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Trajectory Tracking Control for an Input Delayed Delta Robot System through Active Disturbance Rejection ⋆

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Abstract: In this paper, the problem of trajectory tracking problem in a Delta Robot affected by known and fixed input time delays is tackled. The proposed method relies on purely linear high gain disturbance observation and linear feedback control techniques in combination with the classic Smith Predictor control scheme using only a simplified model of the Robot. The disturbance observer is based on a Generalized Proportional Integral Observer, which estimates along with the disturbance function, some of its time derivatives. This set of time derivatives and the disturbance inputs are taken for a different approach of the Smith predictor control scheme, in which, by means of a power series expansion, a prediction of the lumped disturbance is used for the output feedback control task. Experimental results show the effectiveness of the strategy in a trajectory tracking task.

Keywords: Delta Robot, Delay Control, Smith predictor, GPI observer, Active Disturbance Rejection Control.

1. INTRODUCTION

Many facts explain the arising of time delays in robotic control systems: dead periods between sensor and system outputs, communication time delays, time elapsed when computing control inputs, execution time delays in digital control systems around one sampling cycle **?**. The variety of applications of control schemes with robustness against time delay effects has been increasing thanks to the development of network technologies, tele-operation systems, **?**, **?**, etc. where there has been a growing in since the early nineties **?**. One of the most important contributions in control for time delay systems is given by Smith **?**, the so-called "Smith Predictor". Since the effectiveness of the Smith Predictor depends on the precise knowledge of the system plant, in some cases, the response may be poor in presence of nonmodeled dynamics or disturbances. Thus, the problem of compensating external disturbance and internal unknown dynamics for complex systems with input delays is still a challenging control problem.

A combination of the Smith Predictor and disturbance linear observers of Generalized Proportional Integral (GPI) nature has shown to be a good alternative for a class of differentially flat systems (see **?**). This controller consists in an observer based output feedback control with the following features:

- A linear Extended Luenberger Like observer which estimates the nonlinearities, nonmodeled dynamics, state dependent perturbations and external disturbance inputs, taken as a generalized lumped disturbance variable term. This observer also obtains the system phase variables.
- A linear observer-based state feedback controller, including a perturbation cancelation strategy.
- A classical Smith Predictor control scheme on the resulting simplified, dominantly linear, input output model (which is possible by virtue of the flatness property).

The main idea of the controller is the fact that the disturbance observer can predict the lumped disturbance input, which allows to approximately reduce the original nonlinear delayed input tracking control problem to that of a perturbed linear delayed input tracking problem, suitable for the application of the classical Smith Predictor control scheme **?**. Besides, the problem of the coupled dynamics for the robotic system is also resolved, being the observer a class of "computed torque estimator", reducing the problem to controlling n second order chains of integrators,

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regardless of the coupled dynamics. The control approach is based on the philosophy of active disturbance rejection control (ADRC) (see ?, ?), incorporating a disturbance prediction stage (for another interesting approach to ADRC for time delay systems, the reader is referred to ?).

In this article, a Smith Predictor based active disturbance rejection control strategy is proposed for the trajectory tracking task in a delta robot affected by known and fixed time delayed inputs. The rest of the article is given as follows: Section 2 introduces the problem formulation and the preliminary results for the proposed control approach. Section 3 describes the control strategy for the theoretical Delta robot model, including the input time delay; Section 4 depicts the experimental control results obtained on a laboratory test bed of the robotic system. Finally, Section 5 contains the conclusions and suggestions for further research.

2. PROBLEM FORMULATION

Consider the following perturbed nonlinear single-input single-output, smooth, nonlinear system,

$$y^{(n)} = \psi(t, y, \dot{y}, ..., y^{(n-1)}) + Ku(t-T) \tag{1}$$

The following hypotheses are assumed:

- T is the fully known and fixed input deadtime of delay.
- $y(t)$ is uniformly absolutely bounded and is differentially flat see ?
- For all bounded solutions, $y(t)$, of (1), obtained by means of suitable control input u, the additive, lumped disturbance input $\psi(\cdot)$ is uniformly absolutely bounded with bounded finite time derivatives.
- K is perfectly known.
- The system may be affected by some additive bounded zero mean deterministic noise.

