Supervisory Control of an Automated Manufacturing System based on Behavioral Constraints

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Abstract
This paper studies the design of supervisory controllers with a minimum number of monitor places for an Automated Manufacturing System modeled as a safe Petri Net using a class of specifications modeled as *Behavioral Constraints*. A set of linear inequalities are induced by the Behavioral Constraints and applying equivalence among inequalities, the set of inequalities is reduced. Using the Invariant Based Control Design method, a supervisor Petri Net with a minimum number of monitor places is designed. The implementation is illustrated with the representation of the resulting supervisor as a ladder diagram to be implemented in a PLC.

Keywords: Supervisory Control Theory, Manufacturing Execution Systems, Petri Nets.

1. INTRODUCTION

The design of supervisory controllers (SCs) in Petri Nets (PNs) using the Invariant Based Control Design (IBCD) method (Moody and Antsaklis (1998), Iordache and Antsaklis (2006)) has been previously considered for the avoidance of forbidden states (Giua et al. (1992)) and for constraining the system behavior using linear inequalities induced by Behavioral Constraints (BCs) (Yamalidou and Kantor (1991)). Synthesizing SCs as PNs with a minimum number of monitor places avoiding forbidden states has been studied in Dideban and Alla (2008). Núñez and Sánchez (2015) proposed an approach to synthesize supervisory controllers based on Behavioral Constraints (SCBCs). A set of linear inequalities are induced by the BCs, which are reduced to a minimal representation applying an algebraic equivalence among inequalities. Applying the IBCD method, a supervisor Petri Net with a minimum number of monitor places is designed. This paper studies the implementation of a supervisory controller based on Behavioral Constraints (SCBC) with a minimum number of monitor places of an Automated Manufacturing System (AMS) modeled as a safe Petri Net (PN) and it is illustrated with the representation of the resulting supervisor as a ladder diagram. Section 2 introduces useful definition and theorems to design SCBC and the properness analysis. Section 3 presents the AMS used for this work and the model as a safe PN. Section 4 introduces the specifications to impose in the AMS and the translation to BCs, then shows the design of a supervisor using IBCD method to synthesize a PN supervisor and establishes that the resulting supervisor is in fact live, non-conflicting and controllable. Section 5 introduces the representation of the synthesized supervisor as a ladder diagram to be implemented on a PLC.

2. BASIC DEFINITIONS

This section introduces the definitions and theorems used in this paper to design a SCBC.

Definition 1. (Núñez and Sánchez (2015)) Let N be a *safe* PN with firing vector $Q = [q_1\ q_2\ \cdots\ q_l]$. Predicate variable $A : Q \to \{True, False\}$ associated to a firing transition t_i is defined with the rule

$$A(q_i) = \begin{cases} True \text{ if } q_i = 1 \\ False \text{ if } q_i = 0 \end{cases}$$

Definition 2. (Núñez and Sánchez (2015)) Let a system (N, M) with N a *safe* PN and marking vector $M = [m_1\ m_2\ \cdots\ m_l]$. Predicate variable $\Theta : M \to$ {True, False} associated to a marking place m_i is defined with the rule

$$\Theta(m_i) = \begin{cases} True \text{ if } m_i = 1 \\ False \text{ if } m_i = 0 \end{cases}$$

Definition 3. (Núñez and Sánchez (2015)) A Behavioral Constraint (BC) is defined with the following logic structure

$$A(q_a) \to \Phi \tag{1}$$

with A being a predicate variable associated to firing transition t_a and Φ a formula in normal conjunctive form, composed by predicate variables associated to marking places, that is

$$\Phi = \phi_1 \wedge \phi_2 \wedge \dots \phi_n \tag{2}$$

with

$$\phi_i(z_r) = \Theta(m_{r_1}) \vee \Theta(m_{r_2}) \vee \dots \Theta(m_{r_l}) \tag{3}$$

with r_j as the place index in N and

$$z_r = m_{r_1} + m_{r_2} + \dots + m_{r_l} \tag{4}$$

$$\phi(z) = \begin{cases} True \text{ if } z \geq 1 \\ False \text{ if } z = 0 \end{cases}$$

