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Adaptive Polynomial Identification and Optimal Tracking Control for Polynomial Systems ⋆

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Abstract:
This paper proposes an adaptive polynomial identifier and a robust nonlinear optimal tracking control scheme for polynomial systems. In order to identify the dynamics of an uncertain and disturbed nonlinear system, the identified parameters are adapted online using a Kalman-Bucy filter algorithm. Based on the polynomial identifier, an infinite-time state-feedback optimal tracking controller is synthesized, which includes an integral term to provide robustness to the control strategy. This algorithm is an adaptive and robust control scheme. The effectiveness of the proposed identifier-controller strategy is illustrated via simulation for the identification and control of the blood glucose-insulin system by using the Bergman minimal model (BeM) to describe the dynamics of type 1 diabetic patients.

Keywords: Adaptive identification, polynomial systems, Kalman filter, nonlinear optimal control, glucose-insuline dynamics, type 1 diabetic patient.

1. INTRODUCTION

Nonlinear systems can be represented in a polynomial form and there are different polynomial basis that can be used to approximate the real systems into a polynomial form, such as Chebyshev polynomials (Madan and Seneta, 1987), Legendre polynomials (Funaro, 1992), etc., even more, there are different nonlinear systems with a natural polynomial structure. This paper considers only the systems that are in a polynomial structure and the identification scheme, which uses the measurements of the system state variables, can provide a polynomial model for a nonlinear system, which is assumed to be unknown.

The identification of dynamic models out of experimental data has very often been motivated and supported by the presumed ability to use the resulting models as a basis for model-based control design, whereby, the parameter identification of the systems is crucial for its control and stability. In system identification, emphasis has long been on aspects of consistency and efficiency, related towards the reconstruction of the real model that underlies the measurement data. In this case, the optimal identification has played a prevailing role in solving problems whose solutions would be hard to find, especially when the system information is unknown and is incapable of modeling complex dynamical systems effectively (Van Den Hof and Schrama, 1995).

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It is always of interest to synthesize efficient controllers in order to obtain a satisfactory behavior of the system variables, and save as much energy as possible to produce the desired motion. A feasible approach with respect to these two objectives is the optimal control. The optimal control approach has the aim of achieving an adequate performance of a control system by minimizing a meaningful cost functional, which is established to evaluate the response of the state variables and the control input expenditures. Different control strategies have been proposed to provide nonlinear feed-back controllers, including the state-dependent Riccati equation (SDRE) approach for state-dependent coefficient factorized (SDCF) nonlinear systems (Banks et al., 2007; Cloutier et al., 1996; Erdem, 2001). This control technique provides an effective algorithm for synthesizing nonlinear feedback controllers.

Indeed, the state-dependent Riccati equation (SDRE) technique is a systematic way for synthesizing nonlinear feedback controllers, which mimics the controller synthesis as done for the linear case. Applications of this technique can be seen in (Parrish and Ridgely, 1997) and (Zhang et al., 2005) and for state estimation in (Mracek and Cloutier, 1998) and (Souza, 2012). Additionally, in (Ornelas-Tellez et al., 2013) is established a mechanism for a class of nonlinear systems such that they can be transformed into a state-dependent factorized systems, via state variable embedding, and then using the SDRE approach to solve the optimal control.

This contribution proposes an optimal polynomial identification scheme to identify unknown nonlinear systems. The identifier parameters are adapted using a Kalman-Bucy filter. Once the nonlinear system is identified, an optimal tracking controller is synthesized based on the identifier. Additionally, an integral term is included to the optimal control scheme in order to deal with constant disturbances and parameter uncertainties. The proposed control scheme is illustrated via simulation for the glucose dynamical model developed by (Bergman et al., 1979).

This manuscript is organized as follows. Section 2 describes the polynomial identification for nonlinear systems. In Section 3 the robust optimal tracking control for polynomial systems to achieve trajectory tracking of the system output toward time-varying references is presented. The application of the identification-control scheme for the Glucose level regulation in diabetic patients is proposed in Section 4. Finally, Section 5 presents the conclusions.