We formulate the problem as follows:

Given a desired flat output reference trajectory, $y^*(t)$, devise a Smith predictor based linear output feedback controller for system (1) so that regardless of the lumped disturbance signal $\psi(t, y(t), \dot{y}(t), ..., y^{(n-1)}(t))$, the differential flat output y tracks the desired reference signal $y^*(t)$ even if in an approximate fashion (the tracking error, $e(t) = y - y^*(t)$, and its first, n, time derivatives, globally asymptotically exponentially converge towards a small as desired neighborhood of the origin in the reference trajectory tracking error phase space).

2.1 A GPI approach for the flat output tracking problem

Let us consider the nonlinear system (1)

$$y^{(n)} = v(t-T) + \psi(t, y, \dot{y}, ..., y^{(n-1)}) \tag{2}$$

where $v(t-T) = Ku(t-T)$. With the aim of constructing an observer which simultaneously estimates the lumped perturbation input: $\psi(t, y(t), \dot{y}(t), ..., y^{(n-1)}(t))$ and the state variables, $\{\dot{y}, \ddot{y}, \cdots y^{(n-1)}\}$, the system is taken as the disturbed linear system: $y^{(n)} = v(t-T) + \psi(t)$, where $\psi(t)$, is obtained with an internal approximating model at the observer.

Notice that the unknown disturbance input, $\psi(t)$, in the simplified system (2), can be expressed in terms of the delayed input v, the system output y, and a finite number of its time derivatives. That is, $\psi(t) = y^{(n)} - v(t-T) = y^{(n)} - Ku(t-T)$, which implies that $\psi(t)$ may be estimated with an unknown input observer.

Remark 1. Since the output $y(t)$ is possibly corrupted by a zero mean deterministic noise with unknown statistic parameters, to ease its effects on the on-line computation of the time derivatives, an integration of the measured signal, $y(t)$, denoted by $y_0(t)$ is carried out as suggested in ?.

The observer is given as follows:

$$\begin{aligned}
\dot{\hat{y}}_0 &= \hat{y}_1 + \lambda_{p+n}(y_0 - \hat{y}_0) \\
\dot{\hat{y}}_1 &= \hat{y}_2 + \lambda_{p+n-1}(y_0 - \hat{y}_0) \\
\dot{\hat{y}}_2 &= \hat{y}_3 + \lambda_{p+n-2}(y_0 - \hat{y}_0) \\
&\vdots \\
\dot{\hat{y}}_n &= v(t-T) + \hat{z}_1 + \lambda_p(y_0 - \hat{y}_0) \\
\dot{\hat{z}}_1 &= \hat{z}_2 + \lambda_{p-1}(y_0 - \hat{y}_0) \\
&\vdots \\
\dot{\hat{z}}_{p-1} &= \hat{z}_p + \lambda_1(y_0 - \hat{y}_0) \\
\dot{\hat{z}}_p &= \lambda_0(y_0 - \hat{y}_0) \\
\hat{\xi}(t) &= \hat{z}_1
\end{aligned} \tag{3}$$

To show the observer convergence, notice that the integral estimation error: $\tilde{e} = y_0 - \hat{y}_0$, satisfies the disturbed linear dynamics

$$\tilde{e}^{(p+n+1)} + \lambda_{p+n}e^{(p+n)} + \cdots + \lambda_0\tilde{e} = \xi^{(p+1)}(t) \tag{4}$$

Since $\xi(t)^{(p+1)}(t)$ is assumed to be uniformly absolutely bounded, then there exist coefficients λ_k such that $\tilde{e}$ converges to a small vicinity of zero, provided the roots of the associated characteristic polynomial in the complex variable s:

$$s^{p+n+1} + \lambda_{p+n}s^{p+n} + \cdots + \lambda_1 s + \lambda_0 \tag{5}$$

are all located deep into the left half of the complex plane. The further away from the imaginary axis, of the complex plane, are these roots located, the smaller the neighborhood of the origin, in the estimation error phase space, where the estimation error $\tilde{e}$ will remain ultimately bounded. Clearly, if $\tilde{e}$, and its time derivatives, converge towards a neighborhood of the origin, then $z_j - \xi^{(j-1)}$, $j = 1, 2, ...$, also converge towards a small vicinity of zero.

From the observer structure, the variable $\hat{z}_1$ denotes an internal model of the disturbance input $\psi(t)$ (see ?).

