On the Leaderless Consensus of EL-Systems using Energy Shaping Controllers

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Abstract: This paper deals with the problem of achieving leaderless consensus (LC) of multiple Euler-Lagrange (EL) systems using the energy shaping plus damping injection principles of passivity–based control. The novel decentralized controller solves the LC-problem in networks of fully-actuated EL-systems with interconnecting time-varying delays and without employing velocity measurements. The paper also presents simulations, with ten agents, to show the performance of the novel controller.

Keywords: Consensus; Euler-Lagrange Systems; Passivity-based Control

1. INTRODUCTION

It is widely known that the Euler-Lagrange (EL) equations of motion describe the behavior of a wide number of physical systems—including mechanical, electrical and electromechanical systems (Ortega et al., 1998). The first results on consensus (synchronization) of a particular class of EL-agents were reported in (Chopra and Spong, 2005), the case of general, nonidentical, EL-systems with delays was first reported in (Nuño et al., 2011). Since then, a plethora of different controllers have been proposed to solve consensus problems, from simple Proportional plus Damping (P+D) schemes (Ren, 2009; Nuño et al., 2013b,a) to more ellaborate adaptive (Chung and Slotine, 2009; Nuño et al., 2011; Abdessameud et al., 2015; Chen et al., 2015) and sliding-mode controllers (Klotz et al., 2015).

Most of the previous reported schemes require velocity measurements for their implementation. Few controllers do not rely on velocities, among them are the following: using a velocity filter, in (Aldana et al., 2014), and a bounded controller, in (Ren, 2009), the leaderless consensus is solved for undelayed networks of EL-systems; the work of Abdessameud et al. (2012) solves the consensus problem, for the attitude of rigid bodies by using a *virtual* system for each agent and, in (Abdessameud and Tayebi, 2013), for linear second-order systems. Zheng and Wang (2012) solves the leaderless consensus problem for linear heterogeneous—first and second order systems—but without interconnecting delays.

Recently, in (Nuño, 2015, 2016), a solution to the leaderless consensus problem with time-varying interconnection delays is proposed. The solution incorporates the Immersion and Invariance velocity observer reported in (Astolfi et al., 2010). The main drawback of this scheme is that the implementation of the observer requirer the *exact knowledge* of the complete EL-dynamics, which in several practical scenarios is unrealistic.

In this paper we design a position feedback consensus controller for EL-systems following the energy shaping plus damping injection methodology where the energies of the system and the controller are added to make the resulting total energy a suitable Lyapunov function and damping is added to achieve asymptotic stability (Ortega et al., 1998). The roots of this procedure can be traced back to the early work of Lagrange and Dirichlet and, in modern times, to the seminal paper (Takegaki and Arimoto, 1981). In (Ortega and Spong, 1989) it was proved that passivity is the key property underlying the stabilization mechanism and the, now widely popular, term passivity-based control (PBC) was coined. The key feature of PBC that we exploit in this paper is that the damping needed to ensure asymptotic stability—that for EL-systems is usually achieved feeding-back the velocity, *i.e.*, the d term in P+d controllers—can be *injected through the controller* without velocity measurements. The history of this important observation—in the context of robotics—may be found in (Ortega et al., 1998, 2016).

Adopting the previous procedure leads in this paper to novel *decentralized* controllers that solve the leaderless consensus problem in networks of fully-actuated EL-systems with interconnecting time-varying delays and without employing velocity measurements. To the best of the authors' knowledge, this is the first work that provides a globally asymptotically stable (GAS) solution to this challenging problem without requiring the knowledge of the complete dynamics of the agents.

The following *notation* is used throughout the paper. $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$. $|\mathbf{x}|$ stands for the standard Euclidean norm of vector $\mathbf{x}$. $\mathbf{I}_k$ represents the identity matrix of size $k \times k$. $\mathbf{1}_k$ is a column vector of size k with all entries equal to one. For any function $\mathbf{f}$:

$\mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\mathbf{f}\|_\infty := \sup_{t\geq 0} |\mathbf{f}(t)|$, $\mathcal{L}_2$-norm as $\|\mathbf{f}\|_2 := (\int_0^\infty |\mathbf{f}(t)|^2 dt)^{1/2}$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_\infty < \infty\}$ and $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_2 < \infty\}$, respectively. The argument of all time dependent signals is omitted, *e.g.*, $\mathbf{x} \equiv \mathbf{x}(t)$, except for those which are time-delayed, *e.g.*, $\mathbf{x}(t - T(t))$. The subscript $i \in \bar{N} := \{1, ..., N\}$, where N is the number of nodes of the network.