Theorem 4. (Núñez and Sánchez (2015)) Let $A(q_a)$ and $\Theta(m_{k_1})$, $\Theta(m_{k_2}) \dots \Theta(m_{k_l})$ be variables as in definitions 1 and 2. Let

$$A(q_a) \rightarrow \Theta(m_{k_1}) \wedge \Theta(m_{k_2}) \wedge \cdots \wedge \Theta(m_{k_n}) \\ \wedge[\Theta(m_{j_1}) \vee \Theta(m_{j_2}) \vee \cdots \vee \Theta(m_{j_m})] \quad (5)$$

Be a BC for restricting the system behavior. There is a supervisor representing by a PN composed by 1 monitor place, which incidence matrix is determined using IBCD with the linear inequality

$$m[nq_a - m_K] + [q_a - m_J] \leq 0 \quad (6)$$

with $m_K = m_{k_1} + m_{k_2} + \cdots + m_{k_n}$ and $m_J = m_{j_1} + m_{j_2} + \cdots + m_{j_m}$ and $m > 0$

Corollary 5. (Núñez and Sánchez (2015)) Let equation

$$A(q_a) \rightarrow \Theta(m_{k_1}) \wedge \Theta(m_{k_2}) \wedge \cdots \wedge \Theta(m_{k_n}) \quad (7)$$

be a BC for restricting the system behavior. There is a supervisor representing by a PN composed by 1 monitor place, which incidence matrix is determined using IBCD with the linear inequality

$$[nq_a - m_K] \leq 0 \quad (8)$$

with $m_K = m_{k_1} + m_{k_2} + \cdots + m_{k_n}$

Definition 6. (Controlled Siphon). (Iordache and Antsaklis (2006)) Let R be a siphon in a net N with M_R its marking vector. R is a controlled siphon if for all marking $M_R^{'}$ reachable from M_{0R}, $|M_R^{'}| \geq 1$. Otherwise, it's an uncontrolled siphon. That is, a controlled siphon is a siphon that never gets unmarked.

Definition 7. (Núñez and Sánchez (2015)) Let N be a safe net and M its marking vector. Let C be the net that represents a supervisor for N and M_c the marking vector of C.

System Under Supervision (SUS) is defined as

$$(N||C, [MM_c]) \quad (9)$$

with $N||C$ represents the synchronization of nets N and C with marking vector $[M\ M_c]$.

Theorem 8. (Núñez and Sánchez (2015)) Let $A(q_a) \rightarrow \Phi$ be a BC and C a net representing a supervisor for N.

SUS of C is live if and only if there is no 2 predicate variables of formula Φ in conjunction, such that associated places belong to a minimal S-invariant of N.

Theorem 9. (Núñez and Sánchez (2015)) Let $A(q_1) \rightarrow \Phi_1$, $A(q_2) \rightarrow \Phi_2$, $\cdots$ $A(q_n) \rightarrow \Phi_n$ be BCs that satisfy conditions of theorem 8. Let C be the net representing the supervisor of all the constraints.

The set of constraints are non-conflicting if and only if, net C does not contain an uncontrolled siphon.

Theorem 10. (Núñez and Sánchez (2015)) Let $A(q_a) \rightarrow \Phi$ be a BC imposed to net N.

Constraint is admissible (RW-controllable) if and only if transition t_a is controllable.

3. AUTOMATED MANUFACTURING SYSTEM DESCRIPTION AND MODELING

The AMS used in this work is described in this section. The system is a pneumatic punching center and its topology is illustrated in Fig. 1. The process begins when a piece arrives to the storage unit, then the input piston pushes the piece into the slot 1 of the rotatory table. The motor then turns 90° clockwise, and the piece advances to slot 2. The piece is processed by the punching machine at slot 2, then the motor turns 90° again, and the piece is moved to slot 3. The piece at slot 3 is pushed by output piston to a conveyor belt. The AMS is modeled using the strategy described in Rivera-Rangel et al. (2005). Each elementary components of the AMS is modeled as a PN module. For each component, a place is added to the model for each discrete value. The set of events are defined as the necessary events to change the discrete value in a component and a transition is added to the model for each event. For the initial marking, a token is added to the associated place of the initial discrete value of each component. The rest of the places remain with no tokens. The elementary components and their allowed discrete values are shown in Table 1. The list of events and corresponding transitions is shown in Table 2, *uc* *c* indicating uncontrollable and controllable transitions, respectively.

