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Improving control of quadrotors carrying a manipulator arm

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Abstract: This paper proposes a simple solution for asymptotical stabilization of a quadrotor carrying a manipulator arm. The manipulator arm is attached to the lower part of the fuselage of a quadrotor to increase the type of missions achievable. The motion of the arm induces torques and disturbances to the quadrotor causing the loss of stability. This work deals with the stabilization of the quadrotor under such disturbances, by means of a bounded quaternion-based feedback. A set of nonlinear control laws are obtained. First, the attitude control stabilizes the quadrotor to a desired position and angular velocities. Then, using a suitable virtual control, the translational dynamics becomes into three independent chains integrators allowing the formulation of a nonlinear control. Both controls consist in saturation functions and allow the stabilization of the quadrotor. Simulations show the effectiveness of the proposed algorithm.

Keywords: Nonlinear control, bounded stabilization, quadrotor helicopter, manipulator.

1. INTRODUCTION

Aerial manipulation has been an active area of research in recent times, mainly because the active tasking of Unmanned Aerial Vehicles (UAVs) increases the employability of these vehicles for various applications. For active tasking one would consider manipulation, grasping and transporting etc.
Unlike fixed wings UAVs that are incapable of driving their velocity to zero, VTOL (Vertical Take-Off and Landing) vehicles such as helicopters with four rotors (afterwards called quadrotors) are ideally suited to the task of aerial manipulation or grasping. However, there are many challenges in aerial grasping for quadrotors. The biggest challenge arises from their limited payload. While multiple robots can carry payloads with grippers (Mellinger et al. (2010)) or with cables (Michael et al. (2011)), their end effectors and grippers have to be light weight themselves and capable of grasping complex shapes. Secondly, the dynamics of the robot are significantly altered by the addition of payloads. Indeed this is also an attraction in assembly because aerial robots can use this to sense disturbance forces and moments. However, for payload transport, it is necessary that the robots are able to estimate the inertia of the payload and adapt to it to improve tracking performance.
Numerous approaches have been proposed to deal with such a problem. In Orsag et al. (2013b) and Khalifa et al. (2012), a Newton-Euler approach is used to model and control a manipulator based quadrotor. In Orsag et al. (2013a), a Lyapunov based model Reference Adaptive Control is used to stabilize a quadrotor with multi degree of freedom (DOF) manipulator. However, the stability analysis is carried out with a linear approach and only rigid body dynamics of the quadrotor were considered due to the complexity of the system.
In Ghadiok et al. (2011), indoor experiments are performed with a quadrotor equipped with a gripper, where an IR camera is used to grip an object with LED placed on it. In Ghadiok et al. (2012), the experiments are extended to outdoor, using a GPS system and a Kalman filter to improve the precision in the position system. But both contributions are limited to the use of a 1-DOF gripper, which reduces the precision of manipulation.
In Lipiello and Ruggiero (2012a,b), Cartesian impedance control and redundancy are studied using Euler-Lagrange formulation. Jimenez-Cano et al. (2013) presents a Newton-Euler approach to model and control a quadrotor through a Variable Parameter Integral Backstepping (VPIB). However, the parametrization of the system is made through Euler angles, which present attitude estimation singularities.
The contribution of this paper is centered on the asymptotical stabilization of a quadrotor carrying a manipulator arm, attending the problem of the estimation of the mass of a payload, which add robustness to the proposed approach. Also, contrary to the above mentioned research, the design of the attitude control law uses the quaternion parametrization, which avoid the presence of singularities.
The paper is structured as follows. In section 2, the attitude model of the quadrotor with the manipulator arm is given. Then, the attitude control design is formulated in section 3, where its stability is proved. The section 4 is

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devoted to the load mass estimation. Section 5 presents simulation results, which show the effectiveness of the proposed algorithm. Finally, in section 6 some conclusions are presented.