2. DYNAMIC MODEL AND CONTROL OBJECTIVE

2.1 Node Dynamics

Consider a network of N, *fully-actuated* n–DoF, EL-systems of the form

$$\frac{d}{dt}\left(\nabla_{\dot{\mathbf{q}}_i}\mathfrak{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\right) - \nabla_{\mathbf{q}_i}\mathfrak{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) = \boldsymbol{\tau}_i,$$

where $\mathfrak{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)$ is the Lagrangian that is defined as

$$\mathfrak{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) = {}^s\mathcal{K}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) - {}^s\mathcal{U}_i(\mathbf{q}_i),$$

with ${}^s\mathcal{K}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) := \frac{1}{2}\dot{\mathbf{q}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i$, the kinetic energy and ${}^s\mathcal{U}_i(\mathbf{q}_i)$ the potential energy. $\mathbf{q}_i, \dot{\mathbf{q}}_i \in \mathbb{R}^n$ are the generalized position and velocity, respectively, $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ is the generalized inertia matrix, which is positive definite and bounded, and $\boldsymbol{\tau}_i \in \mathbb{R}^n$ is the vector of external forces.

Each agent's EL-equations of motion can be written as

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \nabla_{\mathbf{q}_i}{}^s\mathcal{U}_i(\mathbf{q}_i) = \boldsymbol{\tau}_i \qquad (1)$$

where $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal forces matrix, defined via the Christoffel symbols of the first kind. Piling up the vectors $\mathbf{q}_i$ and $\boldsymbol{\tau}_i$ as

$$\mathbf{q} := \mathrm{col}(\mathbf{q}_i), \quad \boldsymbol{\tau} := \mathrm{col}(\boldsymbol{\tau}_i), \quad \forall i \in \bar{N}.$$

the Hamiltonian (total energy) of the complete N EL-systems is

$${}^s\mathcal{T}(\mathbf{q},\dot{\mathbf{q}}) = {}^s\mathcal{K}(\mathbf{q},\dot{\mathbf{q}}) + {}^s\mathcal{U}(\mathbf{q}),$$

where

$${}^s\mathcal{K}(\mathbf{q},\dot{\mathbf{q}}) := \sum_{i\in\bar{N}} {}^s\mathcal{K}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i), \quad {}^s\mathcal{U}(\mathbf{q}) := \sum_{i\in\bar{N}} {}^s\mathcal{U}_i(\mathbf{q}_i),$$

are the total kinetic and potential energies, respectively.

All the agents dynamics can be compactly written as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \nabla_{\mathbf{q}}{}^s\mathcal{U}(\mathbf{q}) = \boldsymbol{\tau}. \qquad (2)$$

where we defined the overall inertia and Coriolis and centrifugal forces matrices as $\mathbf{M}(\mathbf{q}) := \mathrm{blockdiag}\{\mathbf{M}_i(\mathbf{q}_i)\}$, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) := \mathrm{blockdiag}\{\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\}$.

The following well-known property of EL-systems is instrumental for the sequel (Duindam et al., 2009; Hatanaka et al., 2015; Ortega et al., 1998).

Fact 1. The system (2) defines a cyclo–passive [1] operator $\Sigma_s : \boldsymbol{\tau} \to \dot{\mathbf{q}}$ with storage function ${}^s\mathcal{T}(\mathbf{q},\dot{\mathbf{q}})$. More precisely, ${}^s\dot{\mathcal{T}}(\mathbf{q},\dot{\mathbf{q}}) = \boldsymbol{\tau}^\top\dot{\mathbf{q}}$. ◁

2.2 Interconnection Topology

It is assumed that the EL-agents exchange information according to some prespecified constant pattern. This is characterised by N sets $\mathcal{N}_i \subset \bar{N}$, which identify the set of agents transmitting information to the ith agent. This interconnection of the N EL-agents is modeled via the Laplacian matrix $\mathbf{L} := \{L_{ij}\} \in \mathbb{R}^{N\times N}$, whose elements are defined as

$$L_{ij} = \begin{cases} \sum_{j\in\mathcal{N}_i} a_{ij} & i = j \\ -a_{ij} & i \neq j \end{cases} \qquad (3)$$

where $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$ otherwise (Cao and Ren, 2011).

The following assumption on the interconnection topology is imposed throughout the paper. Its motivation will become clear in the sequel—see point **O1** in Section 3.

A1. The EL-agents interconnection graph is *undirected and connected*.

By construction, $\mathbf{L}$ has a zero row sum. Moreover, Assumption **A1**, ensures that $\mathbf{L}$ is symmetric, has a single zero-eigenvalue and the rest of its spectrum is strictly positive. Thus, $\mathrm{rank}(\mathbf{L}) = N - 1$. Therefore, exists $\alpha \in \mathbb{R}$ such that $\ker(\mathbf{L}) = \alpha\mathbf{1}_N$.