The model for z_1 is hypothesized as an element of a family of fixed degree time-polynomials, say of order $p-1$?. The model takes a *self updating* character when incorporated as part of an extended linear Luenberger type observer. The observer injection gains are tuned such that the estimation error converges to a small neighborhood of the origin, whose size, as well as the convergence time depend on the order of the internal model p, and the gain selection.

Thus, the self-updating residual function, $r(t)$, in the approximation, $\xi(t) = z_1 + r(t)$, and its finite time derivatives, say $r^{(p)}(t)$, are uniformly absolutely bounded. Let us denote by y_j the estimate of $y^{(j-1)}$ for $j = 1, ..., n$.

2.2 A Smith Predictor GPI controller

From (2), the called forward system is defined as follows:

$$y_f^{(n)}(t) = v(t) + \psi(t + T, y_f, \dot{y}_f, ..., y_f^{(n-1)}), \qquad (6)$$

where $\psi(t+T, y_f, \dot{y}_f, ..., y_f^{(n-1)})$ is the *predicted disturbance input* to be estimated (approximately) by means of the estimated states of the original system. A Taylor series expansion allows the following disturbance input predictor:

$$\psi(t + T) = \psi(t) + \dot{\psi}(t)T + \frac{1}{2!}\ddot{\psi}(t)T^2 + \ldots \qquad (7)$$

Then, using the input disturbance estimator and a truncated version of (7), an approximate disturbance input predictor is given as follows:

$$\widehat{\psi}(t + T) = z_1(t) + z_2(t)T + \frac{1}{2!}z_3(t)T^2 + \ldots + \frac{1}{(p-1)!}z_p T^{p-1} \qquad (8)$$

As performed in polynomial series approximation, a higher value of p allows a better approximation but, on the other hand, the numerical complexity of the observer is increased. It is proposed the following control law for the forward system using the disturbance predictor estimation

$$v(t) = -\hat{\psi}(t + T) + y^*(t)^{(n)} - \sum_{j=0}^{n-1}\left(\kappa_j[y_f^{(j)} - (y^*(t))^{(j)}] + e_f^{(j)}\right) \qquad (9)$$

$$e_f^{(j)}(t) = \hat{y}^{(j)}(t) - y_f^{(j)}(t - T) \qquad (10)$$

where $\hat{y}^{(j)}(t)$, $j = 0, \ldots, n-1$ are supplied by the GPI observer and $y_f^{(j)}(t)$ are, through algebraic manipulations, available for measurement. The terms $e_f^{(j)}$ are introduced in order to handle possible errors in the disturbance prediction, as a part of the Smith Predictor methodology. This terms use the difference between the plant output, say $y(t)$ and the time delayed forward output $y_f(t - T)$ to compensate possible differences between the delayed system and the delayed forward model. Figure 1 shows an schematic of the control design:

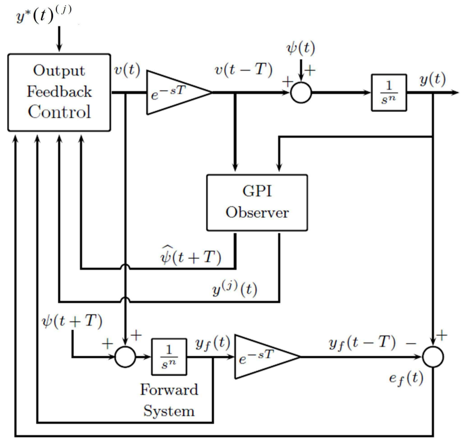

Fig. 1. Control scheme.

3. CONTROLLING THE DELTA ROBOT

Consider the Delta type robot proposed by L.W. Tsai et al. ?, ?, which consists in a three degree of freedom (DOF) parallel robot. It is characterized by: 1) An easy solution for the direct kinematics problem and 2) The position and orientation of the moving platform are naturally decoupled. Figure 2 depicts the Delta Robot. The dynamic model with delayed input is given as follows:

$$M(\Theta)\ddot{\Theta}(t) + G(\Theta, t) - R(\Theta, P(t))\lambda = \tau(t - T) \qquad (11)$$

where $\Theta = [\theta_{11},\, \theta_{12},\, \theta_{13}]^T$ is the actuated articular position vector, $P(t) = [p_x(t),\, p_y(t),\, p_z(t)]^T$ represents the