2. SYSTEM MODELING

2.1 Unit quaternion and attitude kinematics

Consider two orthogonal right-handed coordinate frames: the body coordinate frame, $B(x_b, y_b, z_b)$, located at the center of mass of the rigid body and the inertial coordinate frame, $N(x_n, y_n, z_n)$, located at some point in the space (for instance, the earth NED frame). The rotation of the body frame B with respect to the fixed frame N is represented by the attitude matrix $R \in SO(3) = \{R \in \mathbb{R}^{3\times3} : R^T R = I, \det R = 1\}$.
The cross product between two vectors $\xi, \varrho \in \mathbb{R}^3$ is represented by a matrix multiplication $[\xi^\times]\varrho = \xi \times \varrho$, where $[\xi^\times]$ is the well known skew-symmetric matrix.

The n-dimensional unit sphere embedded in $\mathbb{R}^{n+1}$ is denoted as $\mathbb{S}^n = \{x \in \mathbb{R}^{n+1} : x^T x = 1\}$. Members of $SO(3)$ are often parameterized in terms of a rotation $\beta \in \mathbb{R}$ about a fixed axis $e_v \in \mathbb{S}^2$ by the map $\mathcal{U} : \mathbb{R}\times\mathbb{S}^2 \to SO(3)$ defined as

$$\mathcal{U}(\beta, e_v) := I_3 + \sin(\beta)[e_v^\times] + (1 - \cos(\beta))[e_v^\times]^2 \tag{1}$$

Hence, a unit quaternion, $q \in \mathbb{S}^3$, is defined as

$$q := \begin{pmatrix} \cos\frac{\beta}{2} \\ e_v \sin\frac{\beta}{2} \end{pmatrix} = \begin{pmatrix} q_0 \\ q_v \end{pmatrix} \in \mathbb{S}^3 \tag{2}$$

where $q_v = (q_1 \quad q_2 \quad q_3)^T \in \mathbb{R}^3$ and $q_0 \in \mathbb{R}$ are known as the vector and scalar parts of the quaternion respectively. The quaternion q represents an element of $SO(3)$ through the map $\mathcal{R} : \mathbb{S}^3 \to SO(3)$ defined as

$$\mathcal{R} := I_3 + 2q_0[q_v^\times] + 2[q_v^\times]^2 \tag{3}$$

Remark 2.1. $R = \mathcal{R}(q) = \mathcal{R}(-q)$ for each $q \in \mathbb{S}^3$, *i.e.* even quaternions q and $-q$ represent the same physical attitude.

Denoting by $w = (w_1 \quad w_2 \quad w_3)^T$ the angular velocity vector of the body coordinate frame, B relative to the inertial coordinate frame N expressed in B, the kinematics equation is given by

$$\begin{pmatrix} \dot{q}_0 \\ \dot{q}_v \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -q_v^T \\ I_3 q_0 + [q_v^\times] \end{pmatrix} w = \frac{1}{2}\Xi(q)w \tag{4}$$

The attitude error is used to quantify mismatch between two attitudes. If q defines the current attitude quaternion and q_d is the desired quaternion, i.e. the desired orientation, then the error quaternion that represents the attitude error between the current orientation and the desired one is given by

$$q_e := q_d^{-1} \otimes q = (q_{e_0} \; q_{e_v}^T)^T \tag{5}$$

where q^{-1} is the complementary rotation of the quaternion q which is given by $q^{-1} := (q_0 \quad -q_v^T)^T$ and $\otimes$ denotes the quaternion multiplication, Shuster (1993).

2.2 Model of a quadrotor carrying a manipulator arm

The attitude dynamics and kinematics for the quadrotor have been reported in many works *e.g.* Castillo et al. (2004), Guerrero-Castellanos et al. (2008). In these works the model considers that the quadrotor mass distribution is symmetric. However, the mass distribution of a quadrotor with a manipulator is no longer symmetrical and varies with the movement of the arm. Consider a quadrotor with a manipulator arm with n links attached to its lower part. If the dynamics of the arm is neglected, the attitude kinematics and dynamics is given by

$$\begin{pmatrix} \dot{q}_0 \\ \dot{q}_v \end{pmatrix} = \frac{1}{2}\Xi(q)\boldsymbol{w} \tag{6}$$