In the paper we also consider the fact that the information exchange between agents is subject to time-delays. For these interconnection delays we assume the following.

A2. The communication, from the j-th agent to the i-th agent, is subject to a variable time-delay $T_{ji}(t)$ with a known upper-bound ${}^*T_{ji}$. Hence, it holds that

$$0 \leq T_{ji}(t) \leq {}^*T_{ji} < \infty. \qquad (4)$$

Furthermore, $\dot{T}_{ji}(t)$ is bounded.

2.3 Control Objective

Consider a network of N EL-systems of the form (1). Assume that velocities are *not available* for measurement. Further, suppose that the interconnection graph fulfills Assumptions **A1** and **A2**. Design a *decentralized* controller to solve the following consensus problem.

(LC) Leaderless Consensus Problem. The network has to asymptotically reach a consensus position. That is, *there exists* a constant $\mathbf{q}_c \in \mathbb{R}^n$ such that, for all $i \in \bar{N}$,

$$\lim_{t\to\infty} |\dot{\mathbf{q}}_i(t)| = 0, \qquad \lim_{t\to\infty} \mathbf{q}_i(t) = \mathbf{q}_c. \qquad (5)$$

Before going through the main result, let us present the following lemma that serves as instrumental in the proof.

Lemma 1. (Nuño et al., 2009). For any vector signals $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, any variable time-delay $0 \leq T(t) \leq {}^*T < \infty$ and any constant $\alpha > 0$, the following inequality holds

$$-\int_0^t \mathbf{x}^\top(\sigma)\int_{-T(\sigma)}^0 \mathbf{y}(\sigma+\theta)d\theta d\sigma \leq \frac{\alpha}{2}\|\mathbf{x}\|_2^2 + \frac{{}^*T^2}{2\alpha}\|\mathbf{y}\|_2^2.$$

◇

3. PASSIVITY-BASED CONTROLLER DESIGN

In the standard PBC methodology, the controller is another EL-system with its own generalized coordinates and Lagrangian function, that we interconnect with the plant to be controlled via a power–preserving interconnection (Ortega et al., 1998). In this way, the plant and controller

[1] The difference between cyclo–passive and passive operators is that the storage function of the former is not necessarily bounded from below.

energies and dampings are *added* up in the overall system, being able then to shape the energy and add the required damping.

3.1 Design of the EL–Controller

Denote the generalized coordinates of the controller as $\boldsymbol{\theta} \in \mathbb{R}^{Nn}$ and select its total energy function as

$$^c\mathcal{T}(\mathbf{q},\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) := {}^c\mathcal{K}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) + {}^c\mathcal{U}(\mathbf{q},\boldsymbol{\theta}) \tag{6}$$

where

$$^c\mathcal{K}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = \frac{1}{2}\dot{\boldsymbol{\theta}}^\top \mathbf{M}_c \dot{\boldsymbol{\theta}}$$

is the controller's kinetic energy with $\mathbf{M}_c \in \mathbb{R}^{Nn\times Nn}$ its *constant* positive semi-definite inertia matrix and $^c\mathcal{U}(\mathbf{q},\boldsymbol{\theta})$ the potential energy. Applying the EL-equations of motion we get the controllers dynamics

$$\mathbf{M}_c\ddot{\boldsymbol{\theta}} + \mathbf{D}\dot{\boldsymbol{\theta}} + \nabla_{\boldsymbol{\theta}}{}^c\mathcal{U}(\mathbf{q},\boldsymbol{\theta}) = \mathbf{0}_{Nn}. \tag{7}$$

where $\mathbf{D} := \text{blockdiag}\{d_i\mathbf{I}_n\} > 0$ is an $Nn \times Nn$ *damping* matrix.

It is important to underscore the following:

(i) The controllers potential energy *depends* on the plants generalized coordinates $\mathbf{q}$ that, as shown below, is instrumental to interconnect the plant and the controller EL-systems.
(ii) Damping has been added to the controllers dynamics, hoping that it will *propagate* to the system to achieve asymptotic stability, see (Ortega et al., 1998) for a discussion on this property.

The controller dynamics (7) verifies the following obvious input–output property.