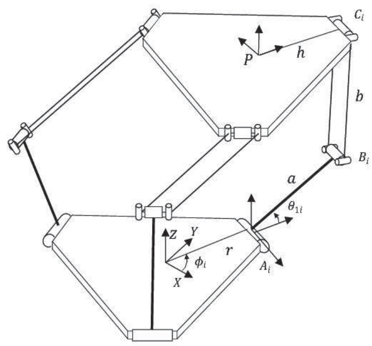

Fig. 2. Schematics of the Delta Robot

cartesian positions, $\lambda = [\lambda_1,\, \lambda_2,\, \lambda_3]^T$, denotes the Lagrange multipliers, $\tau(t-T) = [\tau_1(t-T), \tau_2(t-T), \tau_3(t-T)]^T$ is the vector of the delayed external torques and forces, $M(\Theta)$ denotes the generalized inertia matrix, $G(\Theta, t)$ contains the gravitational terms, and $R(\Theta, P(t))$ is the restriction vector which indicates the coupling restrictions between the mobile platform, the parallel links and the Delta Robot arms. In this case, the coupling dynamics is computed by the Lagrange Multipliers. $M(\Theta), G(\Theta, t), R(\Theta, P(t))$ are given as follows:

$$M(\Theta) = \begin{bmatrix} (\frac{1}{3}m_a + m_b)a^2 & 0 & 0 \\ 0 & (\frac{1}{3}m_a + m_b)a^2 & 0 \\ 0 & 0 & (\frac{1}{3}m_a + m_b)a^2 \end{bmatrix}$$

$$G(\Theta, t) = \begin{bmatrix} (\frac{1}{2}m_a + m_b)ga\cos(\theta_1) \\ (\frac{1}{2}m_a + m_b)ga\cos(\theta_2) \\ (\frac{1}{2}m_a + m_b)ga\cos(\theta_3) \end{bmatrix}$$

$$R(\Theta, P) = \begin{bmatrix} 2a\left[(p_x\cos(\phi_1) + p_y\sin(\phi_1) + h - r)\sin(\theta_1) - p_z\cos(\theta_1)\right] \\ 2a\left[(p_x\cos(\phi_2) + p_y\sin(\phi_2) + h - r)\sin(\theta_2) - p_z\cos(\theta_2)\right] \\ 2a\left[(p_x\cos(\phi_3) + p_y\sin(\phi_3) + h - r)\sin(\theta_3) - p_z\cos(\theta_3)\right] \end{bmatrix}$$

The terms λ_i, $i = 1, 2, 3$, are obtained by the following system of equations:

$$2\sum_{i=1}^{3}\lambda_i(p_x+h\cos(\phi_i)-r\cos(\phi_i)-a\cos(\phi_i)\cos(\theta_i)) = (m_p+3m_b)\ddot{p}_x - f_{px}$$
$$2\sum_{i=1}^{3}\lambda_i(p_y+h\sin(\phi_i)-r\sin(\phi_i)-a\sin(\phi_i)\cos(\theta_i)) = (m_p+3m_b)\ddot{p}_y - f_{py} \quad (12)$$
$$2\sum_{i=1}^{3}\lambda_i(p_z-a\sin(\theta_i) = (m_p+3m_b)\ddot{p}_z+(m_p+3m_b)g - f_{px}$$

where f_{px}, f_{py}, and f_{pz}, are the external forces for the mobile platform in the x, y, and z axes, m_a is the input link mass, m_b denotes the parallel bars link mass, and m_p is the mobile platform mass. Last model is fully linearizable through a static feedback, being θ_i, $i = 1, 2, 3$ the set of flat outputs.

3.1 GPI Active Disturbance Rejection Controller

Suppose that $G(\Theta)$, $R(\Theta, P)$ are nonmodeled dynamics, and each actuator is affected by the train gear disturbances ($\eta(t) = [\eta_1\ \eta_2\ \eta_3]^T$). Lumping last terms leads to the following disturbance vector.

$$\psi(t) = M^{-1}(\Theta)\left[-G(\Theta)+R(\Theta,P)\right]+\eta(t) \quad (13)$$