$$J\dot{\boldsymbol{w}} = -\boldsymbol{w}^\times J\boldsymbol{w} + \Gamma_T \tag{7}$$

where $J \in \mathbb{R}^{3\times3}$ is the symmetric positive definite constant inertial matrix of the rigid body expressed in the body frame B and $\Gamma_T \in \mathbb{R}^3$ is the vector of applied torques. Γ_T depends on the couples generated by the actuators (control couples), aerodynamic couples such as gyroscopic couples, gravity gradient or, as in the case of the present work, the couple generated by the movement of a robot manipulator placed under the body. In the present work only the control couples, gyroscopic couples and this one generated by the manipulator is considered in the control design. Consequently,

$$\Gamma_T = \Gamma + \Gamma_{arm} + \Gamma_G \tag{8}$$

where Γ and Γ_G will be described in the section 2.3. On the other hand, the vector Γ_{arm} is the torque generated by the total propulsive force being applied at the quadrotor geometric center which is displaced from the center of mass, Sangbum (2002). This torque can be computed by

$$\Gamma_{arm} = m_q g \zeta_c \times \mathcal{R}(q)e_3 \tag{9}$$

where m_q is the mass of the quadrotor, g is the acceleration due to gravity, $\zeta_c = (\zeta_{cx} \; \zeta_{cy} \; \zeta_{cz})^T \in \mathbb{R}^3$ is the position of center of mass of the quadrotor with respect to the pivot point, $\mathcal{R}(q)$ is the rotation matrix and $e_3 = (\,0 \;\, 0 \;\, 1\,)^T$. The center of mass can be computed by

$$\zeta_c = \frac{1}{m_a}\left[\sum_{i=1}^{n} m_i \varrho_i + m_l \varrho_l\right] \tag{10}$$

where $m_a = \Sigma_{i=1}^n m_i + m_l$ is the total mass of the manipulator and the load; m_i is the mass of each link of the manipulator and m_l is the mass of the load. Finally ϱ_i and ϱ_l are the position vector of each link of the manipulator and the load, respectively, both with respect to the reference body frame given by the quadrotor.

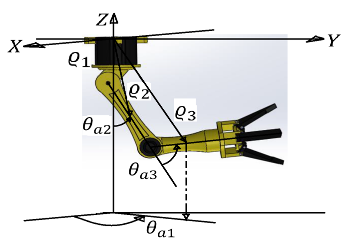

Fig. 1. Manipulator arm with three degrees of freedom.

For our case, let's consider the scheme in Fig. 1, which shows an anthropomorphic arm manipulator. This system has three degrees of freedom and then, the corresponding ϱ_i, where $i = \{1, 2, 3\}$, is given by

$$\begin{aligned}
\varrho_1 &= [\,0\ 0\ 0\,]^T \\
\varrho_2 &= [\,l_{c1}\sin\theta_{a2}\cos\theta_{a1}\ \ l_{c1}\sin\theta_{a2}\sin\theta_{a1}\ \ -l_{c1}\cos\theta_{a2}\,]^T \\
\varrho_3 &= [(l_1 + l_{c2})\sin(\theta_{a2}+\theta_{a3})\cos\theta_{a1} \\
&\quad (l_1 + l_{c2})\sin(\theta_{a2}+\theta_{a3})\sin\theta_{a1}\ \ (-l_1 - l_{c2})\cos(\theta_{a2}+\theta_{a3})]^T \\
\varrho_l &= [(l_1 + l_2)\sin(\theta_{a2}+\theta_{a3})\cos\theta_{a1} \\
&\quad (l_1 + l_2)\sin(\theta_{a2}+\theta_{a3})\sin\theta_{a1}\ \ (-l_1 - l_2)\cos(\theta_{a2}+\theta_{a3})]^T
\end{aligned} \tag{11}$$

where l_{c1} and l_{c2} are the distances from the respective joint axes to the center of mass of each link, l_1 and l_2 are the total length of the link, and θ_{ai} measures the angular displacement from z and x axes.
Due to space constraints the detail of the derivation of the dynamics of the manipulator arm coupled with the quadrotor dynamics is not developed here. However, it is worth to mention that the detailed model is used for the closed-loop simulations presented in the section 5.