Fact 2. System (7) defines a cyclo–passive operator $\Sigma_c : \dot{\mathbf{q}} \to \nabla_{\mathbf{q}}{}^c\mathcal{U}(\mathbf{q},\boldsymbol{\theta})$ with storage function $^c\mathcal{T}(\mathbf{q},\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$. More precisely, $^c\dot{\mathcal{T}}(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = \dot{\mathbf{q}}^\top\nabla_{\mathbf{q}}{}^c\mathcal{U}(\mathbf{q},\boldsymbol{\theta}) - \dot{\boldsymbol{\theta}}^\top\mathbf{D}\dot{\boldsymbol{\theta}}$. ◁

3.2 Interconnection and Stability Analysis

The next step in the PBC design is to interconnect the plant with the controller via

$$\boldsymbol{\tau} = -\nabla_{\mathbf{q}}{}^c\mathcal{U}(\mathbf{q},\boldsymbol{\theta}) \tag{8}$$

It is clear from Facts 1 and 2 that the resulting system is the negative feedback interconnection of two passive subsystems. Consequently, the total (desired) energy function of the closed–loop system is the sum of energy of the system plus the energy of the controller, that is,

$$^d\mathcal{T}(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) := {}^s\mathcal{T}(\mathbf{q},\dot{\mathbf{q}}) + {}^c\mathcal{T}(\mathbf{q},\boldsymbol{\theta},\dot{\boldsymbol{\theta}}), \tag{9}$$

and it, clearly, verifies

$$^d\dot{\mathcal{T}}(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = -\dot{\boldsymbol{\theta}}^\top\mathbf{D}\dot{\boldsymbol{\theta}} \leq 0. \tag{10}$$

The controller dynamics (7) is now selected to, first, ensure that there exists an equilibrium point of the overall system where the control objective is achieved, say $(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = (\mathbf{q}_\star,\mathbf{0}_{Nn},\boldsymbol{\theta}_\star,\mathbf{0}_{Nn})$ and, second, to render it stable (in the sense of Lyapunov). Towards this end, we postulate the total energy $^d\mathcal{T}(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ as a Lyapunov function. From (10) we see that it is a *nonincreasing* function, therefore it only remains to make this function positive definite, which is tantamount to proving that it has a unique and isolated minimum at the equilibrium point.

The PBC design is completed establishing *asymptotic* stability of the equilibrium. Since, almost invariably, the Lyapunov function is not strict—as seen in (10)—this is done invoking LaSalle's invariance principle. In particular, it is necessary to prove that $\dot{\boldsymbol{\theta}}$ is a *detectable* output for the interconnected system. Namely, that $\dot{\boldsymbol{\theta}}(t) \equiv \mathbf{0}_{Nn}$ implies that

$$\lim_{t\to\infty}(\mathbf{q}(t),\dot{\mathbf{q}}(t),\boldsymbol{\theta}(t),\dot{\boldsymbol{\theta}}(t)) = (\mathbf{q}_\star,\mathbf{0}_{Nn},\boldsymbol{\theta}_\star,\mathbf{0}_{Nn}). \tag{11}$$

3.3 Application of PBC to Networked Systems

In the context of this work —*i.e.*, position feedback design of a network system with communication delays—there are three obstacles to be overcome to complete the PBC design.

R1. The interconnection topology imposes constraints on the choice of the controllers potential energy—that should satisfy (8). These constraints appear in two different forms, on one hand, the i–th agent only knows the positions of its neighbours $\mathcal{N}_i$. On the other hand, as seen from the right hand side of (8), the interconnection forces are generated by the *gradient* of a potential function. For simplicity, these forces are assumed generated by *linear springs* interconnecting the agents. Hence, this translates into a symmetry condition on the Laplacian, explaining the need of Assumption **A1**, see (Nuño et al., 2013a; Arcak, 2007; Proskurnikov et al., 2015).

R2. Due to the absence of velocity measurements, damping can only be injected through the controller, as done in (10). If velocity is available for measurement there is no need to implement a dynamic controller and the simple P+d control

$$\boldsymbol{\tau}_i = \nabla_{\mathbf{q}_i}{}^s\mathcal{U}_i(\mathbf{q}_i) - p_i\sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{q}_i - \mathbf{q}_j) - d_i\dot{\mathbf{q}}_i,$$

with $p_i > 0$, solves the (**LC**) problem, see (Nuño et al., 2013b).

R3. The presence of the communication delays renders unfeasible the implementation of the controller (8), as the position information of the neighbouring agents is delayed.

We deal with all these obstacles in the section below, where the PBC solution to the (**LC**) is presented.

4. MAIN RESULT

For the sake of clarity we briefly discuss first the case of undelayed interconnections and later state a proposition for the delayed case.