2. POLYNOMIAL IDENTIFICATION

2.1 Nonlinear System

Consider the problem of approximation of a disturbed nonlinear system, whose dynamical behavior is given by

$$\begin{aligned} \dot{\mathcal{X}} &= F(\mathcal{X}, u) + \bar{\Gamma}, \qquad \mathcal{X}(t_0) = \mathcal{X}_0 \\ \mathcal{Y} &= \mathcal{C}(\mathcal{X}) \end{aligned} \tag{1}$$

where $\mathcal{X} \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the system input, $\mathcal{Y} \in \mathbb{R}^p$ is the system output; F and $\mathcal{C}$ are a smooth vector fields of appropriate dimensions. $\bar{\Gamma}$ is an unknown and bounded disturbance term representing modeling errors, uncertain parameters and unmodeled dynamics. In this contribution, the considered approximation problem consists of determining an adaptive polynomial identifier, which approximates the dynamical behavior of an dynamical system of the form (1).

2.2 Polynomial Identifier

In order to approximate system (1), the polynomial identifier is proposed as

$$\begin{aligned} \dot{x} &= f(x,\theta) + B(x,\theta)\, u + \Gamma(\theta), \qquad x(t_0) = x_0 \\ y &= h(x,\theta) \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^p$ is the system output; functions $f(x,\theta)$, $B(x,\theta)$ and $h(x,\theta)$ are smooth maps of appropriate dimensions, where θ is the identifier parameters vector, which is on-line adapted such that convergence of identifier (2) toward system (1) is achieved. Term $\Gamma(\theta)$ represents possible non-state-dependent additional parameters for the identifier.

One of the most important features of the polynomial structure considered in this work, specifically for functions $f(x,\theta)$ and $h(x,\theta)$ in (2), is that they always admit the state-dependent coefficient factorization as $f(x,\theta) = A(x,\theta)\, x$ and $h(x,\theta) = C(x,\theta)\, x$, respectively [1]. Then, system (2) can be rewritten as

$$\begin{aligned} \dot{x} &= A(x,\theta)\, x + B(x,\theta)\, u + \Gamma(\theta) \\ y &= C(x,\theta)\, x. \end{aligned} \tag{3}$$

It is worth mentioning that, due to identification process, matrices $A(x,\theta)$, $B(x,\theta)$ and $C(x,\theta)$ are proposed to be linear with respect to the entries of vector θ and polynomial with respect to x. In this sense, different polynomial basis can be used to approximate the vector field $F(\mathcal{X}, u)$ in (1), such as Chebyshev polynomials, Legendre polynomials, etc. In addition to this, there are different nonlinear systems with a natural polynomial structure (Basin et al., 2011; Bold et al., 2003; Gokdere and Simaan, 1997).

2.3 Adaptation Algorithm

Least-squares (LS) algorithms are successful procedures to adjust parameters, which can be derived by different methods. Thus, for the on-line adaptation of the identifier parameters we apply an LS algorithm, the so-called *Kalman-Bucy filter* (KBF) (Sastry and Bodson, 2011), used as a state estimator (Bellgardt et al., 1986), for which the parameters become the states to be estimated. The main objective of the KBF is to determine the optimal values for the parameter vector θ such that the identification error $e_i = x - \mathcal{X}$ is minimized.

The dynamics of the unknown parameters vector θ^*, which would produce the minimum identification error, can be described by

$$\dot{\theta}^* = 0 \tag{4}$$

with output

$$y_i^* = x^* \tag{5}$$

where the ideal state vector x^* depends on θ^* in accordance to its respective dynamics from (2). Assuming that the right-hand sides of (4) and (5) are perturbed by zero mean white Gaussian noises of spectral intensities $\Psi \in \mathbb{R}^{2n\times 2n}$ and $1/g \in \mathbb{R}$, respectively, the KBF is given by

$$\begin{aligned} \dot{\theta} &= -g\, \Phi\, w\, e_i \\ \dot{\Phi} &= \Psi - g\, \Phi\, w\, w^T \Phi, \qquad \Psi,\ g > 0 \end{aligned} \tag{6}$$