2.3 Actuator model

The collective input (or throttle input) is the sum of the thrusts of each rotor f_1, f_2, f_3, f_4. Therefore, the reactive couple Q_j generated in the free air by rotor j due to the motor drag and the total thrust T produced by the four rotors can be, respectively, approximated by

$$Q_j = k s_j^2 \tag{12}$$

$$T = \sum_{j=1}^{4} f_j = b \sum_{j=1}^{4} s_j^2 \tag{13}$$

where s_j represents the rotational speed of rotor j. $k > 0$ and $b > 0$ are two parameters depending on the density of air, the radius, the shape, the pitch angle of the blade and other factors (Castillo et al. (2004)). The vector of gyroscopic couples Γ_G is given by

$$\Gamma_G = \sum_{j=1}^{4} J_r(\boldsymbol{w} \times \boldsymbol{z}_b)(-1)^{j+1} s_j \tag{14}$$

where J_r is the inertia of the so-called rotor (composed of the motor rotor itself with the gears). The components of the control torque $\Gamma \in \mathbb{R}^3$ generated by the rotors are given by $\Gamma = [\Gamma_1\ \Gamma_2\ \Gamma_3]^T$, with

$$\Gamma_1 = d(f_2 - f_4) = db(s_2^2 - s_4^2) \tag{15}$$

$$\Gamma_2 = d(f_1 - f_3) = db(s_1^2 - s_3^2) \tag{16}$$

$$\Gamma_3 = Q_1 + Q_2 + Q_3 + Q_4 = k(s_1^2 + s_2^2 + s_3^2 + s_4^2) \tag{17}$$

3. ATTITUDE CONTROL DESIGN

3.1 Problem statement

The objective is to design a control law which drives the quadrotor to attitude stabilization under the torques and moments exerted to this from the movement of a manipulator arm attached to its lower part. In other words, let q_d denote the constant quadrotor stabilization orientation, then the control objective is described by the following asymptotic conditions

$$q_e \to [\pm 1\ 0\ 0\ 0]^T,\ w \to 0 \text{ as } t \to \infty \tag{18}$$

Furthermore, it is known that actuator saturation reduces the benefits of the feedback. When the controller continuously outputs infeasible control signals that saturate the actuators, system instability may follow. Then, besides the asymptotic stability, the control law also takes into account the physical constraints of the control system, in order to apply only feasible control signals to the actuators.

3.2 Attitude control with manipulator arm

In this subsection, a control law that stabilizes the system described by (6) and (7) is proposed. The goal is to design a control torque that is bounded.

Definition 3.1. Given a positive constant M, a continuous, nondecreasing function $\sigma_M : \mathbb{R} \to \mathbb{R}$ is defined by

$$\begin{aligned}
&(1)\sigma_M = s \text{ if } |s| < M; \\
&(2)\sigma_M = \text{sign}(s)M \text{ elsewhere;}
\end{aligned} \tag{19}$$

Note that the components of Γ_{arm_i} are always bounded, *i.e.* $|\,\Gamma_{arm_i}\,| < \delta_i$. Then, one has the following result.

Theorem 3.2. Consider a rigid body rotational dynamics described by (6) and (7) with the following bounded control inputs $\Gamma = (\Gamma_1\ \Gamma_2\ \Gamma_3)^T$ such that

$$\Gamma_i = -\sigma_{M_{i2}}\left(\Gamma_{arm_i} + \sigma_{M_{i1}}(\lambda_i[w_i + \rho_i q_i])\right) \tag{20}$$

with $i \in \{1, 2, 3\}$ and where $\sigma_{M_{i1}}$ y $\sigma_{M_{i2}}$ are saturation functions. Assuming $\delta_i < M_{i2} - M_{i1}$ and $M_{1i} \geq 3\lambda_i\rho_i$. λ_i and ρ_i are positive parameters. Then the inputs (20) asymptotically stabilize the rigid body to the origin $(1\ 0^T\ 0^T)^T$ (*i.e.* $q_0 = 1, q_v = 0$ and $w = 0$) with a domain of attraction equal to $\mathbb{S}^3 \times \mathbb{R}^3 \setminus (-1\ 0^T\ 0^T)^T$.

