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Splines smoothing assisted least-squares identification of robotic manipulators ⋆

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Abstract: This article presents a procedure that can be used to identify parameters of robotic manipulators that can be described by Euler-Lagrange equations of motion. This approach requires only measurements of angular position and torques acting in joints of the robot. To obtain smooth data for identification, angular measurements are smoothed by cubic splines. This allows analytical calculations of corresponding angular velocities and accelerations. Estimation of robot parameters can be then posed as an overdetermined linear problem. This approach is demonstrated on a simulation of a two-degree of freedom robotic manipulator. Results show that the procedure is much more robust compared to other classically used techniques while preserving high accuracy.

Keywords: Parameter identification, Mechanical systems, Robotic manipulators

1. INTRODUCTION

The need to build a reliable mathematical models for a complicated machinery such as robotic manipulators is driven by their extensive industrial use. Precision of mathematical model usually directly affects performance of algorithms that are used to control the robots. There are several ways how a model can be build, depending on how much information about the modeled phenomena is available. Most mathematical models are composed of two parts, structure, i.e. the underlying equations of the model and parameters that correspond to a particular model structure. When both structure and parameters are known, the corresponding model is called white-box model. When the structure or parameters are at least partially known, the corresponding model is called grey-box model. When both structure and parameters are unknown we term the model as black-box, detailed explanation can be found in Ljung (1999). In case of robotic manipulators we are dealing with identification of a grey-box model. The structure of model can be determined using classical mechanics and Euler-Lagrangian formalism, the parameters could be, in principle, determined by direct calculations. In practice however, precise physical model and precise measurements of all the parameters are rarely supplied from construction process. Moreover phenomenas like friction and actuator uncertainties further complicate the model building. Therefore it is often more preferable to identify the model directly from data measured by sensors of the robot.

Most approaches to estimation of the parameters of robotic manipulators exploit the fact that the estimation problem can be written as a problem of linear regression, if all of the configuration angles, angular velocities, angular accelerations and torques are available, e.g. Atkeson et al. (1986). Robotic manipulators are often equipped with rotary encoders or other sensors that can measure the configuration angles very accurately. Measurements of torques are usually also available, although these are usually less precise. However measurements of angular velocities and accelerations are usually not directly available. A most common remedy to this problem is to calculate the finite differences approximation of angular velocities and accelerations. Such approximation is however very sensitive to measurement noise and even in case of very accurate angular measurements, resulting velocity and acceleration estimates are very noisy. Therefore digital filtering is in turn employed to smooth these estimates. Main advantages of such approach is that the procedure is very simple, for example application, see Gautier and Poignet (2000).

Different approach based on frequency filtering was proposed in Olsen and Petersen (2001) and Olsen et al. (2002). This approach also exploits the linear structure of the identification problem, however instead of calculating the approximation of the velocities and accelerations, authors first calculate a smoothed estimate of configuration angles and then perform analytical calculation of corresponding velocities and accelerations. Main drawback of this method is that this method works well only in closed loop, therefore a controller with reasonable tracking performance must be available a priory.

Our approach rely on the linear structure of the problem as well. To obtain the smooth identification data we exploit the splines smoothing algorithm designed by Reinsch (1967) to smooth the angular data. Corresponding veloci-

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ties and accelerations can be then computed analytically. We demonstrate this procedure on a simulation of a two degree of freedom robotic manipulator. Comparison with other smoothing and filtering techniques is also carried out.

2. IDENTIFICATION PROCEDURE

Robotic manipulators are usually modeled as mechanical chains of rigid bodies, sometimes called links, connected with joints. Position of links, denoted as $\boldsymbol{q}$, constituting the chain are usually defined via relative or absolute angles. Positions $\boldsymbol{q}$ together with velocities $\dot{\boldsymbol{q}}$ constitute the state vector $\boldsymbol{x}$. When initial state $\boldsymbol{x}(0)$ is defined, subsequent robot motion can be calculated by integration of the Euler-Lagrange (EL) equations of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\dot{\boldsymbol{q}}}\right) - \frac{\partial L}{\partial \boldsymbol{q}} = \boldsymbol{\tau}, \tag{1}$$

where $\boldsymbol{\tau}$ denotes vector of torques. Scalar quantity L is denoted as Lagrangian and can be calculated as

$$L = T - V, \tag{2}$$

where T is the kinetic and K is the potential energy of the this mechanical system. When the kinetic energy is quadratic,

$$K(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{D}(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{3}$$

EL equations can be written in following matrix form

$$\boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{4}$$

where

$$\boldsymbol{G}(\boldsymbol{q}) = \frac{\partial V(\boldsymbol{q})}{\partial(\boldsymbol{q})}, \tag{5}$$

$$\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = (\frac{\partial}{\partial \boldsymbol{q}}(\boldsymbol{D}(\boldsymbol{q})\dot{\boldsymbol{q}})) - \frac{1}{2}(\frac{\partial}{\partial \boldsymbol{q}}(\boldsymbol{D}(\boldsymbol{q}))\dot{\boldsymbol{q}})^T. \tag{6}$$

Matrix $\boldsymbol{D}$ contains terms related to inertia forces, matrix $\boldsymbol{C}$ contains terms related to centrifugal and Coriolis forces and matrix $\boldsymbol{G}$ contains terms describing gravitational forces. For excellent treatment of classical mechanics, see Landau and Lifshitz (1976).