A simple, natural choice for the controller energy (6) is $\mathbf{M}_c = \mathbf{I}_{Nn}$ and

$$^c\mathcal{U}(\mathbf{q},\boldsymbol{\theta}) = -{}^s\mathcal{U}(\mathbf{q}) + \frac{1}{2}(\mathbf{q}-\boldsymbol{\theta})^\top\mathbf{K}(\mathbf{q}-\boldsymbol{\theta}) + \frac{1}{2}\boldsymbol{\theta}^\top(\mathbf{PL}\otimes\mathbf{I}_n)\boldsymbol{\theta},$$

where $\mathbf{K} := \text{blockdiag}\{k_i\mathbf{I}_n\} > 0$ is the $Nn \times Nn$ matrix of the springs stiffness coefficients and $\mathbf{P} := \text{diag}\{p_i\} > 0$ is a $N \times N$ gain matrix. With this choice we cancel the potential energy of the agents and interconnect them through linear springs. [2]

It is easy to show that the desired energy (9) has a global minimum at

$$(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = ((\mathbf{1}_N \otimes \mathbf{q}_c), \mathbf{0}_{Nn}, (\mathbf{1}_N \otimes \mathbf{q}_c), \mathbf{0}_{Nn}),$$

where $\mathbf{q}_c \in \mathbb{R}^n$ that, as is well known (Nuño et al., 2011), coincides (in the undelayed case) with the average of the initial conditions of the agents positions. Consequently, ${}^d\mathcal{T}(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ is a Lyapunov function and the equilibrium is stable. Some signal chasing allows us to prove (11), hence, it is a GAS equilibrium.

The control signal (8) and the controller dynamics (7) of the ith-EL-system are given by

$$\boldsymbol{\tau}_i = \nabla_{\mathbf{q}_i} {}^s\mathcal{U}_i(\mathbf{q}_i) - k_i(\mathbf{q}_i - \boldsymbol{\theta}_i) \tag{12}$$

and

$$\ddot{\boldsymbol{\theta}}_i = -d_i\dot{\boldsymbol{\theta}}_i - k_i(\boldsymbol{\theta}_i - \mathbf{q}_i) - p_i \sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j),$$

respectively. Clearly, this controller is decentralized and its implementation does not require velocity measurements. When communication delays are present, (12) remains unaltered. However, the controller dynamics changes to

$$\ddot{\boldsymbol{\theta}}_i = -d_i\dot{\boldsymbol{\theta}}_i - k_i(\boldsymbol{\theta}_i - \mathbf{q}_i) - p_i \sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j(t - T_{ji}(t))), \tag{13}$$

for which we can state our first main result.

Proposition 1. Consider the network of EL-agents (2) with the interconnection graph verifying Assumptions **A1** and **A2**. The controller (12), (13) solves the (**LC**) problem provided that the gains are set as

$$2d_i > p_i \sum_{j\in\mathcal{N}_i} a_{ij}\left(\alpha_i + \frac{{}^*T_{ij}^2}{\alpha_j}\right), \quad \forall i \in \bar{N}, \tag{14}$$

for any $\alpha_i > 0$.

Proof. Using the properties of the Laplacian matrix, as in Nuño et al. (2013b), it is easy to show that the time derivative of the desired energy function (9)—evaluated along (2), (12) and (13)—is given by

$$\begin{aligned}{}^d\dot{\mathcal{T}} = &-\dot{\boldsymbol{\theta}}^\top\mathbf{D}\dot{\boldsymbol{\theta}} + \dot{\boldsymbol{\theta}}^\top(\mathbf{PL}\otimes\mathbf{I}_n)\boldsymbol{\theta}\\ &-\sum_{i\in\bar{N}} p_i \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\boldsymbol{\theta}}_i^\top(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j(t - T_{ji}(t))).\end{aligned}$$

Since

$$\dot{\boldsymbol{\theta}}^\top(\mathbf{PL}\otimes\mathbf{I}_n)\boldsymbol{\theta} = \dot{\boldsymbol{\theta}}^\top \begin{bmatrix} p_1 \sum_{j\in\mathcal{N}_1} a_{1j}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_j) \\ \vdots \\ p_N \sum_{j\in\mathcal{N}_N} a_{Nj}(\boldsymbol{\theta}_N - \boldsymbol{\theta}_j)\end{bmatrix}$$

we have that

$$\dot{\boldsymbol{\theta}}^\top(\mathbf{PL}\otimes\mathbf{I}_n)\boldsymbol{\theta} = \sum_{i\in\bar{N}} p_i \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\boldsymbol{\theta}}_i^\top(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j),$$

then ${}^d\dot{\mathcal{T}}$ can be written as

$${}^d\dot{\mathcal{T}} = -\sum_{i\in\bar{N}}\left(d_i|\dot{\boldsymbol{\theta}}_i|^2 + p_i\sum_{j\in\mathcal{N}_i} a_{ij}\dot{\boldsymbol{\theta}}_i^\top(\boldsymbol{\theta}_j - \boldsymbol{\theta}_j(t - T_{ji}(t)))\right)$$