Proof. Consider the candidate Lyapunov function V, which is positive definite.

$$\begin{aligned}
V &= \frac{1}{2}\boldsymbol{w}^T J \boldsymbol{w} + \kappa((1 - q_0)^2 + \boldsymbol{q}^T\boldsymbol{q}) \\
&= \frac{1}{2}\boldsymbol{w}^T J \boldsymbol{w} + 2\kappa(1 - q_0)
\end{aligned} \tag{21}$$

where J is defined as before, and $\kappa > 0$ must be determined. The derivative of (21) after using (6) and (7)is given by

$$\begin{aligned}
\dot{V} &= \boldsymbol{w}^T J \dot{\boldsymbol{w}} - 2\kappa\dot{q}_0 \\
&= \boldsymbol{w}^T(-\boldsymbol{w}^\times J\boldsymbol{w} + \Gamma + \Gamma_{arm} + \Gamma_G) + \kappa\boldsymbol{q}^T\boldsymbol{w} \\
&= \underbrace{w_1(\Gamma_1 + \Gamma_{arm_1}) + \kappa q_1 w_1}_{\dot{V}_1} \\
&\quad + \underbrace{w_2(\Gamma_2 + \Gamma_{arm_2}) + \kappa q_2 w_2}_{\dot{V}_2} \\
&\quad + \underbrace{w_3(\Gamma_3 + \Gamma_{arm_3}) + \kappa q_3 w_3}_{\dot{V}_3}
\end{aligned} \tag{22}$$

$\dot{V}$ is the sum of the three terms $(\dot{V}_1, \dot{V}_2, \dot{V}_3)$. First $\dot{V}_1$ is analyzed. From Γ_1 in (20) and equation (22), one gets

$$\dot{V}_1 = w_1(-\sigma_{M_{12}}(\Gamma_{arm_1} + \sigma_{M_{11}}(\lambda_1[w_1 + \rho_1 q_1])) + \Gamma_{arm_1}) + \kappa q_1 w_1 \quad (23)$$

if we choose $\delta_1 < M_{12} - M_{11}$, $\sigma_{M_{12}}$ is always operating in its linear region so the $\dot{V}_1$ becomes

$$\dot{V}_1 = -w_1\sigma_{M_{11}}(\lambda_1[w_1 + \rho_1 q_1]) + \kappa q_1 w_1 \quad (24)$$

Assume that $|w_1| > 2\rho_1$, that is $w_1 \in]2\rho_1, +\infty[$. Since $|q_1| \leq 1$, it follows that $|w_1 + \rho_1 q_1| \geq \rho_1 + \epsilon$ for any $\epsilon > 0$ sufficiently small. Therefore, $w_1 + \rho_1 q_1$ has the same sign as w_1. From equation (24) and the norm condition on the quaternion, $\dot{V}_1$ takes the following form

$$\begin{aligned}\dot{V}_1 &= -w_1\sigma_{M_{11}}(\lambda_1[w_1 + \rho_1 q_1]) + \kappa w_1 q_1 \\ &\leq -|w_1|\sigma_{M_{11}}(\lambda_1(\rho_1 + \epsilon)) + \kappa|w_1|\end{aligned} \quad (25)$$