Table 1. Elementary components and discrete values of AMS

Component	Discrete Value	Place
Storage Unit	No piece in storage	P_1
	Piece in storage	P_2
Input piston	Input piston in	P_3
	Input piston out	P_4
Rotor slot 1	No piece in slot 1	P_5
	Piece in slot 1	P_6
Rotor slot 2	No piece in slot 2	P_7
	Piece in slot 2	P_8
Rotor slot 3	No piece in slot 3	P_9
	Piece in slot 3	P_{10}
Punching machine	Machine withdrawn	P_{11}
	Machine active	P_{12}
Output piston	Output piston in	P_{13}
	Output piston out	P_{14}
Rotor	Rotor in load position	P_{15}
	Rotor not in load position	P_{16}
Valve A	Valve A closed	P_{17}
retract input piston	Valve A open	P_{18}
Valve B	Valve B closed	P_{19}
activate input piston	Valve B open	P_{20}
Valve C	Valve C closed	P_{21}
retract output piston	Valve C open	P_{22}
Valve D	Valve D closed	P_{23}
activate output piston	Valve D open	P_{24}
Valve E	Valve E closed	P_{25}
activate punching machine	Valve E open	P_{26}
Rotatable Motor	Motor off	P_{27}
	Motor on	P_{28}

For the causal relationships in the AMS, self-looped arcs are added to the model.

- A piece can arrive to slot 1 only if input piston is out and there is a piece in storage, adding arcs from $P2$ and $P4$ to $T5$.
- Input piston can be activate only if valve A is open, and it can be retract only if valve B is open, adding arcs from $P18$ to $T4$ and from $P20$ to $T3$.
- Punching machine can be activate only if valve E is on, adding an arc from $P26$ to $T11$.