Employing the relation

$$\boldsymbol{\theta}_j - \boldsymbol{\theta}_j(t - T_{ji}(t)) = \int_{t-T_{ji}(t)}^t \dot{\boldsymbol{\theta}}_j(\theta)d\theta,$$

we get

$${}^d\dot{\mathcal{T}} = -\sum_{i\in\bar{N}}\left(d_i|\dot{\boldsymbol{\theta}}_i|^2 + p_i\sum_{j\in\mathcal{N}_i} a_{ij}\dot{\boldsymbol{\theta}}_i^\top\int_{t-T_{ji}(t)}^t \dot{\boldsymbol{\theta}}_j(\theta)d\theta\right).$$

Integrating ${}^d\dot{\mathcal{T}}$, from 0 to t, invoking Lemma 1 and following the same steps as in (Nuño et al., 2013b), it can be shown that setting the controller's gains such that (14) is satisfied then there exists $\lambda_i > 0$ such that

$${}^d\mathcal{T}(0) \geq {}^d\mathcal{T}(t) + \sum_{i\in\bar{N}} \lambda_i\|\dot{\boldsymbol{\theta}}_i\|_2^2.$$

This last, and the fact that ${}^d\mathcal{T}(t) \geq 0$, for all $t \geq 0$, ensures that $\dot{\boldsymbol{\theta}}_i \in \mathcal{L}_2$ and ${}^d\mathcal{T} \in \mathcal{L}_\infty$.

Since ${}^d\mathcal{T}$ is positive definite and radially unbounded with respect to to $\dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_i, |\mathbf{q}_i - \boldsymbol{\theta}_i|, |\boldsymbol{\theta}_i - \boldsymbol{\theta}_j|$ then all these signals are bounded. $\dot{\boldsymbol{\theta}}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ ensures that $|\boldsymbol{\theta}_i - \boldsymbol{\theta}_j(t - T_{ji}(t))| \in \mathcal{L}_\infty$. With all these bounded signals it follows from (13) that $\ddot{\boldsymbol{\theta}}_i \in \mathcal{L}_\infty$. Barbalăt's Lemma allows us to conclude that $\lim_{t\to\infty} \dot{\boldsymbol{\theta}}_i(t) = \mathbf{0}_n$.

Now, differentiating (13) yields

$$\begin{aligned}\frac{d}{dt}\ddot{\boldsymbol{\theta}}_i = &-d_i\ddot{\boldsymbol{\theta}}_i - k_i(\dot{\boldsymbol{\theta}}_i - \dot{\mathbf{q}}_i)\\ &- p_i\sum_{j\in\mathcal{N}_i} a_{ij}\left(\dot{\boldsymbol{\theta}}_i - (1 - \dot{T}_{ji})\dot{\boldsymbol{\theta}}_j(t - T_{ji}(t))\right)\end{aligned} \tag{15}$$

A2 and $\ddot{\boldsymbol{\theta}}_i, \dot{\boldsymbol{\theta}}_i, \dot{\mathbf{q}}_i \in \mathcal{L}_\infty$ ensure that $\frac{d}{dt}\ddot{\boldsymbol{\theta}}_i \in \mathcal{L}_\infty$. Therefore, $\ddot{\boldsymbol{\theta}}_i$ is uniformly continuous and, since

$$\lim_{t\to\infty}\int_0^t \ddot{\boldsymbol{\theta}}_i(\sigma)d\sigma = \lim_{t\to\infty}\dot{\boldsymbol{\theta}}_i(t) - \dot{\boldsymbol{\theta}}_i(0) = -\dot{\boldsymbol{\theta}}_i(0),$$

we have that $\lim_{t\to\infty}\ddot{\boldsymbol{\theta}}_i(t) = \mathbf{0}_n$. Invoking the same arguments, it can be established that $\lim_{t\to\infty}\frac{d}{dt}\ddot{\boldsymbol{\theta}}_i(t) = \mathbf{0}_n$. Consequently, from (15), $\lim_{t\to\infty}\dot{\mathbf{q}}_i(t) = \mathbf{0}_n$.

The proof is completed showing, first, that the controllers generalized coordinates $\boldsymbol{\theta}$ converge to a consensus point. Second, proving that the systems generalized coordinates $\mathbf{q}$ converge to $\boldsymbol{\theta}$. For the first step we use the fact that

$$\boldsymbol{\theta}_i - \boldsymbol{\theta}_j(t - T_{ji}(t)) = \boldsymbol{\theta}_i - \boldsymbol{\theta}_j + \int_{t-T_{ji}(t)}^t \dot{\boldsymbol{\theta}}_j(\theta)d\theta,$$

and since $\lim_{t\to\infty}\dot{\boldsymbol{\theta}}_i(t) = \mathbf{0}_n$, from (13), it holds that

$$\lim_{t\to\infty}\sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{\theta}_i(t) - \boldsymbol{\theta}_j(t)) = \mathbf{0}_n.$$

In matrix form and making use of the Laplacian, this last expression can be written as

$$\lim_{t\to\infty}(\mathbf{L}\otimes\mathbf{I}_n)\boldsymbol{\theta}(t) = \mathbf{0}_{Nn}.$$

[2] The exact cancelation of the potential energy entails some robustness problems. If the associated forces satisfy some suitable bounds this can be avoided replacing the cancelation by a domination with quadratic functions, as usually done in robotics (Ortega et al., 1998).