Thus, the dynamics governing the simplified disturbed system is

$$\ddot{\Theta} = M^{-1}(\Theta)\tau(t-T)+\psi(t) \quad (14)$$

The relation between the motor torque and the voltage input is:

$$\tau(t-T) = \begin{bmatrix}(K_1N/R_a)V_1(t-T)\\(K_1N/R_a)V_2(t-T)\\(K_1N/R_a)V_3(t-T)\end{bmatrix} \quad (15)$$

Using (15), (14),and $M^{-1}(\Theta)$ we have:

$$\ddot{\Theta} = \frac{1}{(\frac{1}{3}m_a+m_b)a^2}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}(K_1N/R_a)V_1(t-T)\\(K_1N/R_a)V_2(t-T)\\(K_1N/R_a)V_3(t-T)\end{bmatrix}+\psi(t) \quad (16)$$

Equation (16) consists of a set of three decoupled, disturbed systems of the form (2). It is, then, possible to define three independent control inputs of the form (9), to solve the robust trajectory tracking problem. According to the procedure of section 2.1, the forward disturbed system is proposed as

$$\ddot{\Theta}_f = \frac{1}{(\frac{1}{3}m_a+m_b)a^2}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}(K_1N/R_a)V_1(t)\\(K_1N/R_a)V_2(t)\\(K_1N/R_a)V_3(t)\end{bmatrix}+\psi(t+T) \quad (17)$$

where $\psi(t+T)$ is the vector of disturbed input signals to be estimated, in an online fashion, using the truncated power series expansions in combination with the GPI observers. It is assumed that ψ is unknown, but uniformly absolutely bounded. Let us define $\hat{e}_{01}, \hat{e}_{02}$, and $\hat{e}_{03}$ the estimates of the integral tracking errors, $e_{01} = \int_0^t(\theta_1(\tau)-\theta_1^*(\tau))d\tau$,$e_{02} = \int_0^t(\theta_2(\tau)-\theta_2^*(\tau))d\tau$, and $e_{03} = \int_0^t(\theta_3(\tau)-\theta_3^*(\tau))d\tau$. Now, consider $\hat{e}_{11}, \hat{e}_{12}$, and $\hat{e}_{13}$ the estimates of the tracking errors $e_{11} = \theta_1(t)-\theta_1^*(t)$,$e_{12} = \theta_2(t)-\theta_2^*(t)$, $e_{13} = \theta_3(t)-\theta_3^*(t)$. In a similar fashion, let $\hat{e}_{21}, \hat{e}_{22}$, and $\hat{e}_{23}$ be the estimates of the velocity tracking error states, given by $e_{21} = \dot{\theta}_1(t)-\dot{\theta}_1^*(t)$,$e_{22} = \dot{\theta}_2(t)-\dot{\theta}_2^*(t)$, and $e_{23} = \dot{\theta}_3(t)-\dot{\theta}_3^*(t)$ respectively. The reconstruction errors, associated to the tracking errors are defined as follows: $\tilde{e}_{01} = e_{01}-\hat{e}_{01}$, $\tilde{e}_{02} = e_{20}-\hat{e}_{02}$ and $\tilde{e}_{03} = e_{03}-\hat{e}_{03}$. Thus, the set of GPI observers for the reference tracking errors in the input delayed Delta robot are proposed as:

$$\begin{aligned}\dot{\hat{e}}_{0j} &= \hat{e}_{1j}+\lambda_{(p+2)j}\tilde{e}_{0j}\\ \dot{\hat{e}}_{1j} &= \hat{e}_{2j}+\lambda_{(p+1)j}\tilde{e}_{0j}\\ \dot{\hat{e}}_{0j} &= (K_1N/R_a)V_j(t-T)+\hat{z}_{1j}+\lambda_{pj}\tilde{e}_{0j}\\ \dot{\hat{z}}_{1j} &= \hat{z}_{2j}+\lambda_{(p-1)j}\tilde{e}_{0j}\\ \dot{\hat{z}}_{2j} &= \hat{z}_{3j}+\lambda_{(p-2)j}\tilde{e}_{0j}\\ &\vdots\\ \dot{\hat{z}}_{pj} &= \lambda_{0j}\tilde{e}_{0j}\\ j &= 1,2,3\end{aligned} \quad (18)$$