2.1 Least-squares estimation of model parameters

Equations of motion (1) that describe the motion of kinematic chain are parametrized by various physical quantities. Each link can be characterized by its length l_i, position l_{c_i} and weight m_{c_i} of its center of mass, and by corresponding inertia I_i. In the case the links are modeled by more complicated shapes some additional parameters might be present. Moreover it is usually desired to identify parameters of some friction model as well. To simplify identification of these unknown coefficients more convenient parametrization $\boldsymbol{\theta}$ can be introduced.

Due to practical reasons, we assume that input-output identification data are composed from measurements of configuration angles $\boldsymbol{q}$ and input torques $\boldsymbol{\tau}$. An intuitive way to obtain some estimate on parameters $\boldsymbol{\theta}$ would be to choose the estimate $\hat{\boldsymbol{\theta}} = \arg\min J(\boldsymbol{\theta})$,

$$J = \sum_{i=1}^{n}((\boldsymbol{q}(t_i) - \hat{\boldsymbol{q}}(\boldsymbol{\theta}, t_i))^2 + (\boldsymbol{\tau}(t_i) - \hat{\boldsymbol{\tau}}(\boldsymbol{\theta}, t_i))^2). \tag{7}$$

Unfortunately minimization of (7) is a complex nonlinear problem that can be solved only approximately by iterative methods. The main difficulties that arise during the minimization are following. Calculations are very time consuming. Minimization often results in local solutions that doesn't correspond to physically realisable parametrizations. Calculation is interrupted because the equations are non-integrable with the estimate provided by minimization routine. Due to these reasons, it is a common practise to assume that $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ are available and to exploit the fact that equations (4) admits a linear structure

$$\boldsymbol{\tau}(t) = \boldsymbol{Z}(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), \ddot{\boldsymbol{q}}(t))\boldsymbol{\theta}. \tag{8}$$

where matrix $\boldsymbol{Z}$ can be determined from matrices $\boldsymbol{D}$, $\boldsymbol{C}$ and $\boldsymbol{G}$. By taking sequences of measurements of $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, $\ddot{\boldsymbol{q}}$ and $\boldsymbol{\tau}$ during the time interval $\langle t_0, t_n\rangle$, (8) can be written as follows.

$$\boldsymbol{y}_\tau = \boldsymbol{X\theta} + \boldsymbol{e} \tag{9}$$

where vector

$$\boldsymbol{y}_\tau = [\tau_1(t_0), \ldots, \tau_1(t_n), \ldots, \tau_m(t_0), \ldots, \tau_m(t_n)]^T, \tag{10}$$

is composed of m sequences of torque measurements, corresponding to m actuators acting at each joint of the chain. Matrix $\boldsymbol{X}$ is composed of corresponding rows of matrix $\boldsymbol{Z}$.

In case when $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, $\ddot{\boldsymbol{q}}$ are known precisely, i.e. are measured without error and torque measurements are independent and identically distributed, error $\boldsymbol{e}$ will also be independent and identically distributed. Then optimal least-squares solution of (9) is given by

$$\hat{\boldsymbol{\theta}} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}_\tau. \tag{11}$$

However event though configuration angles $\boldsymbol{q}$ can be often measured very precisely and with high sampling frequency, velocities $\dot{\boldsymbol{q}}$ and accelerations $\ddot{\boldsymbol{q}}$ are usually not available and have to be estimated from $\boldsymbol{q}$. Errors in the measurement and estimation of the explanatory variables violate the assumptions made on normality of $\boldsymbol{e}$. Due to these reasons we can expect the resulting estimate $\hat{\boldsymbol{\theta}}$ to be biased and to have large variance.

2.2 Data smoothing and estimation

By far the most common approach to the estimation of $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ is to use the algorithm of finite differences. It is a well known fact that by using this approximation one amplifies the noise contained in the angular measurements. Therefore digital filtering is often employed to smooth the resulting estimate of the data. Smoothing the data will decrease the variance of the estimate $\hat{\boldsymbol{\theta}}$, however due to the distortion introduced by the digital filter and by the finite difference approximation, we can still expect the resulting estimate to be biased.