Taking

$$\kappa < \min(M_{11}, \lambda_1\rho_1 + \epsilon) \quad (26)$$

one can assure the decrease of V_1, i.e. $\dot{V}_1 < 0$. Consequently, w_1 enters $\Phi_1 = \{w_1 : |w_1| \leq 2\rho_1\}$ in finite time t_1 and remains in it thereafter. In this case, $(w_1 + \rho_1 q_1) \in [-3\rho_1, 3\rho_1]$.
Let M_{11} verify the next inequality $M_{11} \geq 3\lambda_1\rho_1$, equation (26) then becomes:

$$\kappa < \lambda_1\rho_1 + \epsilon \quad (27)$$

For $t_2 > t_1$, the argument of $\sigma_{M_{11}}$ will be bounded as follows

$$|\lambda_1(w_1 + \rho_1 q_1)| \leq 3\lambda_1\rho_1 \leq M_{11} \quad (28)$$

Consequently, σ_{M1} operates in a linear region

$$\Gamma_1 = -\lambda_1[w_1 + \rho_1 q_1] \quad (29)$$

As a result, (24) becomes

$$\dot{V}_1 = -\lambda_1 w_1^2 - \lambda_1\rho_1 w_1 q_1 + \kappa w_1 q_1 \quad (30)$$

Choosing $\kappa = \lambda_1\rho_1$ which satisfies inequality (27), one obtains

$$\dot{V}_1 = -\lambda_1 w_1^2 \leq 0 \quad (31)$$

The same argument is applied to $\dot{V}_2$ and $\dot{V}_3$, (22) becomes

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \\ &= -(\lambda_1 w_1^2 + \lambda_2 w_2^2 + \lambda_3 w_3^2) \leq 0\end{aligned} \quad (32)$$

In order to complete the proof, the LaSalle Invariance Principle is invoked. All the trajectories converge to the largest invariant set $\bar{\Omega}$ in $\Omega = \{(\boldsymbol{q}, \boldsymbol{w}) : \dot{V} = 0\} = \{(\boldsymbol{q}, \boldsymbol{w}) : \boldsymbol{w} = 0\}$. In the invariant set, $J\dot{\boldsymbol{w}} = -[\lambda_1\rho_1 q_1 \; \lambda_2\rho_2 q_2 \; \lambda_3\rho_3 q_3]^T = 0$ that is, $\bar{\Omega}$ is reduced to the origin. This ends the proof of the asymptotic stability of the closed loop system.

4. LOAD MASS ESTIMATION

4.1 Problem statement

The objective is to design a control law with the inner-outer loop configuration, which allows the system to estimate the mass that it is carrying, having the attitude stabilization problem solved and also could be able to stabilize the quadrotor to a coordinate desired position. In other words, once the control law has stabilized the attitude of the system, $\lim_{t\to\infty}(q, \boldsymbol{w}) = (q_d, \boldsymbol{0})$, this could be able to stabilize the quadrotor in a desired position, $\lim_{t\to\infty}(\boldsymbol{p}, \boldsymbol{v}) = (\boldsymbol{p}_d, \boldsymbol{0})$, and this stabilization must be kept even under the disturbances from the manipulator arm.

4.2 Estimation strategy

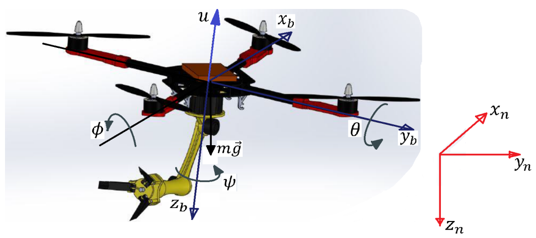

Fig. 2. Schematic configuration of a quadrotor carrying a manipulator arm.