where θ is the estimated value for θ^* and vector w contains the selected polynomial basis to approximate the vector field $F(\mathcal{X}, u)$. Indeed, Ψ and g are fixed design parameters of the algorithm to ensure the identification error convergence. Matrix Φ is called the covariance matrix and acts in the θ updating law as a directional adaption gain. The initial condition for Φ is $\Phi(0) > 0$, whereas for θ is arbitrary. $\Phi(0)$ is usually chosen to reflect the confidence in the initial estimate of $\theta(0)$. In general, it is recommended to select a large value for $\Phi(0)$. Note that the adaptation algorithm (6) must be implemented for each system state variable.

[1] For instance, the polynomial scalar system $\dot{x} = -x + x^3$ can be presented as $\dot{x} = a(x)x$, with $a(x) = (x^2 - 1)$.

3. ROBUST OPTIMAL TRACKING CONTROL FOR POLYNOMIALS SYSTEMS

In order to synthesize the optimal controller, we will use the salient feature of the state-dependent representation for (3), such that an analytical solution for the optimal tracking control, via the Riccati equation, can be obtained. In optimal tracking control, the output of the system is required to track a desired trajectory as close as possible in an optimal sense and with minimum control effort expenditure (Anderson and Moore, 1990a). In order to introduce the trajectory tracking, the error is defined as

$$\begin{aligned} e &= r - y \\ &= r - C(x,\theta)\,x \end{aligned} \tag{7}$$

where r is the desired reference to be tracked by the system output y. The quadratic cost functional J to be minimized, associated with system (3), is defined as

$$J = \frac{1}{2}\int_{t_0}^{\infty} \left(e^T Q\, e + u^T R\, u\right)\, dt \tag{8}$$

where Q and R are symmetric and positive definite matrices. Q is a matrix weighting the performance of the state vector x, meanwhile R is a matrix weighting the control effort expenditure; hence these matrices are used to establish a trade-off between state performance and control effort (Kirk, 1970). If more importance is given to the system state performance, one can select a higher value for Q or reduce R. If one is more interested in saving control energy, it is suggested a lower value for Q is selected or R is increased (Athans and Falb, 1966b) and (Anderson and Moore, 1990b). Particularly in (Athans and Falb, 1966b), the entries of these matrices are selected such that physical constraints for states and control signals are included in the control scheme.

3.1 Optimal Tracking Control

At this point, the optimal tracking control solution is established, by assuming the nonexistence of the disturbance (i.e. $\Gamma(\theta) = 0$) for (3), and omitting the parameter dependence of θ to simplify the notation in all system functions. After that, the robust optimal tracking control is stated in the next section.

By considering that the state is available for feedback, the optimal tracking solution is established as the following theorem.

Theorem 1. Assume that system (3), with $\Gamma(\theta) = 0$, is state dependent controllable and state-dependent observable. Then the optimal control law

$$u^*(x) = -R^{-1}B^T(x)\ \left(P(x)\,x - z(x)\right) \tag{9}$$

achieves trajectory tracking for system (3) along the desired trajectory r, where $P(x)$ is the solution of the equation

$$\begin{aligned} \dot{P}(x) = &-C^T(x)\ Q\,C(x) + P(x)\,B(x)\,R^{-1}\,B^T(x) \\ &\times P(x) - A^T(x)\,P(x) - P(x)\,A(x) \end{aligned} \tag{10}$$

and $z(x)$ is the solution of the equation

$$\begin{aligned} \dot{z}(x) = &-\left[A(x) - B(x)\,R^{-1}B^T(x)\,P(x)\right]^T z(x) \\ &-C^T(x)\ Q\,r \end{aligned} \tag{11}$$

with boundary conditions $P(x(\infty)) = 0$ and $z(x(\infty)) = 0$, respectively. Control law (9) is optimal in the sense that it minimizes the cost functional (8), which has an optimal value function given as

$$J^* = \frac{1}{2}x^T(t_0)\,P(x(t_0))\,x(t_0) - z^T(x(t_0))\,x(t_0) + \varphi(t_0) \tag{12}$$

where φ is the solution to the scalar differentiable function

$$\dot{\varphi} = -\frac{1}{2}r^T Q\,r + \frac{1}{2}z^T\,B(x)\,R^{-1}B^T(x)\,z \tag{13}$$

with $\varphi(\infty) = 0$.