We propose a slightly different approach. Instead of calculating the estimates of $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ approximately by finite differences we calculate a smooth approximation of angular data $\boldsymbol{q}$ by an algorithm of Reinsch (1967) based on cubic spline smoothing. Resulting approximation of the angular data will be given by cubic spline functions that are continuous functions with continuous first and second derivatives that can be calculated analytically, therefore no additional error will be introduced. This approach also distorts the signal as during the smoothing process one has to choose a trade-off between the accuracy of the fit and between the smoothness of the approximation, however the distortion might be less severe comparing to the filtered finite differences approximation.

According to Reinsch (1967) the problem of smoothing can be posed as follows. We assume that a sequence of angular measurements, corresponding to j-th robot link, $q_j(t_i)$ is given, so that

$$t_0 < t_1 < \ldots < t_n. \tag{12}$$

For the sake of simplicity we will however drop the index j. We will search for a smoothing function $\hat{q}(t)$ that shall

$$\text{minimize} \int_{t_0}^{t_n} \ddot{g}(t)^2 \,\mathrm{d}t \tag{13}$$

among all functions $g(t)$ so that

$$\sum_{i=0}^{n} \frac{g(t_i) - q(t_i)}{\delta q(t_i)} \leq S, \quad g \in C^2[t_0, t_n], \tag{14}$$

holds. If available, one should use an estimate of standard deviation of $q(t_i)$ for $\delta q(t_i)$. Parameter S controls the degree of smoothing. For $S = 0$ the problem changes to a problem of interpolation and as S approaches infinity the problem changes to a problem of least-squares fitting of a straight line to the data.

The solution of (13) and (14) can be found by standard methods of the calculus of variations. Thus, by introducing the auxiliary variable z together with the Lagrangian parameter p, we have to look for the minimum of the functional

$$\int_{t_0}^{t_n} \ddot{g}(t)^2 dt + p\left\{\sum_{i=0}^{n}\left(\frac{g(t_i) - q(t_i)}{\delta q(t_i)}\right) + z^2 - S\right\} \tag{15}$$

The optimal curve $\hat{q}(t)$ can be determined from corresponding Euler-Lagrangian equations and following conditions can be found

$$\hat{q}^{''''}(t) = 0, \quad t_i < t < t_{i+1}, \quad i = 0, \ldots, n-1 \tag{16}$$

$$\hat{q}^{(k)}(t_i)_- - \hat{q}^{(k)}(t_i)_+ = \left\{\begin{array}{lll} 0 & \text{if } k = 0,1 & (i = 1, \ldots, n-1) \\ 0 & \text{if } k = 2 & (i = 0, \ldots, n) \\ 2p\dfrac{\hat{q}(t_i) - q(t_i)}{\delta q(t_i)^2} & \text{if } k = 3 & (i = 0, \ldots, n) \end{array}\right\}. \tag{17}$$

These can be satisfied by a cubic spline

$$\hat{q}(t) = a_i + b_i(t - t_i) + c_i(t - t_i)^2 + d_i(t - t_i)^3, \quad t_i \leq t < t_{i+1}. \tag{18}$$

By inserting (18) into (17) we obtain the relations between spline coefficients

$$c_0 = c_n = 0, \tag{19}$$

$$d_i = (c_{i+1} - c_i)/(3h_i) \tag{20}$$

$$b_i = (a_{i+1} - a_i)/h_i - c_i h_i - d_i h_i^2, \tag{21}$$

where $h_i = t_{i+1} - t_i$ for $i = 0, \ldots, n-1$. And also additionally

$$\boldsymbol{N}\boldsymbol{c} = \boldsymbol{Q}^T \boldsymbol{a}, \tag{22}$$

$$\boldsymbol{Q}\boldsymbol{c} = p\boldsymbol{M}^{-2}(\boldsymbol{y} - \boldsymbol{a}), \tag{23}$$

where

$$\boldsymbol{c} = [c_1, \ldots, c_{n-1}]^T, \tag{24}$$

$$\boldsymbol{y}_q = [q(t_0), \ldots, q(t_n)]^T, \tag{25}$$

$$\boldsymbol{a} = [a_0, \ldots, a_n]^T. \tag{26}$$

Matrix $\boldsymbol{M}$ is a diagonal $(n+1) \times (n+1)$ matrix with following entries

$$M_{ii} = \delta q(t_{i-1}), \quad i = 1, \ldots, n+1. \tag{27}$$

Matrix $\boldsymbol{N}$ is a square $(n-1)\times(n-1)$ symmetric tridiagonal matrix defined for $i = 1, \ldots, n-1$ as