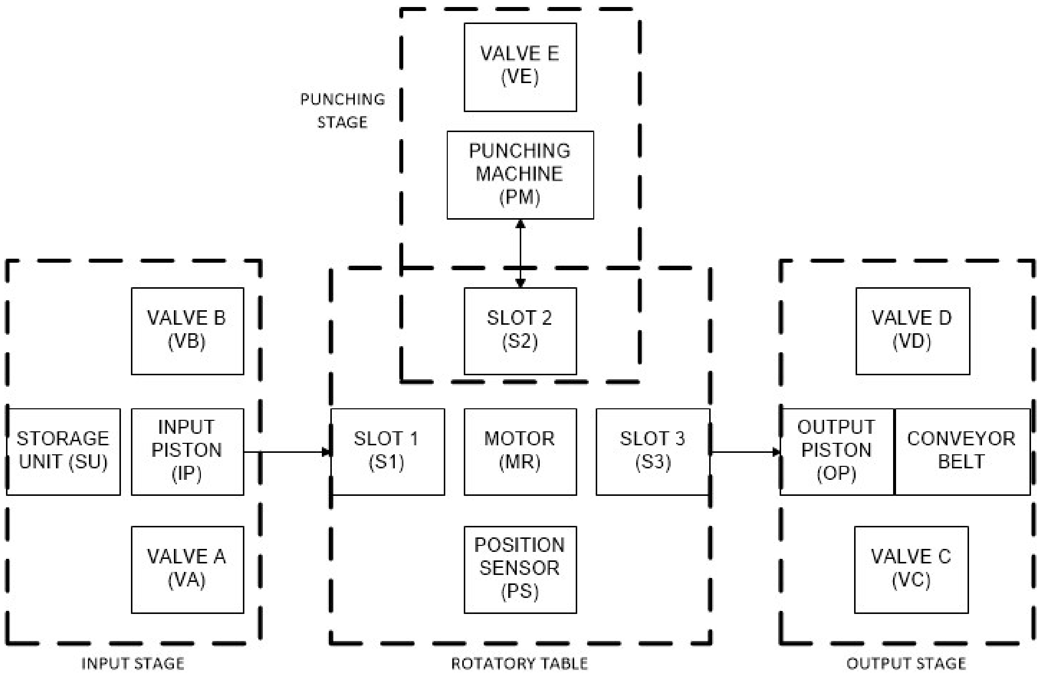

Figure 1. AMS Topology

Table 2. Events and corresponding transitions of AMS

Event	Transition	Type
Piece arrives to storage	T_1	uc
Piece leaves from storage	T_2	uc
Activate input piston	T_3	uc
Retract input piston	T_4	uc
Piece arrives to slot 1	T_5	uc
Piece leaves from slot 1	T_6	uc
Piece arrives to slot 2	T_7	uc
Piece leaves from slot 2	T_8	uc
Piece arrives to slot 3	T_9	uc
Piece leaves from slot 3	T_{10}	uc
Activate punching machine	T_{11}	uc
Retract punching machine	T_{12}	uc
Activate output piston	T_{13}	uc
Retract output piston	T_{14}	uc
Rotor arrives to load position	T_{15}	uc
Rotor leaves from load position	T_{16}	uc
Open valve A	T_{17}	c
Close valve A	T_{18}	c
Open valve B	T_{19}	c
Close valve B	T_{20}	c
Open valve C	T_{21}	c
Close valve C	T_{22}	c
Open valve D	T_{23}	c
Close valve D	T_{24}	c
Open valve E	T_{25}	c
Close valve E	T_{26}	c
Turn on motor	T_{27}	c
Turn off motor	T_{28}	c

- Output piston can be activate only if valve C is open, and it can be retract only if valve D is open, adding arcs from $P22$ to $T14$ and from $P24$ to $T13$.

$$d = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \tag{10}$$

$$D_p = blockdiag\{\mathbf{d}\} \tag{11}$$

The resulting model is shown in Fig. 2. The corresponding net is live and 1-bounded, i. e. is a safe net. The incidence matrix d of each PN module is shown in Eq. 10. Hence, the incidence matrix D_p of the entire system is presented as a $28x28$ block matrix in Eq. 11. The initial marking vector m of each module is shown in Eq. 12. Hence the initial marking vector M_o of all the AMS is presented in a block vector in Eq. 13.

$$m = [\,1\ \ 0\,] \tag{12}$$

$$M_o^T = [\,m|m|m|m|m|m|m|m|m|m|m|m|m|m\,] \tag{13}$$

4. SUPERVISORY CONTROLLER DESIGN

The specifications to be imposed in the AMS are described in this section. For the purpose of this work, four safety specifications are defined to ensure the AMS functionality. According with definition 3, each specification have a corresponding BC.

(1) If turning on the motor is enabled, then there is a manufacturing piece in slot 1 or in slot 2 and both piston and punching machine are withdrawn. The corresponding BC is shown in Eq. 14.
(2) If turning on the B valve to activate input piston is enabled, then there is a piece in storage and rotor is in load position.The corresponding BC is shown in Eq. 15.
(3) If turning on the D valve to activate output piston is enabled, then there is a piece in slot 3. The corresponding BC is shown in Eq. 16.
(4) If turning on the E valve to activate punching machine is enabled, then there is a piece in slot 2. The corresponding BC is shown in Eq. 17.

$$A(q_{27}) \rightarrow \Theta(m_3) \wedge \Theta(m_{13}) \wedge \Theta(m_{11}) \wedge [\Theta(m_6) \vee \Theta(m_8)] \tag{14}$$