Considering an approximation parameter sufficiently large, say $p = 5$, the linear dominant part of the each injection error dynamics, is defined by the following characteristic polynomials expressed in terms of the Laplace operator, s:

$$p_{oi}(s) = s^8+\lambda_{7i}s^7+\lambda_{6i}s^6+\lambda_{5i}s^5+\lambda_{4i}s^4+ \lambda_{3i}s^3+\lambda_{2i}s^2+\lambda_{1i}s+\lambda_{0i} \quad (19)$$

The observer gain parameters λ_{ji}, for $i = 1, 2, 3$, and $j = 1, 2\cdots, 7$ are chosen in such a way that each characteristic polynomial of the injection dominant dynamics has its roots in the left half complex plane, sufficiently far of the imaginary axis. To achieve the last objective, Hurwitz polynomials of the form:

$$p_{oi}(s) = (s^2+2\zeta_i\omega_i s+\omega_i^2)^4 \quad (20)$$

are proposed as dominant characteristic polynomials of the closed loop dynamics. Using the truncated Taylor series expansion to predict the lumped disturbance functions $\psi_j(t+T)$, the following estimator is proposed:

$$\hat{\psi}_j(t+T) = \hat{z}_{1j}+\hat{z}_{2j}T+\hat{z}_{3j}\frac{T^2}{2!}+\hat{z}_{4j}\frac{T^3}{3!}+\hat{z}_{5j}\frac{T^4}{4!} \quad (21)$$

Now, let us define the errors associated to the prediction process and the Smith predictor control design:

$$e_{rr} = \begin{bmatrix}\theta_1-\theta_{1f}(t-T)\\\theta_2-\theta_{2f}(t-T)\\\theta_3-\theta_{3f}(t-T)\end{bmatrix},\quad e_f = \begin{bmatrix}\theta_{1f}(t)-\theta_1^*(t)\\\theta_{2f}(t)-\theta_2^*(t)\\\theta_{3f}(t)-\theta_3^*(t)\end{bmatrix} \quad (22)$$

Finally, the output feedback controller is given by:

$$V_j(t) = \frac{R_a(\frac{1}{3}m_a+m_b)a^2}{K_1N}\Big[-\hat{\psi}_j(t+T) -\kappa_{1j}\frac{d}{dt}(e_{fj}+e_{rrj})-\kappa_{0j}(e_{fj}+e_{rrj})\Big] \quad (23)$$

The controller includes a compensation for the disturbance prediction functions $\psi_j(t+T)$, $j = 1, 2, 3$. The cancelation is carried out through the disturbance observer extended states and the use of the tracking velocities $\dot{\hat{\theta}}_{1j}$. The controller gain parameters, κ_{0i}, κ_{1i}, are chosen such that the associated dominant characteristic polynomials of the closed loop systems: $p_j(s) = s^2+\kappa_{1j}s+\kappa_{0j}$, locate their roots deep into the left half of the complex plane. In particular, it can be proposed a location of the form:

$$s^2+2\zeta_{cj}\omega_{ncj}s+\omega_{ncj}^2 \quad (24)$$

with $\zeta_{cj}, \omega_{ncj} > 0$, to emulate stable responses of second order systems.

4. EXPERIMENTAL RESULTS ON A DELTA ROBOT

The position of each arm was obtained by means of incremental encoders, of 2000 pulses per revolution, to measure the angular position of each gear box shaft. The position data was sent to the main controller by means of a data acquisition board Sensoray Model 626. The control strategy was implemented in the Matlab-Simulink platform, and the devised control signals were transferred to the actuators through three power amplifiers Sanyo: Model STK4050II. The sampling time was set to be 0.001 [s], the input delay parameter $T = 0.010$ [s] was implemented by software using The Transport Delay block. The delta robot actuators were three dc geared motors NISCA: Model NC5475B. The motor parameters are: A torque constant $K_1 = 0.0724$ $[N - m/A]$, the armature resistance is $R_a = 2.983$ $[\Omega]$, the back electromotive force constant, $K_2 = 0.0687\,[N - m - s/rad]$, and the gear relation is $N = 16$. The delta robot system parameters