$$N_{ii} = 2(h_{i-1} + h_i)/3, \quad N_{i+1,i} = N_{i,i+1} = h_i/3. \tag{28}$$

Matrix $\boldsymbol{Q}$ is tridiagonal $(n+1)\times(n-1)$ matrix with entries defined for $i = 1, \ldots, n-1$ as

$$Q_{ii} = 1/h_{i-1} \tag{29}$$

$$Q_{i+1,i} = (-1/h_{i-1} - 1/h_i) \tag{30}$$

$$Q_{i+2,i} = 1/h_i. \tag{31}$$

A left hand multiplication of (23) by $\boldsymbol{Q}^T\boldsymbol{M}^2$ separates the variable $\boldsymbol{c}$,

$$(\boldsymbol{Q}^T\boldsymbol{M}^2\boldsymbol{Q} + p\boldsymbol{N})\boldsymbol{c} = \boldsymbol{Q}^T\boldsymbol{q} \tag{32}$$

$$\boldsymbol{a} = \boldsymbol{y}_q - p^{-1}\boldsymbol{M}^2\boldsymbol{Q}\boldsymbol{c}. \tag{33}$$

Thus, if parameter p is known, coefficients $\boldsymbol{c}$ and $\boldsymbol{a}$ can be calculated using (32) and (33). Coefficients b_i and d_i can be calculated from (20) and (21). To obtain the value of parameter p, Reinsch (1967) proposes following algorithm:

Start with $p = 0$.

(1) Cholesky decomposition $\boldsymbol{R}^T\boldsymbol{R}$ of $(\boldsymbol{Q}^T\boldsymbol{M}^2\boldsymbol{Q} + p\boldsymbol{N})$.
(2) Compute $\boldsymbol{u}$ from $\boldsymbol{R}^T\boldsymbol{R}\boldsymbol{u} = \boldsymbol{Q}^T\boldsymbol{y}_q$ and $\boldsymbol{v} = \boldsymbol{M}\boldsymbol{Q}\boldsymbol{u}$, accumulate $e = \boldsymbol{v}^T\boldsymbol{v}$.
(3) *If* $e > S$
  (a) Compute $\boldsymbol{w}$ by solving $\boldsymbol{R}^T\boldsymbol{w} = \boldsymbol{N}\boldsymbol{u}$,
  (b) replace p by $p + (e - (Se)^{\frac{1}{2}})/(\boldsymbol{u}^T\boldsymbol{N}\boldsymbol{u} - p\boldsymbol{w}^T\boldsymbol{w})$,
  (c) restart with step (1).
  Otherwise
(4) Compute $\boldsymbol{a} = \boldsymbol{y}_q - \boldsymbol{M}\boldsymbol{v}$, $\boldsymbol{c} = p\boldsymbol{u}$ and b_i, d_i according to (20), (21).

Calculation of parameter p is a convex problem and therefore this procedure converges globally. For more details and efficient implementation of this algorithm see, Reinsch (1967). Application of this algorithm to the angular data $\boldsymbol{q}(t_i)$ yields analytical expressions for smoothed functions $\hat{\boldsymbol{q}}(t)$ and their time derivatives $\dot{\hat{\boldsymbol{q}}}(t)$ and $\ddot{\hat{\boldsymbol{q}}}(t)$,

$$\hat{q}_j(t) = a_{i,j} + b_{i,j}\Delta t_i + c_{i,j}\Delta t_i^2 + d_{i,j}\Delta t_i^3, \tag{34}$$

$$\dot{\hat{q}}_j(t) = b_{i,j} + 2c_{i,j}\Delta t_i + 3d_{i,j}\Delta t_i^2, \tag{35}$$

$$\ddot{\hat{q}}_j(t) = 2c_{i,j} + 6d_{i,j}\Delta t_i, \tag{36}$$

where

$$\Delta t_i = t - t_i, \quad t_i \leq t < t_{i+1} \quad i = 0, \ldots, n.$$

Consequently estimates $\hat{\boldsymbol{q}}(t_i)$, $\dot{\hat{\boldsymbol{q}}}(t_i)$ and $\ddot{\hat{\boldsymbol{q}}}(t_i)$ are used in (11) to calculate the estimate of the robot parameters $\hat{\boldsymbol{\theta}}$.