$$A(q_{17}) \rightarrow \Theta(m_2) \wedge \Theta(m_{15}) \tag{15}$$

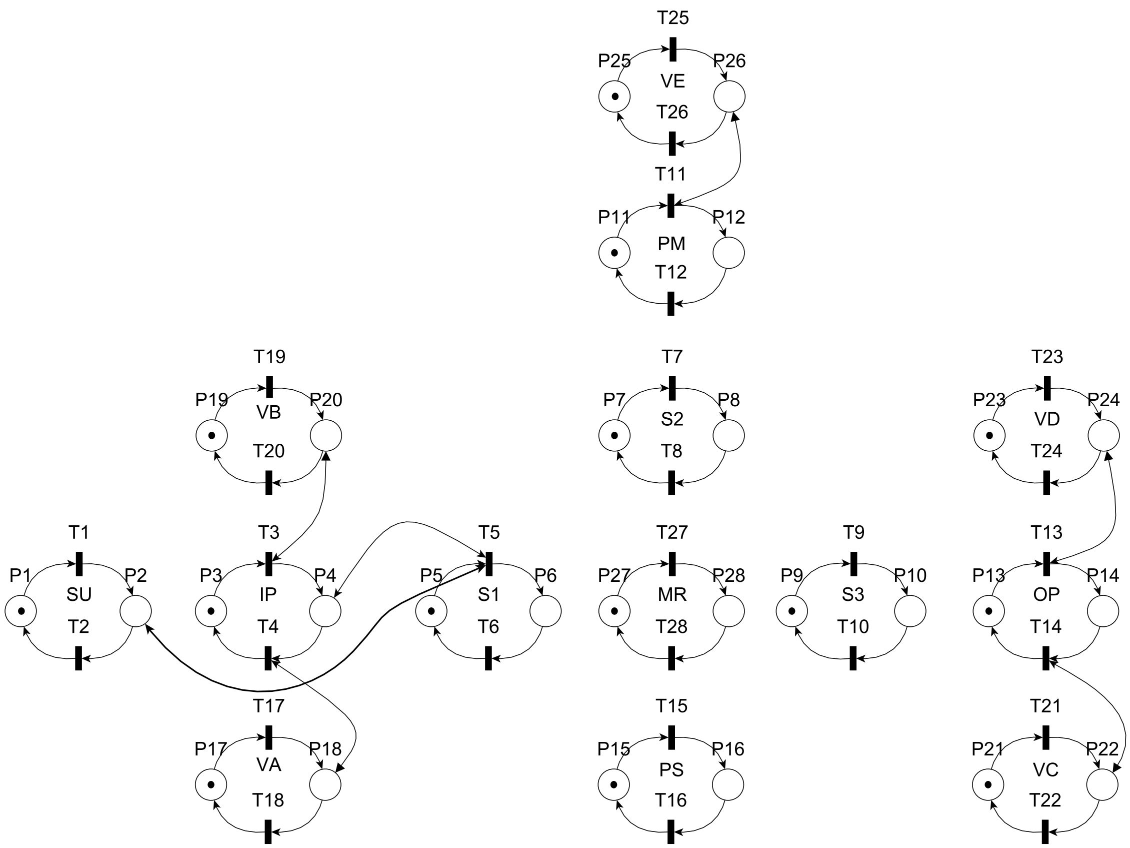

Figure 2. AMS model

$$A(q_{21}) \to \Theta(m_{10}) \tag{16}$$

$$A(q_{25}) \to \Theta(m_8) \tag{17}$$

Applying Theorem 4 and Corollary 5, Eqs. 14-17 are transformed into a set of linear inequalities shown in Eq. 18.

$$\begin{aligned} 7q_{27} - 2m_3 - 2m_{13} - 2m_{11} - m_6 - m_8 \leq 0 \\ 2q_{17} - m_2 - m_{15} \leq 0 \\ q_{21} - m_{10} \leq 0 \\ q_{25} - m_8 \leq 0 \end{aligned} \tag{18}$$

$$L_1 = \begin{bmatrix} 0 & 0 & -2 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & -2 & 0 & -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L = [\, L_1 | 0 \,] \tag{19}$$

$$D_{c1} = \begin{bmatrix} 0 & 0 & -2 & 2 & 1 & -1 & 1 & -1 & 0 & 0 & -2 & 2 & -2 & 2 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_c = [\, D_{c1} | 0 \,] \tag{20}$$