The schematic representation of a quadrotor carrying a manipulator arm can be seen in Fig. 2, where the inertial reference frame $N(x_n, y_n, z_n)$, the body reference frame $B(x_b, y_b, z_b)$, the force u (thrust) and the weight vector is $m\boldsymbol{g}$ are depicted. The dynamics of the whole system is obtained with the Newton-Euler formalism and the kinematics is represented using the quaternions formalism, and is given by

$$\Sigma_T : \begin{cases} \dot{\boldsymbol{p}} = \boldsymbol{v} \\ m_T\dot{\boldsymbol{v}} = -m_T\boldsymbol{g} + R\begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix} \end{cases} \quad (33)$$

$$\Sigma_O : \begin{cases} \dot{q} = \dfrac{1}{2}\Xi(q)\boldsymbol{w} \\ J\dot{\boldsymbol{w}} = -\boldsymbol{w}^{\times}J\boldsymbol{w} + \Gamma_T \end{cases} \quad (34)$$

where $\boldsymbol{p}$ and $\boldsymbol{v}$ are linear position and velocity vectors, m_T is the total mass of the system; the quadrotor, the manipulator and the load, $\boldsymbol{g}$ the acceleration due to gravity, R is the rotation matrix, given in (3).
Note that the rotation matrix R can be given in function of Euler angles, that is

$$R(\phi, \theta, \psi) = \begin{pmatrix} C\psi\ C\theta & S\psi\ C\theta & -S\theta \\ C\psi\ S\theta\ S\phi - S\psi\ C\theta & S\phi\ S\theta\ S\psi + C\psi\ C\phi & C\theta\ S\phi \\ C\psi\ C\phi\ S\theta + S\psi\ S\phi & S\theta\ S\psi\ C\phi - C\psi\ S\phi & C\theta\ C\phi \end{pmatrix}, \quad (35)$$

Taking into account the equations (33) and (34), this system can be seen as a cascade system, where the translational dynamics (33), depends on the attitude (34), but the attitude dynamics does not depend on the translational one. This property will be used to design the control law. Now, assume that using the control law (20) one can stabilize the yaw dynamics, that is $\psi = 0$, then after a sufficiently long time, system (33) becomes:

$$\begin{pmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}, \quad (36)$$

$$\begin{pmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{pmatrix} = \begin{pmatrix} -\frac{u}{m_T}\text{sen}\theta \\ \frac{u}{m_T}\text{sen}\phi \cos\theta \\ \frac{u}{m_T}\cos\phi \cos\theta - g \end{pmatrix}, \quad (37)$$

With an appropriate choice of these target configuration, it will be possible to transform (36)-(37) into three independent linear triple integrators. For this, take

$$\begin{aligned} \phi_d &:= \arctan\left(\frac{r_2}{r_3+g}\right), \\ \theta_d &:= \arcsin\left(\frac{-r_1}{\sqrt{r_1^2+r_2^2+(r_3+g)^2}}\right) \end{aligned} \quad (38)$$

where r_1, r_2 and r_3 will be defined after. Then, choose as positive thrust the input control

$$u = m_T\sqrt{r_1^2+r_2^2+(r_3+g)^2} \quad (39)$$

Let be the state $p = (p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9) = (\int p_x, p_x, v_x, \int p_y, p_y, v_y, \int p_z, p_z, v_z)$, then (36)-(37) becomes:

$$\Sigma_x : \begin{cases} \dot{p}_1 = p_2 \\ \dot{p}_2 = p_3 \\ \dot{p}_3 = r_1 \end{cases} \quad (40)$$

$$\Sigma_y : \begin{cases} \dot{p}_4 = p_5 \\ \dot{p}_5 = p_6 \\ \dot{p}_6 = r_2 \end{cases} \quad (41)$$

$$\Sigma_z : \begin{cases} \dot{p}_7 = p_8 \\ \dot{p}_8 = p_9 \\ \dot{p}_9 = r_3 \end{cases} \quad (42)$$

Note that u will be always positive, and $u \geq mg$, in order to compensate the system's weight. Consequently, even if the mass value of the load m_l is unknown, the presence of a possible steady state error is avoided by the addition of a state in the chains of integrators (40)-(42) and the control law proposed later. Also, once the signal control u given by (39) is computed, it is possible to estimate the value of the unknown mass.
The estimated value of m_l is used to calculate the center of mass of the system given in (10), which allows the computation of the torque (9) generated by the manipulator. Finally this value will be used as Γ_{arm_i} in the attitude control (20).