3. EXAMPLE: TWO DEGREE OF FREEDOM ROBOTIC MANIPULATOR

We will be discussing the identification of a simulated planar robotic manipulator composed of two rigid links that are connected via actuated rotary joints. Schematics of the robot is depicted on fig. 1. Corresponding physical parameters of the robot are listed in tab. 1. Robot parameters that were used in the simulation were based on our laboratory model and are as follows

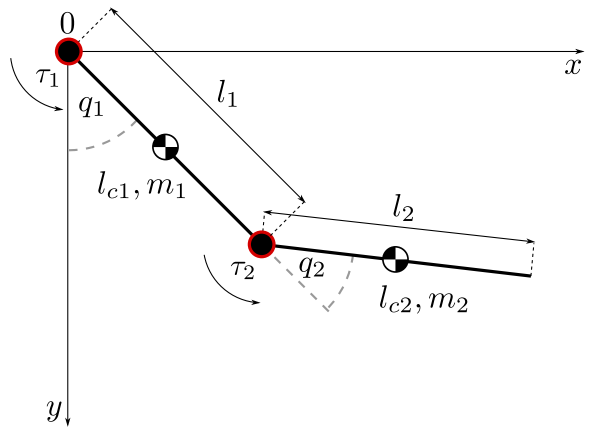

Fig. 1. Schematics of a 3-link robot

Table 1. Parameters of the 2-link robot

l_1, l_2	length of 1st and 2nd link	$[m]$
l_{c_1}, l_{c_2}	center of gravity of 1st and 2nd link	$[m]$
m_1, m_2	mass of 1st and 2nd link	$[kg]$
I_1, I_2	inertia of 1st and 2nd link	$[kg.m^2]$
μ_1, μ_2	viscous friction parameters	$[N.s/m]$
g	gravitational acceleration	$[m.s^{-2}]$

$$\begin{aligned} l_1 &= 0.2675, & l_2 &= 0.2675, \\ l_{c_1} &= 0.0.0398, & l_{c_2} &= 0.0.0392, \\ m_1 &= 0.4347, & m_2 &= 0.4322, \\ I_1 &= 0.0024, & I_2 &= 0.0024, . \end{aligned} \tag{37}$$

Using the Lagrangian formalism following model was obtained

$$\boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{38}$$

where matrices $\boldsymbol{D}$, $\boldsymbol{C}$, $\boldsymbol{G}$ are defined as follows

$$\begin{aligned} \boldsymbol{D} &= \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3 \cos(q_2); & \theta_2 + \theta_3 \cos(q_2) \\ \theta_2 + \theta_3 \cos(q_2); & \theta_2 \end{bmatrix} \\ \boldsymbol{C} &= \begin{bmatrix} -2\theta_3 \sin(q_2)\dot{q}_2 + \theta_6; & -\theta_3 \sin(q_2)\dot{q}_2 \\ \theta_3 \sin(q_2)\dot{q}_1; & \theta_7 \end{bmatrix} \\ \boldsymbol{G} &= \begin{bmatrix} \theta_4 g \sin(q_1) + \theta_5 g \sin(q_1+q_2) \\ \theta_5 g \sin(q_1+q_2) \end{bmatrix}. \end{aligned} \tag{39}$$

Substitution (40) was used to reduce the number of parameters and simplify identification process without degrading the model generality,

$$\begin{aligned} \theta_1 &= m_1 l_{c1}^2 + I_1 + m_2 l_1^2, \\ \theta_2 &= m_2 l_{c2}^2 + I_2, \\ \theta_3 &= m_2 l_1 l_{c2}, \\ \theta_4 &= m_1 l_{c1} + m_2 l_1, \\ \theta_5 &= m_2 l_{c2}, \\ \theta_6 &= \mu_1, \\ \theta_7 &= \mu_2. \end{aligned} \tag{40}$$

The state space vector $\boldsymbol{x}$ is composed of configuration angles $\boldsymbol{q}$ and corresponding angular velocities $\dot{\boldsymbol{q}}$, $\boldsymbol{\tau}$ denotes torques generated by actuators,

$$\boldsymbol{x} = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T, \quad \boldsymbol{\tau} = (\tau_1, \tau_2)^T \tag{41}$$

Associated state space model is given as

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{\tau}, \tag{42}$$

with corresponding vector fields

$$\boldsymbol{f} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \boldsymbol{D}^{-1}(-\boldsymbol{C} - \boldsymbol{G}) \end{bmatrix}, \qquad \boldsymbol{g} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{D}^{-1} \end{bmatrix}. \tag{43}$$

3.1 Smoothing and estimation of identification data

To analyse the performance of the identification algorithm, a Monte Carlo simulation comparison with similar identification techniques was carried out. Three methods were used, all three based on least-squares estimation (11), and the noisy measurements of $\boldsymbol{u}$ and $\boldsymbol{q}$,

$$\boldsymbol{y}_{q,\tau} = [\boldsymbol{q}^T, \boldsymbol{\tau}^T]^T + [\boldsymbol{v}_q^T, \boldsymbol{v}_\tau^T]^T \tag{44}$$

where

$$\boldsymbol{v}_q \sim \mathcal{N}(0, \boldsymbol{R}_q) \quad \boldsymbol{v}_\tau \sim \mathcal{N}(0, \boldsymbol{R}_\tau) \tag{45}$$

with covariance matrices

$$\boldsymbol{R}_q = \begin{bmatrix} \sigma_q^2 & 0 \\ 0 & \sigma_q^2 \end{bmatrix} \quad \boldsymbol{R}_\tau = \begin{bmatrix} 10^{-2} & 0 \\ 0 & 10^{-2} \end{bmatrix}. \tag{46}$$