Proof. For proof details see (Ornelas-Tellez et al., 2013).

Matrix (10) and vector (11) need to be solved backward in time, nonetheless in (Lewis and Syrmos, 1995) and (Weiss et al., 2012) a change of variable is presented such that these equations can be solved forward in time. This is done by multiplying a minus sign to the right-hand side of (10) and (11).

3.2 Robust Optimal Tracking Control

This controller design considers the case when the disturbance term $\Gamma(\theta)$ is part of system (3); then, an integral term of the tracking error e can be included such that the disturbance is rejected. Thus, the integral term for the tracking error is defined as

$$\dot{q} = -e \tag{14}$$

where $q \in \mathbb{R}^p$ is a state vector of integrators for a system with p outputs.

The augmented system, with state vector $x_a = [q^T,\ x^T]^T$, can be established as

$$\begin{aligned} \dot{x}_a &= \begin{bmatrix} \dot{q} \\ \dot{x} \end{bmatrix} \\ &= \begin{bmatrix} -e \\ A(x)x + B(x)u + \Gamma \end{bmatrix} \\ &= \begin{bmatrix} C(x)x - r \\ A(x)x + B(x)u + \Gamma \end{bmatrix}. \end{aligned} \tag{15}$$

System (15) can be rewritten as

$$\dot{x}_a = A_a(x_a)\,x_a + B_a(x_a)\,u + D_a \tag{16}$$

where $A_a(x_a) = \begin{bmatrix} 0 & C(x) \\ 0 & A(x) \end{bmatrix}$, $B_a(x_a) = \begin{bmatrix} 0 \\ B(x) \end{bmatrix}$, $C_a = [\,q \quad C(x)\,]$ and $D_a = \begin{bmatrix} -r \\ \Gamma \end{bmatrix}$.

For system (16), let us consider the problem of minimizing the cost functional

$$J_a = \frac{1}{2}\int_{t_0}^{\infty} \left(q^T Q_I\, q + e^T Q\, e + u^T R\, u\right) \tag{17}$$

$$= \frac{1}{2}\int_{t_0}^{\infty} \left(\left[q^T\ e^T\right] \begin{bmatrix} Q_I & 0 \\ 0 & Q \end{bmatrix} \begin{bmatrix} q \\ e \end{bmatrix} + u^T R\, u\right) \tag{18}$$

where Q_I is a weighting parameter which determines the speed convergence of the error in the control law, and is considered as the integrator gain.

$$u^*(x_a) = -R^{-1}B_a^T(x_a)\,\left(P_a(x_a)\,x_a - z_a(x_a)\right) \tag{19}$$

with $P_a(x_a)$ and $z_a(x_a)$ defined accordingly to the established in Section 3.1. Note that the controller (19) is a feedback of the state x and the integrator q.

4. APPLICATION OF THE OPTIMAL POLYNOMIAL IDENTIFICATION AND ROBUST OPTIMAL TRACKING CONTROL FOR THE GLUCOSE-INSULIN SYSTEM

4.1 Bergman model

The Bergman Model (BeM) is an example of a nonlinear systems with a natural polynomial structure, which describes the glucose-insulin regulatory system. This system is described as (Bergman et al., 1979)