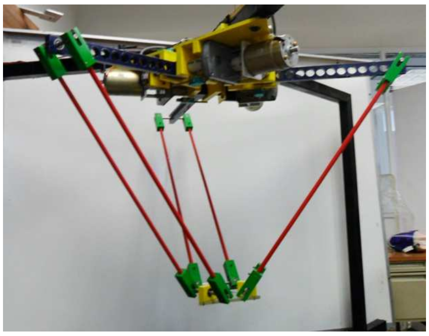

Fig. 3. Delta robot system prototype

are: $a = 0.25$[m], $b = 0.50$ [m], $h = 0.045$ [m], $r = 0.1$ [m], $m_a = 0.168$ [kg], $m_b = 0.3$ [kg], and $m_p = 0.215$ [kg]. The controller was applied to achieve a reference trajectory in the cartesian space, $x - y - z$. To start the main trajectory, the robot tracked a line between the initial point $(0, 0, -400)[mm]$, and the point $(200, 0, -400)[mm]$ in the $x - y - z$ space. Then, the trajectory was a circle centered at the origin of the $x - y$ plane, with radius $200[mm]$, for $z = 400[mm]$. The inverse kinematics was used to find the joint angles, θ_1, θ_2, and θ_3. The initial conditions for the joint variables in the robot were: $[\theta_1(0) = 0$, $\theta_2(0) = 0$, and $\theta_3(0) = 0$. The observer gain parameters were set to be as follows:

$$\begin{bmatrix} \zeta_{o1} \\ \zeta_{o2} \\ \zeta_{o3} \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} \qquad \begin{bmatrix} \omega_{o1} \\ \omega_{o2} \\ \omega_{o3} \end{bmatrix} = \begin{bmatrix} 20 \\ 22 \\ 20 \end{bmatrix}$$

The controller design parameters were specified to be:

$$\begin{bmatrix} \zeta_{c1} \\ \zeta_{c2} \\ \zeta_{c3} \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} \omega_{nc1} \\ \omega_{nc2} \\ \omega_{nc3} \end{bmatrix} = \begin{bmatrix} 13 \\ 13 \\ 13 \end{bmatrix}$$

Figure 3 shows the experimental delta robot test bed. The tracking results in the the $x-y-z$ space, obtained by the proposed Smith predictor based GPI output feedback controller are shown in Figure 4. Figure 5 depicts the behavior of the controller in each actuated joint. Figure 6 shows the control inputs (in voltage) for the tracking process. Last results show that the approximation is acceptable in spite of the input time delay and finally, figures 7, 8 depict the disturbance input predictors.

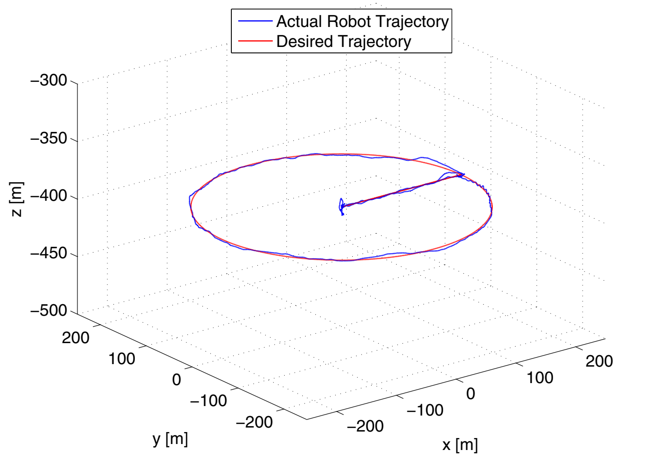

Fig. 4. x, y, z-directions reference trajectory tracking with observer based linear controller