During the simulation three different values of σ_q^2 were used, to simulate measurements with different accuracy. Following variances were used, $\sigma_q^2 = 10^{-8}$, $\sigma_q^2 = 10^{-6}$ and $\sigma_q^2 = 10^{-4}$. The difference between the identification methods is following. The first method is based on data without any smoothing or filtering, where angular velocities $\dot{\boldsymbol{q}}$ and accelerations $\ddot{\boldsymbol{q}}$ were computed via forward difference based on noisy measurements of $\boldsymbol{q}$ as

$$\dot{q}_j(t_i) = \frac{q_j(t_{i+1}) - q_j(t_i)}{T_s}, \tag{47}$$

$$\ddot{q}_j(t_i) = \frac{\dot{q}_j(t_{i+1}) - \dot{q}_j(t_i)}{T_s}, \tag{48}$$

where

$$i = 1, \ldots, n \quad j = 1, 2. \tag{49}$$

where T_s stands for sampling period and n denotes number of time instants in which the signals were measured. Second method computes velocities and accelerations in the same manner, except that the angular data and the noisy velocity and acceleration estimates are then filtered via digital Butter-Worth filters with following transfer functions

$$G_q(z) = \frac{b_{\dot{q}}}{a_{\dot{q}}} = \frac{b_0 + b_1 z^{-1} + \ldots + b_3 z^{-3}}{1 + a_1 z^{-1} + \ldots + a_3 z^{-3}} \tag{50}$$

$$G_{\dot{q}}(z) = \frac{b_{\dot{q}}}{a_{\dot{q}}} = \frac{b_0 + b_1 z^{-1} + \ldots + b_3 z^{-3}}{1 + a_1 z^{-1} + \ldots + a_3 z^{-3}} \tag{51}$$

$$G_{\ddot{q}}(z) = \frac{b_{\ddot{q}}}{a_{\ddot{q}}} = \frac{b_0 + b_1 z^{-1} + \ldots + b_3 z^{-4}}{1 + a_1 z^{-1} + \ldots + a_3 z^{-4}} \tag{52}$$

where the coefficients

$$\begin{aligned} b_q &= [0.0004,\ 0.0012,\ 0.0012,\ 0.0004], \\ a_q &= [1,\ -2.6862,\ 2.4197,\ -0.7302], \\ b_{\dot{q}} &= 10^{-4} * [0.1249,\ 0.3746,\ 0.3746,\ 0.1249], \\ a_{\dot{q}} &= [1,\ -2.9058,\ 2.8159,\ -0.9100], \\ b_{\ddot{q}} &= 10^{-6} * [0.0585,\ 0.2338,\ 0.3507,\ 0.2338,\ 0.0585], \\ a_{\ddot{q}} &= [1,\ -3.9179,\ 5.7571,\ -3.7603,\ 0.9212], \end{aligned} \tag{53}$$

were computed using the MATLAB function `butter`. Forward-backward filtering was realised using the MATLAB function `filtfilt`.

The third method uses the spline smoothing algorithm to first estimate the angular data $\boldsymbol{q}$ and subsequently calculates the derivatives $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ analytically via formulas (34) - (36). Parameter S was chosen equal to number of data points used for smoothing of a signal, but it can be tuned manually to obtain even better smoothing performance.

Data for the identification experiment were obtained using an open loop control where $u_1 = 6sin(t)$ and $u_2 = 4cos(t)$. The time interval of the whole experiment was $\langle 0, T\rangle$, with $T = 10[s]$ and the sampling period $T_s = 0.0025[s]$. However we omitted several data points due to following reasons. We omitted the data from the time interval $\langle 0, 0.25\rangle[s]$ in all three methods due to sharp transition in velocities from initial zero values. We further omitted the time interval $\langle 0, 1.7475\rangle[s]$ when digital filtering was used to smooth the data. Even though forward and backward filtering was used a significant distortion of signal was observed. To reduce the effect of the distortion one can increase the cut-off frequency, but we decided against that as it makes the data much more noisy and simply excluded the data where the filter has not reached its steady state.