$$\dot{G} = -p_1G - XG + p_1G_b + D \tag{20}$$

$$\dot{X} = -p_2X + p_3(I - I_b) \tag{21}$$

$$\dot{I} = -\eta(I - I_b) + \gamma(G - h)t + u. \tag{22}$$

Here G, X and I are plasma glucose concentration, the insulin influence on glucose concentration reduction, and insulin concentration in plasma, respectively. The control input u represents the insulin infusion rate, p_1 is the insulin-independent glucose-utilization rate, p_2 is the rate of decrease of the tissue glucose uptake ability, p_3 is the insulin-dependent increase of the glucose uptake ability and D is the disturbance in glucose levels caused by the meal. The term $\gamma(G-h)t$ represents the pancreatic insulin secretion after a meal intake at $t = 0$. As this work is focused on insulin therapy which is usually administrated to Type 1 diabetes mellitus patients, γ is assumed to be zero to represent the true dynamic of this disease (Fisher, 1991). The parameter η is the first order decay rate for insulin in blood. The parameters to simulate the BeM in silico patients were obtained from (Kaveh and Shtessel, 2008), see Table 1.

Table 1. BeM parameters

Variable	Patient	Units
p_1	0.1082	$1/min$
p_2	0.02	$1/min$
p_3	5.3×10 6	$ml/\mu U min^2$
η	0.2659	$1/min$
G_b	110	mg/dl
I_b	90	$\mu U/ml$

4.2 Polynomial identification for the Bergman minimal model

Since system (20)-(22) has a polynomial structure, we use such structure to design the adaptable identifier. Then, the identifier results as

$$\dot{x}_1 = \theta_1x_1 - x_2x_1 + \theta_2 \tag{23}$$

$$\dot{x}_2 = \theta_3x_2 + \theta_4x_3 + \theta_5 \tag{24}$$

$$\dot{x}_3 = \theta_6x_3 + \theta_7 + u \tag{25}$$

where the state vector is $x = [x_1\ x_2\ x_3]^T = [G\ X\ I]^T$, $A(x,\theta) = \begin{bmatrix} \theta_1 & -x_1 & 0 \\ 0 & \theta_2 & \theta_4 \\ 0 & 0 & \theta_6 \end{bmatrix}$, $B(x,\theta) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $C(x,\theta) = [1\ \ 0\ \ 0]$, and $\Gamma(\theta) = \begin{bmatrix} \theta_2 \\ \theta_5 \\ \theta_7 \end{bmatrix}$, where $\theta = [\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6\ \theta_7]^T$ are the parameters to be identified by the Kalman-Bucy algorithm. System (23)-(25) can be decomposed in the state-dependent coefficient factorization as required in (3), for which the robust optimal tracking control can be applied.

4.3 Robust optimal tracking control applied to the BeM

The BeM's output is the blood glucose level (G), then, only is required add an integrator term q for the augmented system. In this case the augmented system with state vector $x_a = [q^T,\ x^T]^T$ can be established as

$$x_a = \begin{bmatrix} q \\ x \end{bmatrix} = \begin{bmatrix} q \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{26}$$

$$= \begin{bmatrix} -e \\ A(x,\theta)x + B(x,\theta)u + \Gamma \end{bmatrix} \tag{27}$$

$$= \begin{bmatrix} x_1 - r \\ A(x,\theta)x + B(x,\theta)u + \Gamma \end{bmatrix}. \tag{28}$$

System (28) can be rewritten as

$$\dot{x}_a = A_a\left(x_a,\theta\right)x_a + B_a\left(x_a,\theta\right)u + \Gamma \tag{29}$$

where $A_a\left(x_a,\theta\right) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \theta_1 & -x_1 & 0 \\ 0 & 0 & \theta_2 & \theta_4 \\ 0 & 0 & 0 & \theta_6 \end{bmatrix}$, $B_a\left(x_a,\theta\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $C_a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ and $\gamma_a = \begin{bmatrix} -r \\ \theta_2 \\ \theta_5 \\ \theta_7 \end{bmatrix}$, with r as the reference glucose level, and the cost functional to be minimized is (18). For the augmented system, the optimal controller is given by (19). Once the identifier has converged, the optimal control law is applied, this is at time $t \geq 80$.