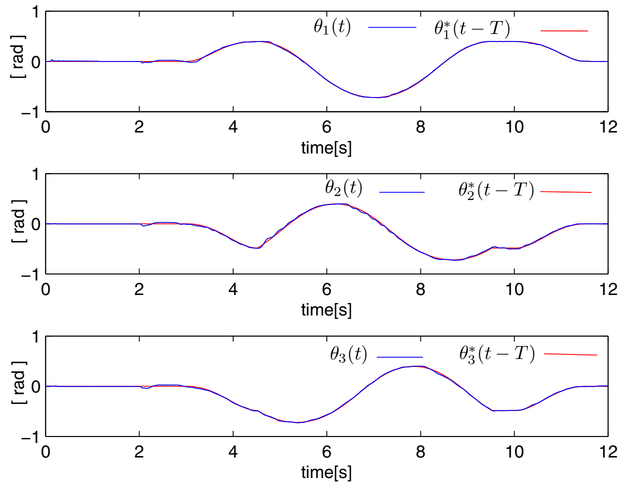

Fig. 5. Tracking behavior for the actuated joints

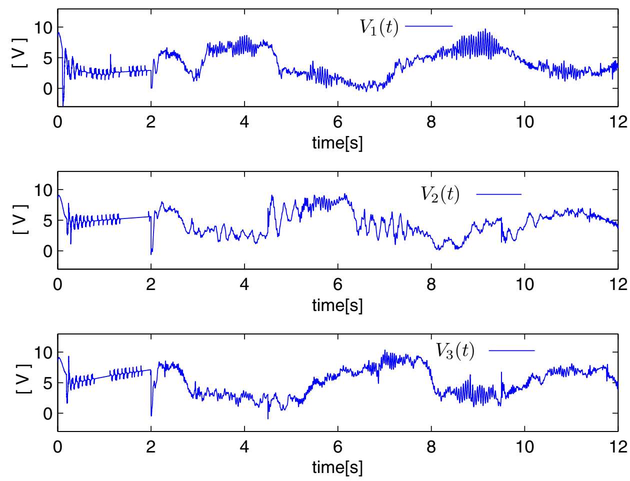

Fig. 6. Voltage control inputs

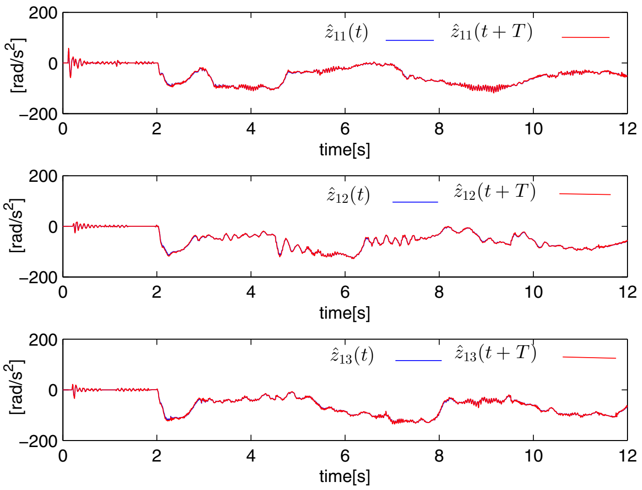

Fig. 7. Disturbance input predictor

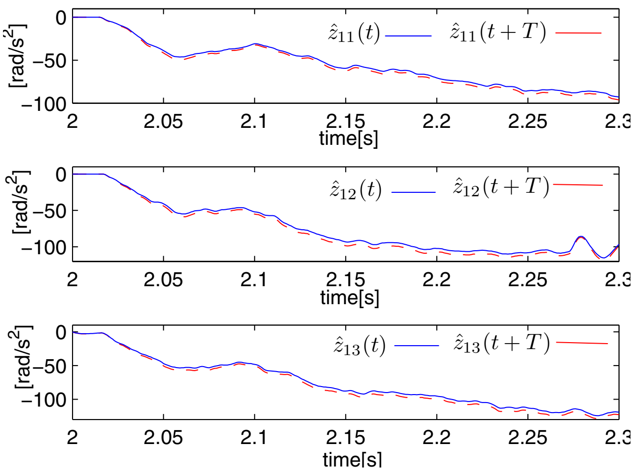

Fig. 8. Zoom on disturbance input predictor

5. CONCLUDING REMARKS

In this article, the observer based linear output feedback control for trajectory tracking on input delayed Delta Robot was solved using only a partial knowledge of its dynamics, which may contain external disturbance inputs. The problem was taken from an input output point of view, where the input-output description of the plant was modeled as a set of linear pure integration systems with a constant input gain matrix and the philosophy was the active disturbance rejection control, in which the problem is to reject all possible effects of additive disturbances (external and internal) lumped in a disturbance additive function. The known and fixed time delay influence was compensated by means of a GPI observer, using a truncated Taylor series expansion to approximately estimate the prediction of the lumped disturbance, which enabled the direct use of the Smith predictor control scheme in the simplified system. The experimental results showed that the tracking error was bounded, allowing good tracking tasks. As a future work, the proposed controller can be extended for the problem of bilateral tele-operation affected by known an fixed input delays. A strategy of control for variable and unknown delay inputs can be formulated in order to implement tele-operation based on Internet communications.

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