The proof of the first claim is completed invoking the properties of the Laplacian.

For the second claim, notice that the closed-loop system (2) and (12) is given by

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{K}(\mathbf{q} - \boldsymbol{\theta}) = \mathbf{0}_{Nn}.$$

The fact that $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_i \in \mathcal{L}_\infty$ and $\lim_{t\to\infty} \dot{\mathbf{q}}_i(t) = \mathbf{0}_n$ implies, by Barbalăt's Lemma, that $\lim_{t\to\infty} \mathbf{q}_i(t) - \boldsymbol{\theta}_i(t) = \mathbf{0}_n$, as required. This completes the proof. □

Remark 1. As it can be clearly seen, from the controller's dynamics (13), the interconnection between the agents only employs part of the controller state. This has allowed us to solve the **LC** problem by inject damping in the controllers dynamics and not on the agents dynamics.

5. SIMULATIONS

This section provides a numerical simulation using a network of ten 2-DoF nonlinear manipulators with revolute joints. The dynamics of each agent follow (1) with the inertia and Coriolis matrices given by

$$\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix} \delta_{1i} + 2\delta_{2i}\mathrm{c}_{2_i} & \delta_{3i} + \delta_{2i}\mathrm{c}_{2_i} \\ \delta_{3i} + \delta_{2i}\mathrm{c}_{2_i} & \delta_{3i} \end{bmatrix},$$

$$\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \delta_{2i} \begin{bmatrix} -\mathrm{s}_{2_i}\dot{q}_{2_i} & -\mathrm{s}_{2_i}(\dot{q}_{1_i} + \dot{q}_{2_i}) \\ \mathrm{s}_{2_i}\dot{q}_{1_i} & 0 \end{bmatrix}$$

and the gravity vector given by

$$\nabla_{\mathbf{q}_i} {}^s\mathcal{U}_i(\mathbf{q}_i) = \begin{bmatrix} \frac{1}{l_{2_i}} g\delta_{3i}\mathrm{c}_{12_i} + \frac{g}{l_{1_i}}(\delta_{1i} - \delta_{3i})\mathrm{c}_{1_i} \\ \frac{1}{l_{2_i}} g\delta_{3i}\mathrm{c}_{12_i} \end{bmatrix},$$

where $\delta_{1i} := l_{2_i}^2 m_{2_i} + l_{1_i}^2(m_{1_i} + m_{2_i})$, $\delta_{2i} := l_{1_i} l_{2_i} m_{2_i}$ and $\delta_{3i} := l_{2_i}^2 m_{2_i}$. c_{2_i}, s_{2_i} and c_{12_i} stand for the short notation of $\cos(q_{2_i})$, $\sin(q_{2_i})$ and $\cos(q_{1_i} + q_{2_i})$, respectively. q_{k_i} and $\dot{q}_{k_i}$ are the joint position and velocity, respectively, of link k of manipulator i, with $k \in \{1, 2\}$. l_{k_i} and m_{k_i} are the respective lengths and masses of each link. g is the acceleration of gravity constant.

The ten-agent network is composed of three different groups of robot manipulators, with equal members at each group. The physical parameters, for each group, are: $m_1 = 4$kg, $m_2 = 2$kg and $l_1 = l_2 = 0.4$m, for Agents 1, 2 and 3; $m_1 = 2.5$kg, $m_2 = 3$kg, $l_1 = 0.3$m and $l_2 = 0.5$m for Agents 4, 5 and 6; $m_1 = 3$kg, $m_2 = 2.5$kg, $l_1 = 0.5$m and $l_2 = 0.2$m for Agents 7, 8, 9 and 10.