4.4 Simulation results

The effectiveness of the proposed identifier-controller strategy is illustrated via simulation. The parameters used in the simulation for the BeM are given in Table 1. The initial conditions for the BeM are $G(0) = 2G_b$, $I(0) = 2I_b$ and $X(0) = 0$. In the process of identification the basis w in (6) are $w_1 = [x_2\ 1]^T$, $w_2 = [x_3\ x_4\ 1]^T$ and $w_3 = [x_4\ 1]^T$. The identifier parameters to ensure the identification error convergence to zero are $\Psi_1 = diag\{0.05,\ 10\}$, $\Psi_2 = diag\{5,\ 5,\ 5\}$, $\Psi_3 = diag\{10,\ 10\}$ and $g_1 = 100$, $g_2 = 10000$ and $g_3 = 10000$. The parameters for the robust optimal tracking controller, which determine the speed convergence of the error in the control law are $Q_I = 0.5$, $Q = 10$ and $R = 1$. The parameters are selected heuristically such that an adequate performance of the control system is achieved. The next phase in the research is to obtain a formal proof to select the optimal parameters of the controller and identifier.

Fig. 1(a) shows the identification for the basal glucose response, corresponding to the Bergman model (G), by means of the proposed identifier x_1.

Fig. 1(b) shows the identification of the effect of active insulin response, corresponding to the Bergman model (X), by means of the proposed identifier x_2.

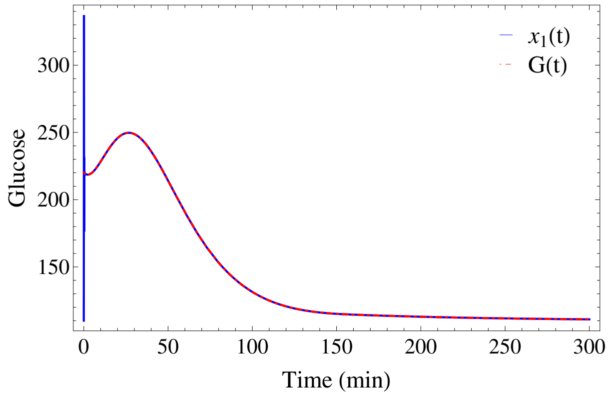
Fig. 1(c) shows the identification of the basal insulin response, corresponding to the Bergman model (I), by means of the proposed identifier x_3.

Fig. 2(a) shows the glucose level regulation with a reference level $r = 110$ mg/dl. Fig. 2(b) shows the control signal (u) that represents the required level of exogenous insulin to keep the blood glucose on the reference level established for a type 1 diabetic patient.

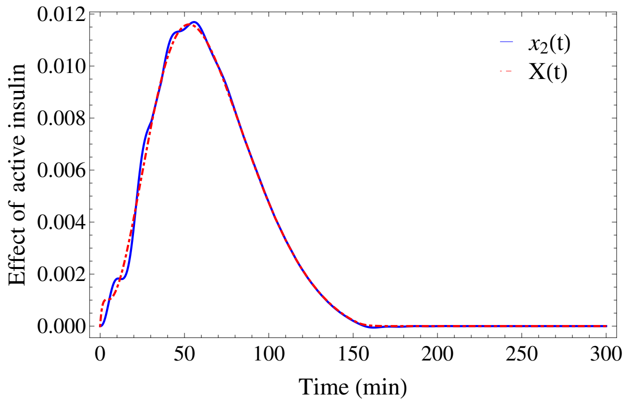
Finally, Fig. 3 illustrates the capabilities of the optimal control scheme for regulating the glucose level to different reference values. Fig. 3 shows the reference levels for different intervals of time (r), i.e., for $t < 250$ the reference level is $r = 115$ mg/dl. The same way, for $250 \leq t < 450$ the reference level is $r = 100$ mg/dl, for $450 \leq t < 650$ the reference level is $r = 120$ mg/dl and the last time interval is $t \geq 650$ with reference level $r = 110$ mg/dl.