Applying IBCD, a PN supervisor is designed. The matrix L is defined in Eq. 19. Using the equation $D_c = -L * D_p$ incidence matrix D_c of supervisor is calculated. Four self-looped arcs are added, one for each BC, connecting the monitor place with the corresponding controllable transition. The weight of each arc is the corresponding coefficient for the transition in the set of induced inequalities shown in Eq. 18. For calculating the initial marking vector M_{oc} of supervisor PN the equation $M_{oc} = -L * M_o$ is used. Initial marking vector for supervisor PN is shown in Eq. 21. The resulting modular supervisors are shown in Fig. 3.

$$M_{oc}^T = [\, 6 \;\, 1 \;\, 0 \;\, 0 \,] \tag{21}$$

4.1 Properness Analysis

This subsection presents an analysis to show that the SCBC designed is in fact proper, i. e. live, non-conflicting controllable. For each BC, there are no 2 places belonging to the same minimal S-invariant. Hence, theorem 8 holds for all the BCs. Therefore, by theorem 9 the PN supervisor must not have an uncontrolled siphon to prove the set of constraints is non-conflicting. A method to determine if a net contains a siphon is proposed in Ezpeleta et al. (1993). Applying this method to the designed supervisor PN, the net does not contains a siphon. Hence, the set of constraints is non-conflicting. The set of constraints must be proven admissible. By theorem 10, transitions T_{27}, T_{17}, T_{21} and T_{25} must be controllable. Hence, the set of constraints is admissible.

5. LADDER DIAGRAM IMPLEMENTATION OF SUPERVISOR CONTROLLER

The implementation of the SCBC is presented in this section. The procedure for the conversion of a PN controller into a ladder diagram is illustrated in Gelen and Uzam (2014). The following rules are established for the

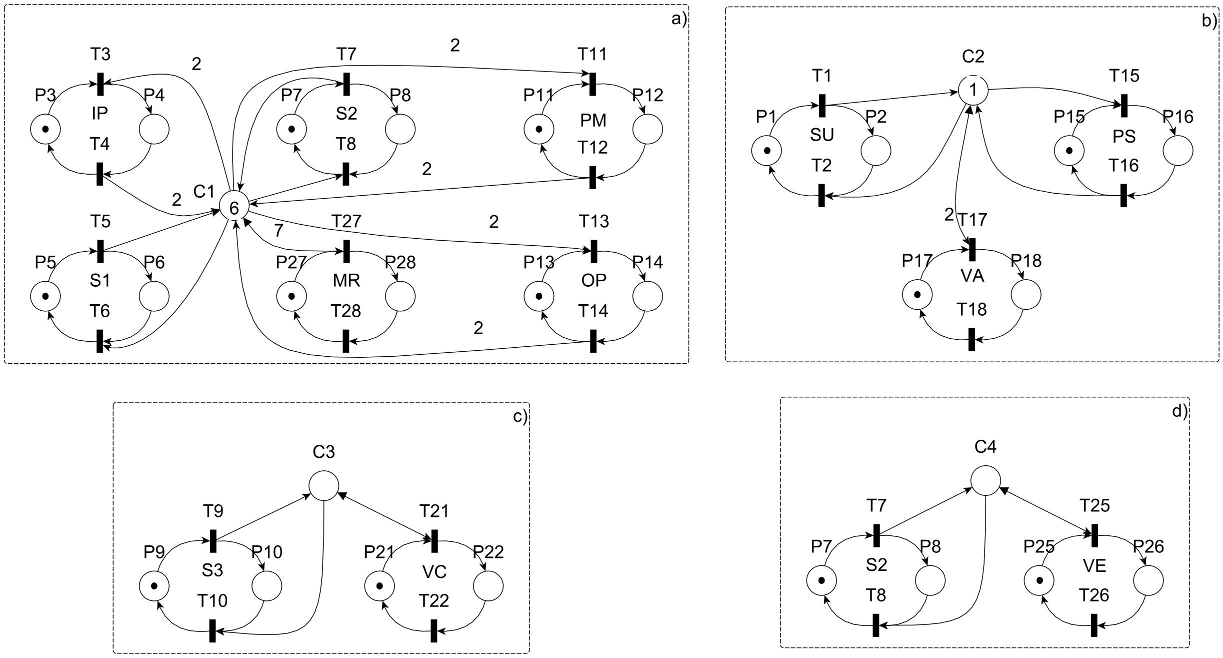

Figure 3. a) Modular supervisor for monitor place $C1$. b) Modular supervisor for monitor place $C2$. c) Modular supervisor for monitor place $C3$. d) Modular supervisor for monitor place $C4$

conversion. Let T_a be a transition in the supervisor PN. Let P_a be an output place of T_a, connected by an arc with weight na. Let P_b be an input place of T_a, connected by an arc with weight nb.

