_3}{P_3}\omega \tag{17}$$

Brayton cycle for gas turbine are not a closed process, therefore the air in the output of the turbine is not reused as input for the compresor. Thus, the cooling dynamics is not necessary to analyze the system behavior.

Using the equations developed previousky, CTPN models are described as it follows.

3.3 Compressor Petri net model

Using Eqs. 13 and 7 the Petri net model shown in Fig. 4 is developed.

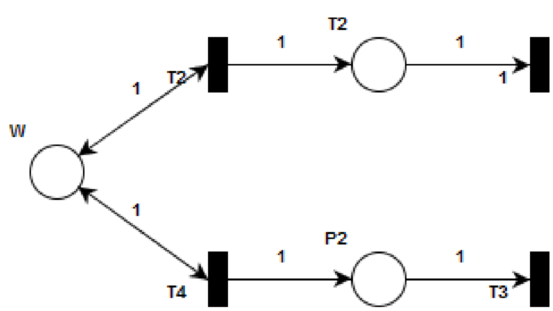

Figure 4. Compressor Petri net model

3.4 Combustion chamber Petri net model

Using Eqs. 14 and 15 the Petri net model shown in Fig. 5 is developed.

3.5 Turbine Petri net model

Using Eqs. 17 and 16 the Petri net model shown in Fig. 6 is developed.

Using the table 1, Eqs 7-17 are translate in terms of marking places, as shown in Eq. 18

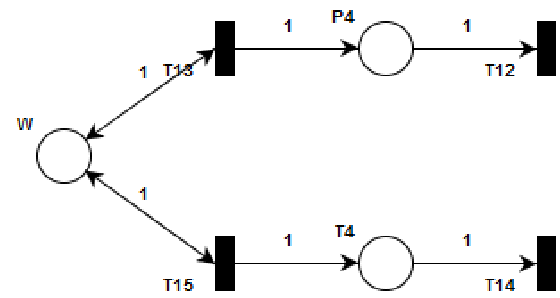

Figure 6. Turbine Petri net model

M_1	P_2
M_2	T_2
M_3	ω
M_4	P_3
M_5	Θ_{Air}
M_6	T_3
M_7	T_0
M_8	P_4
M_9	T_4

Table 1. Place numeration

$$\begin{aligned}
\dot{M}_1 &= -\alpha_{P_2}M_1 + \kappa_{P_2}M_3 \\
\dot{M}_4 &= -\alpha_{P_3}[M_4 - M_1] + \kappa_{P_3}M_5 \\
\dot{M}_8 &= -\alpha_{P_4}M_8 + \kappa_{P_4}M_3 \\
\dot{P}_2 &= -\alpha_{P_2}P_2 + \kappa_{P_2}\frac{T_1}{P_1}M_3 \\
\dot{M}_6 &= -\alpha_{T_3}[M_6 - M_7] + \kappa_{T_3}M_5 \\
\dot{M}_9 &= -\alpha_{P_4}M_9 + \kappa_{P_4}\frac{T_3}{P_3}M_3
\end{aligned} \tag{18}$$

Merging the models shown above, a complete gas turbine Petri net model is obtained with the following incidence matrix C and firing rate matrix Λ, using the place index as shown in Tab. 1.

$$C = \begin{bmatrix}
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix} \tag{19}$$

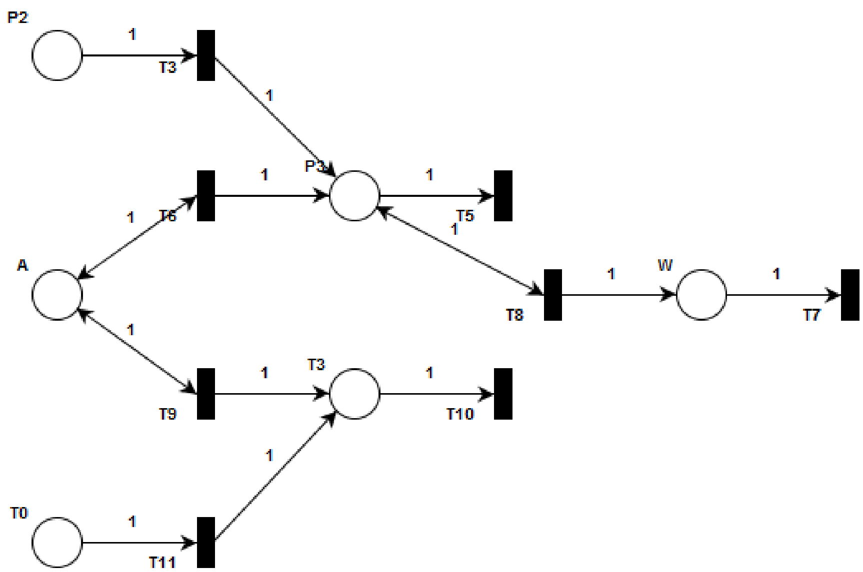

Figure 5. Combustion chamber Petri net model

$$\Lambda = diag \begin{bmatrix} 10 \\ 100 \\ 10 \\ 0.2 \\ 20 \\ 10000000 \\ 5 \\ 36000 \\ 5 \\ 1200 \\ 1.25 \\ 20 \\ 945 \\ 20 \\ 2.63 \end{bmatrix} \tag{20}$$