5. CONCLUSION

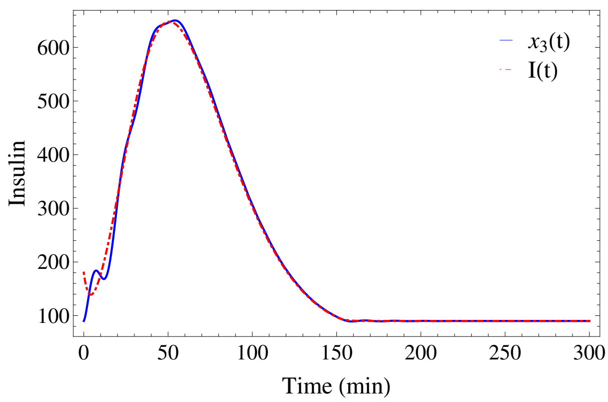
This paper applies an optimal identifier and a robust optimal tracking control for the BeM glucose-insulin system. The simulation results show that the proposed robust optimal control can regulate the glucose level for a type 1 diabetic patient. The robust optimal tracking control proved its effectiveness via simulations, where the glucose level reach the desired reference levels by applying the proposed controller. The proposed identification and control schemes have advantages, such as: they can work with disturbances affecting the system, an adequate identification can be obtained unknown nonlinear systems. Currently, this work is focused on the formal proof of the identifier convergence.



(a) Optimal identification for Glucose



(b) Optimal identification for Effect of active insulin



(c) Optimal identification for Insulin

Fig. 1. Optimal identifier based on the Kalman-Bucy filter applied to the model of Bergman.

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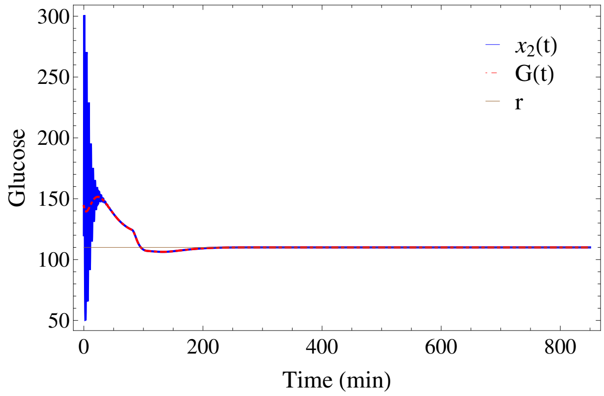
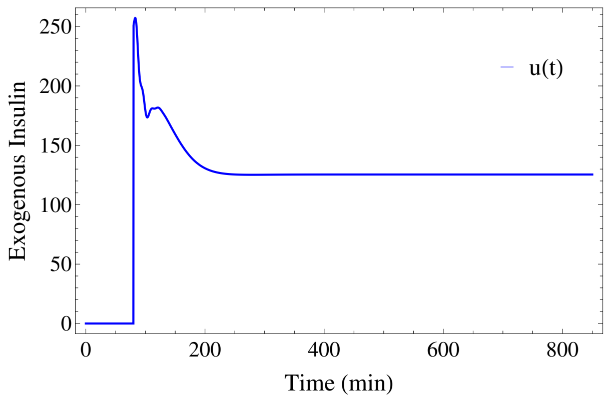


(a) Glucose signal at reference level r.



(b) Control signal to maintain the glucose over the reference level r.

Fig. 2. Adaptive identification and optimal tracking control applied for the BeM to maintain the glucose level over an established reference level.

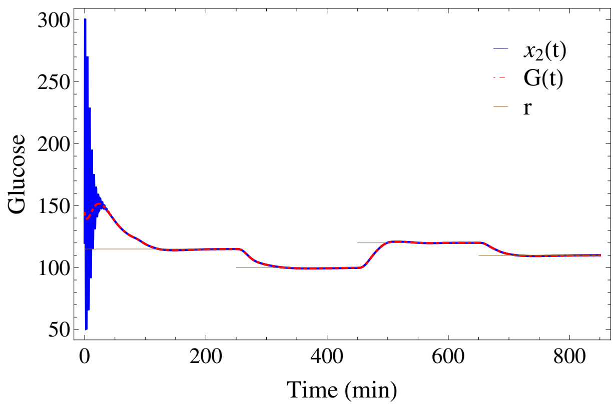


Fig. 3. Adaptive identification and optimal tracking control applied for the BeM to maintain the glucose level over different reference levels.

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