- Each transition T_a is represented as a contact in a ladder segment.
- If P_a is $1-$bounded, then it is represented by a coil with set function. If P_a is not $1-$bounded, then it is represented by an add block, adding na tokens to P_a.
- If P_b is $1-$bounded, then it is represented by a coil with reset function. Also, there is a normally open contact associated to P_b in the segment.
- If P_b is not $1-$bounded, then it is represented by a subtract block, subtracting nb tokens to P_b. Also, there is a comparison contact associated to P_b, with the rule, greater or equal than nb.
- If P_a=P_b (self-loop), then the number of tokens holds. Thus, there are not output blocks associated to P_a in the segment.

The resulting ladder diagram for the SCBC is shown in Figs. 4 and 5.

6. CONCLUSIONS

Modular modeling for a AMS is established as a low-level discrete event representation. Using equivalence among inequalities and modeling safety specifications as BCs, a supervisor PN with a minimum number of monitor places is designed and implemented in a ladder diagram.

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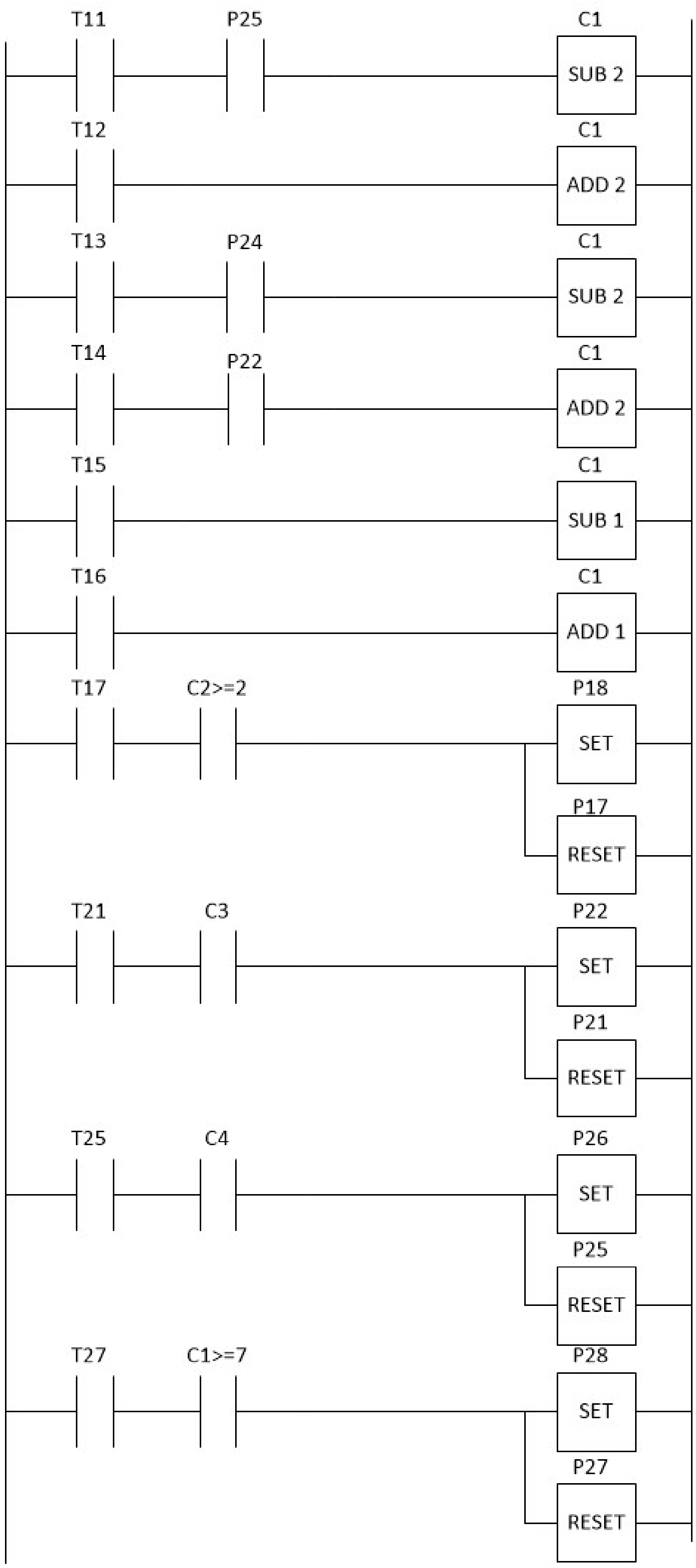