4. SUPERVISORY CONTROLLER DESIGN

There are plenty of safety specifications that can be enforced into the Gas Turbine dynamics, such as regulate the temperature and pressure of every state, or sustain a constant angular speed in the rotor. For the propose of this work, the safety specifications enforces the system to regulate the combustion chamber temperature and keep it lower than 1000 Celsuis degrees. The temperature is selected according to the open loop behavior of the Gas Turbine, reaching a combustion chamber temperature of 1300 Celsuis degrees, the bound may be changed and the design process would not change significantly. The constraint used is this case involve the combustion chamber temperature T_3, which according to Tab. 1 is represented by place 6. Hence, the algebraic constraint is presented in equation 21.

$$M_6 \geq 1000 \tag{21}$$

Therefore, the matrix L and the value b are obtained in order to apply IBCD. The initial marking of place 6 is environmental temperature, 20 Celsuis degrees.

$$L = [\,0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\,] \tag{22}$$

$$b = 1000 \tag{23}$$

Using Eqs. 27 and 26, the incidence matrix D_c and initial marking M_{c0} of the monitor place are calculated, considering C as the system incidence matrix and M_0 as system initial marking.

$$D_c = L * C \tag{24}$$

$$D_c = [\,0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ -1\ 0\ 0\ 0\ 0\,] \tag{25}$$

$$M_{c0} = b - L * M_0 \tag{26}$$

$$M_{c0} = 1000 - 20 = 980 \tag{27}$$

4.1 Admissibility analysis

The SC designed in previous section is only physically implementable if the constraint is admissible. Corollary 6 provides a useful proof in order to determine if the constraint is admissible. First, it is necessary to define the controllable and uncontrollable transitions in the Gas Turbine. The only controllable transition is associated to angle of the gas valve. This transition has been numbered with the index T_3. According with the incidence matrix, its corresponding column is column number 9.

$$D_{uc} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \quad (28)$$

Using Eq. 5

$$L * D_{uc} = [\, 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ -1\ 0\ 0\ 0\ 0\ 0 \,] \quad (29)$$

Since all the entries of this matrix are less than zero, the constraint is admissible. In Fig. 7 a simulation using Matlab toolbox is shown.

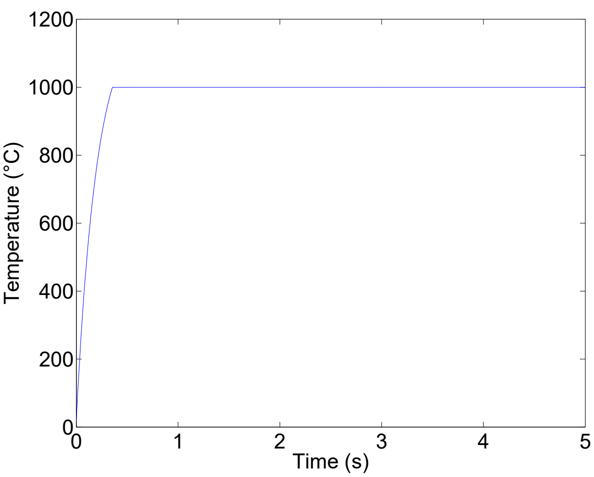

Figure 7. Temperature in combustion chamber

5. CONCLUSIONS AND FURTHER WORK

The design of supervisory controller provides a technique to enforce system behavior into a Gas Turbine. The representation as a CTPN is commonly used for discrete event systems in order to apply supervisory control to constraint the system behavior. Using marking constraints or behavioral constraints, the system can be submit to an specification behavior. The constraints can directly forbid certain place to reach a value, such as temperature must be always bellow 1000 °C or disable some action in order to prevent a bad performance, such as cut the air or the fuel supply if the angular speed is increasing. The formal techniques of supervisory control are widely developed and the approach presented in this work allows the synthesis of supervisory controllers for a gas turbine. Using the approach as a simplified model is significant in order to add non-linear behavior. Another significant application of PNs is developing models to estimate failures and diagnosis on running on-line systems, as is shown in Verma and Kumar (2014). A future research including both PNs approaches improves the reliability of a gas turbine and its control algorithms.

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