Example of exact accelerations data, together with estimate based on filtered finite differences and data calculated via splines smoothing for $\sigma_q = 10^{-6}$ are depicted on fig. 2, fig. 3. We do not plot the noisy finite differences, due to the severe noise. Two interesting phenomena can be observed. The first is that some noise is still present in the estimate obtained via splines smoothing. Level of this noise can be further reduced only by manual tunning of S. However it can be seen that there is otherwise no other significant distortion of the signal. On the other hand the estimate obtained via filtered finite differences is smoother, however a large outliers are present due to the distortion of the signal. Therefore, in case of filtered finite differences, to obtain any reasonable estimate we had to omit a part of the filtered data in the interval $\langle 0, 1.7475\rangle[s]$ as was mentioned before. This interval is marked with green squares in fig. 2 and fig. 3. It should be noted that depending on the data it might not be clear which data to omit. It can be seen that smoothing via splines is much more robust in this matter. In fact it was not necessary to exclude data in this case, except for small interval $\langle 0, 0.25\rangle[s]$ where a sharp transition in velocities is present.

Due to the stochastic nature of the measurements, we performed a Monte Carlo simulation where the open loop control was used to generate $N = 30$ datasets where each set corresponds to data record described before. Each identification procedure was used to generate N estimates of parameter vector $\boldsymbol{\theta}$. A sample mean and a sample variance based on these estimates was calculated as

$$\bar{\hat{\boldsymbol{\theta}}} = \frac{1}{N}\sum_{i=1}^{N}(\boldsymbol{\theta}_i), \tag{54}$$

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\hat{\boldsymbol{\theta}}_i - \bar{\hat{\boldsymbol{\theta}}})^2 \ . \tag{55}$$

Sample means and variances for identified parameters based on raw data, digitally filtered data and data smoothed via spline smoothing, respectively, are summarized in tab. 2 - tab. 7.

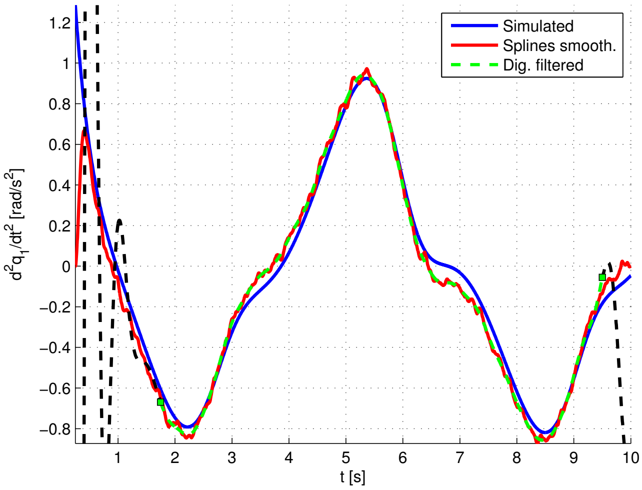

Fig. 2. Angular acceleration $\ddot{q}_1$: blue – simulated values, red – estimate obtained via spline smoothing, dashed black – finite differences with digital filtering, dashed green – finite differences with digital filtering (clipped data)

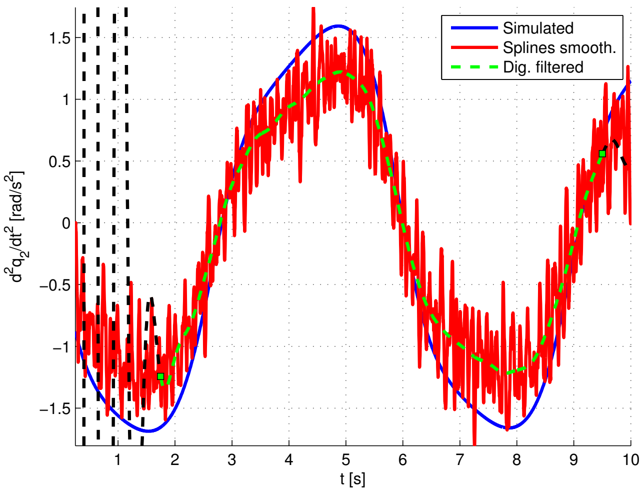

Fig. 3. Angular acceleration $\ddot{q}_2$: blue – simulated values, red – estimate obtained via spline smoothing, dashed black – finite differences with digital filtering, dashed green – finite differences with digital filtering (clipped data)

4. CONCLUSION

A new identification procedure for robotic manipulators was proposed. The motivation behind the method is to circumvent the approximate calculation of derivatives via finite differences and to use analytical calculations instead. To this end a smoothing algorithm based on cubic splines was employed. Simulation results show that the algorithm is much more robust than classically used filtered finite differences and does not generate outliers in the estimates of the derivatives. Procedure is semi-automatic, it requires specification of one parameter only, moreover it was observed that even simple heuristic provide a reasonable