Figure 4. Ladder diagram part 1

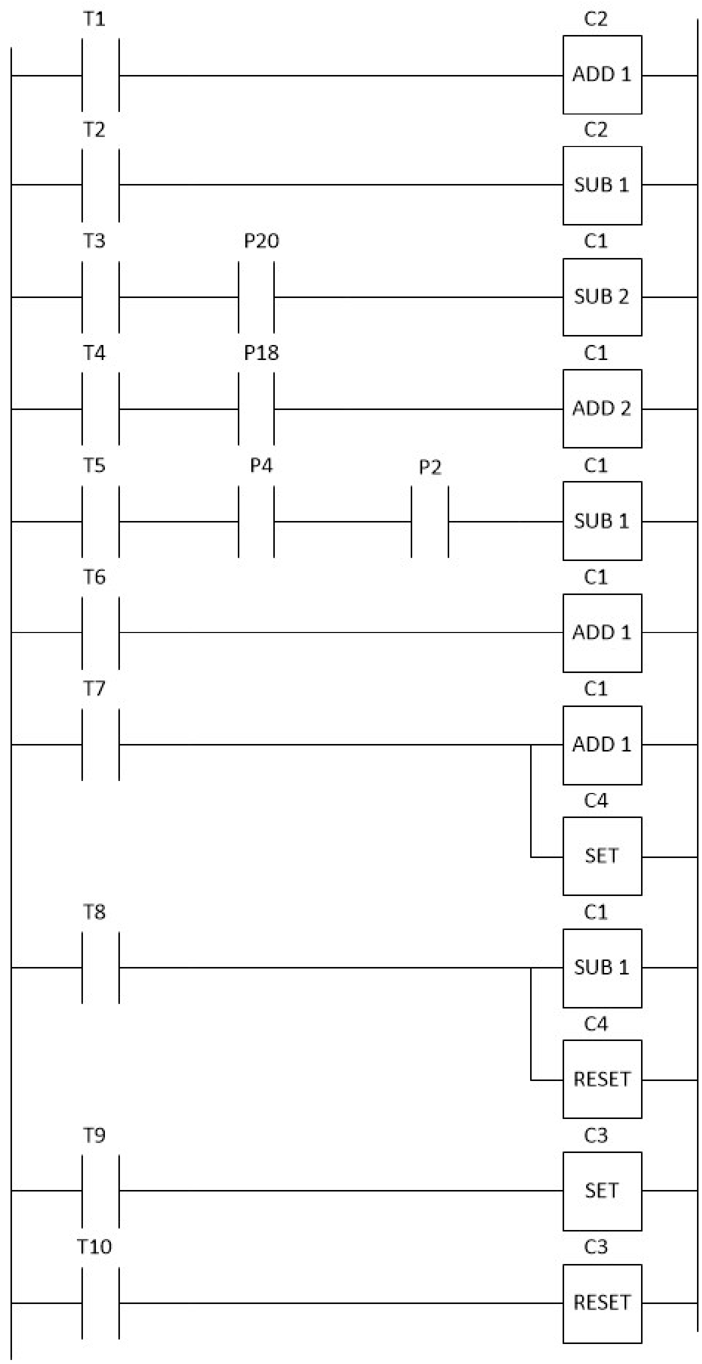

Figure 5. Ladder diagram part 2