The network interconnection has the following Laplacian matrix

$$\mathbf{L} = \begin{bmatrix} 1.4 & 0 & -0.3 & 0 & 0 & 0 & 0 & -0.4 & 0 & -0.7 \\ 0 & 0.9 & 0 & -0.8 & 0 & 0 & 0 & 0 & 0 & -0.1 \\ -0.3 & 0 & 0.5 & 0 & 0 & 0 & 0 & -0.2 & 0 & 0 \\ 0 & -0.8 & 0 & 1 & 0 & 0 & 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0 & -0.5 & 0 & -0.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & -0.4 & -0.2 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0 & 1.4 & 0 & 0 & -0.9 \\ -0.4 & 0 & -0.2 & 0 & 0 & -0.4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.2 & -0.3 & -0.2 & 0 & 0 & 0.7 & 0 \\ -0.7 & -0.1 & 0 & 0 & 0 & 0 & -0.9 & 0 & 0 & 1.7 \end{bmatrix}$$

For simplicity, the variable time-delays for all agents are the same and they emulate an ordinary UDP/IP Internet delay with a normal Gaussian distribution with

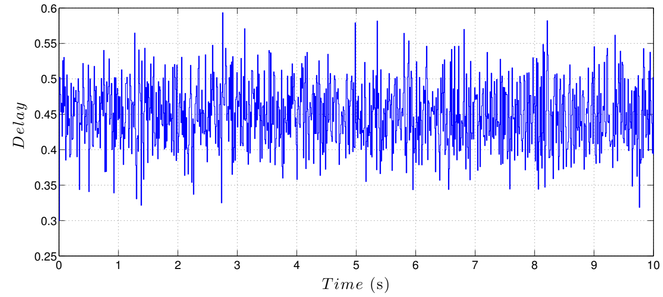

Fig. 1. Emulated UDP/IP Internet delay.

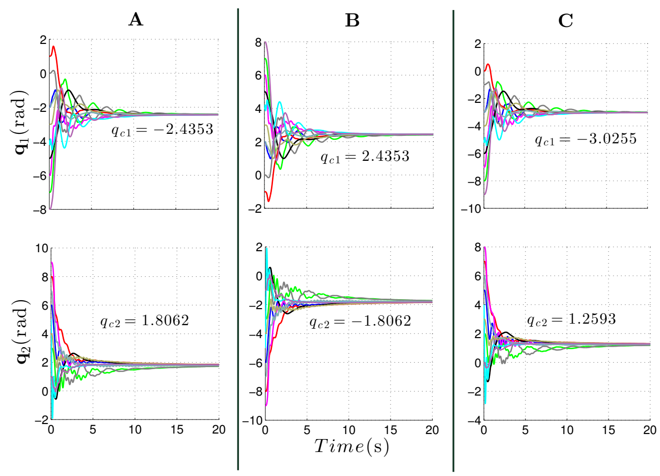

Fig. 2. Simulation results for the novel PBC.

mean, variance and seed equal to 0.45, 0.005 and 0.45, respectively (Salvo-Rossi et al., 2006). Such delays are shown in Fig. 1 for which we have selected ${}^*T_{ij} = 0.65$s. It should be underscored that compared to the real Internet delays in (Nuño et al., 2009), these delays are larger.

The proportional gains p_i have all been set to 10Nm. Setting $\alpha_i = 0.5$, using ${}^*T_{ij} = 0.65$s and $p_i = 10$Nm, condition (14) becomes $d_i > 6.725L_{ii}$. The damping gains are set to: $d_1 = d_7 = 10$, $d_2 = 6.6$, $d_3 = 3.8$, $d_4 = d_8 = 7.3$, $d_5 = 5.9$, $d_6 = 4.5$, $d_9 = 5.2$, $d_10 = 12.2$. The plant-controller interconnection gain has been set to $k_i = 20$Nm.

Fig. 2 depicts the simulation results for the solution of the (**LC**) problem. Columns **A**, **B** and **C** plot the joint positions of the ten EL-agents for different initial conditions. In Column **A** the initial velocities have been set to zero, and

$$\mathbf{q}(0) = [2, 6, -7, 3, 1, 8, 0, 1, -6, 9, -5, 0, -4, 5, -3, 4, -2, 7, -8, 1]$$

with $\boldsymbol{\theta}(0) = \mathbf{q}(0)$. In Column **B** the initial positions are $-\mathbf{q}(0)$ and in Column **C** the initial positions are $\mathbf{q}(0) + \mathbf{1}_{Nn}$.

Fig. 2 shows the positions of all the agents in the network, in the three cases, all the EL-agents find a common agreement position, depicted at each plot. Even with the presence of variable time-delays and without using velocity measurements, Fig. 2 corroborates that the novel PBC solves the (**LC**).

6. CONCLUSIONS

This paper proposes a novel PBC that solves the leaderless consensus problem in networks of multiple EL-agents. The main contribution is the proof that the resulting controller is robust to interconnecting variable time-delays and, more importantly, that it does not require velocity measurements. In contrast with the P+d controllers, the proposed PBC injects the dissipation required for asymptotic stability through the controller dynamics, which then propagates to the system. In the presence of delays, the dissipation has to be increased to compensate for the "losses" induced by the information exchange. The paper also presents a simulation study with ten agents that depict the performance of the novel controller.

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