Table 2. Sample mean of estimated parameters of the 2-link robot, $\sigma_q^2 = 10^{-8}$

Par. no.	True value	Raw Data	Dig. Filt.	CSS
1	0.0340	0.0010	0.0361	0.0359
2	0.0030	0.0005	0.0021	0.0031
3	0.0045	−0.0005	0.0042	0.0043
4	0.1329	0.1282	0.1331	0.1331
5	0.0170	0.0165	0.0168	0.0169
6	0.6189	0.6158	0.6206	0.6210
7	0.6189	0.6200	0.6190	0.6191

Table 3. Sample variance of estimated parameters of the 2-link robot, $\sigma_q^2 = 10^{-8}$

Par. no.	Raw Data	Dig. Filt.	CSS
1	$0.0447 * 10^{-8}$	$0.1894 * 10^{-6}$	$0.0182 * 10^{-5}$
2	$0.0086 * 10^{-8}$	$0.0382 * 10^{-6}$	$0.1022 * 10^{-5}$
3	$0.0283 * 10^{-8}$	$0.0019 * 10^{-6}$	$0.0001 * 10^{-5}$
4	$0.0008 * 10^{-8}$	$0.0015 * 10^{-6}$	$0.0001 * 10^{-5}$
5	$0.0004 * 10^{-8}$	$0.0011 * 10^{-6}$	$0.0011 * 10^{-5}$
6	$0.3448 * 10^{-8}$	$0.0125 * 10^{-6}$	$0.0151 * 10^{-5}$
7	$0.1180 * 10^{-8}$	$0.0004 * 10^{-6}$	$0.0003 * 10^{-5}$

Table 4. Sample mean of estimated parameters of the 2-link robot, $\sigma_q^2 = 10^{-6}$

Par. no.	True value	Raw Data	Dig. Filt.	CSS
1	0.0340	0.0007	0.0330	0.0366
2	0.0030	0.0004	0.0029	0.0024
3	0.0045	−0.0004	0.0045	0.0041
4	0.1329	0.1255	0.1328	0.1331
5	0.0170	0.0183	0.0169	0.0168
6	0.6189	0.4561	0.6207	0.6209
7	0.6189	0.5126	0.6192	0.6192

Table 5. Sample variance of estimated parameters of the 2-link robot, $\sigma_q^2 = 10^{-6}$

Par. no.	Raw Data	Dig. Filt.	CSS
1	10^{-8}	$0.1736 * 10^{-5}$	$0.3771 * 10^{-4}$
2	10^{-8}	$0.0026 * 10^{-5}$	$0.0036 * 10^{-4}$
3	10^{-8}	$0.0007 * 10^{-5}$	$0.0023 * 10^{-4}$
4	$0.0002 * 10^{-4}$	$0.0018 * 10^{-5}$	$0.0032 * 10^{-4}$
5	$0.0001 * 10^{-4}$	$0.0001 * 10^{-5}$	$0.0003 * 10^{-4}$
6	$0.1537 * 10^{-4}$	$0.0013 * 10^{-5}$	$0.0038 * 10^{-4}$
7	$0.0511 * 10^{-4}$	$0.0002 * 10^{-5}$	$0.0002 * 10^{-4}$

Table 6. Sample mean of estimated parameters of the 2-link robot, $\sigma_q^2 = 10^{-4}$

Par. no.	True value	Raw Data	Dig. Filt.	CSS
1	0.0340	10^{-5}	0.0131	0.0454
2	0.0030	10^{-5}	0.0051	−0.0032
3	0.0045	10^{-5}	0.0054	0.0030
4	0.1329	0.1225	0.1308	0.1338
5	0.0170	0.0202	0.0173	0.0158
6	0.6189	0.0179	0.6211	0.6209
7	0.6189	0.0279	0.6188	0.6206

smoothing performance. To achieve higher performance this parameter can be tunned manually or via sophisticated methods based on cross-validation. Of course a usual trade-off between smoothness and accuracy has to be made.

Table 7. Sample variance of estimated parameters of the 3-link robot, $\sigma_q^2 = 10^{-4}$

Par. no.	Raw Data	Dig. Filt.	CSS
1	10^{-10}	$10^{-3} * 0.2434$	0.0016
2	10^{-10}	$10^{-3} * 0.0038$	0.0001
3	10^{-10}	$10^{-3} * 0.0017$	10^{-4}
4	$10^{-6} * 0.0024$	$10^{-3} * 0.0019$	10^{-4}
5	$10^{-6} * 0.0007$	$10^{-3} * 0.0001$	10^{-4}
6	$10^{-6} * 0.1526$	$10^{-3} * 0.0023$	10^{-4}
7	$10^{-6} * 0.9735$	$10^{-3} * 0.0003$	10^{-4}

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