Since the chains of integrators given in (40)-(42) have the same form, a control law can be proposed as in Cruz-José et al. (2012), and can be established by the next lemma:

Lemma 4.1. Taking into account the dynamics expressed in (40)-(42), the control laws with bounded inputs is given by

$$\begin{aligned} r_1 &:= -\varsigma_1\{a_3\sigma_{M1}[\frac{1}{\varsigma_1}(a_2p_1+p_2+p_3)] \\ &\quad + a_2\sigma_{M1}[\frac{1}{\varsigma_1}(a_1p_2+p_3)] + a_1\sigma_{M1}[\frac{1}{\varsigma_1}(p_3)]\}, \\ r_2 &:= -\varsigma_2\{b_3\sigma_{M1}[\frac{1}{\varsigma_1}(b_2p_4+p_5+p_6)] \\ &\quad + b_2\sigma_{M1}[\frac{1}{\varsigma_2}(b_1p_5+p_6)] + b_1\sigma_{M1}[\frac{1}{\varsigma_2}(p_6)]\}, \\ r_3 &:= -\varsigma_3\{c_3\sigma_{M1}[\frac{1}{\varsigma_1}(c_2p_7+p_8+p_9)] \\ &\quad + c_2\sigma_{M1}[\frac{1}{\varsigma_3}(c_1p_8+p_9)] + c_1\sigma_{M1}[\frac{1}{\varsigma_3}(p_9)]\} \end{aligned} \quad (43)$$

where $\sigma_{M_1}(\cdot)$ is defined in (19), with $M_1 = 1, b_{(1,2,3)}, c_{(1,2,3)} > 0$ are tuning parameters defined before, and ς_i are given by

$$\begin{aligned} \varsigma_1 &= \bar{r}_1/(a_1+a_2+a_3), \\ \varsigma_2 &= \bar{r}_2/(b_1+b_2+b_3), \\ \varsigma_3 &= \bar{r}_3/(c_1+c_2+c_3) \end{aligned} \quad (44)$$

Then, the control laws in (43) exponentially stabilize the systems (40)-(42) to the desired position $(p_1, p_2) = (p_{dx}, 0)$, $(p_3, p_4) = (p_{dy}, 0)$ and $(p_5, p_6) = (p_{dz}, 0)$.

5. SIMULATION RESULTS

In order to test the effectiveness of the control law proposed for the system, a set of simulations were performed using MATLAB/Simulink. The parameters of the system used for the simulation are as follows: $m_T = 790g$, $m_l = 110g$, $\max|\Gamma_{1,2}| = 0.4053Nm$, $\max|\Gamma_3| = 0.154Nm$ and $\max|u| = 11.5N$.
The scenario for the simulation is divided in three parts. We consider that the system is taking a mass load and during the first 10 seconds the system is driven to $\boldsymbol{p}_d = (0\ 0\ 1)^T$ to estimate the value of the load. Then, between time 15s and 30s three movements are performed in the manipulator; first, at time 15s, θ_{a2} and θ_{a3} are positioned at 45°, then at time 20s, θ_{a1} and θ_{a2} changes to 90° and θ_{a3} changes to 0°, with this, the manipulator is horizontally extended; at time 25s, the manipulator keeps extended but θ_{a1} changes to 270°. In the last part, at 30s, a disturbance is exerted directly to the manipulator.
In Fig. 3, angular and linear position and velocity are depicted, where attitude stabilization is achieved. Note that even when we consider the quaternion parametrization, Euler angles, given in (35), are used in order to have a better perspective of the behavior of the system. The plots in Fig. 4 show the angular positions of each link in the manipulator, the control torques $\Gamma_{1,2,3}$ which stabilize the quadrotor, as well as the force and control position u. It is shown that the control law ensures the stabilization of the quadrotor to the desired position even with the disturbances exerted from the manipulator.

6. CONCLUSIONS

In this paper, a control law was designed to asymptotically stabilize the attitude and position of a quadrotor carrying a manipulator arm. Moreover, this work has presented a method for aiding the solution through the estimation of the mass that the system is carrying, and consequently

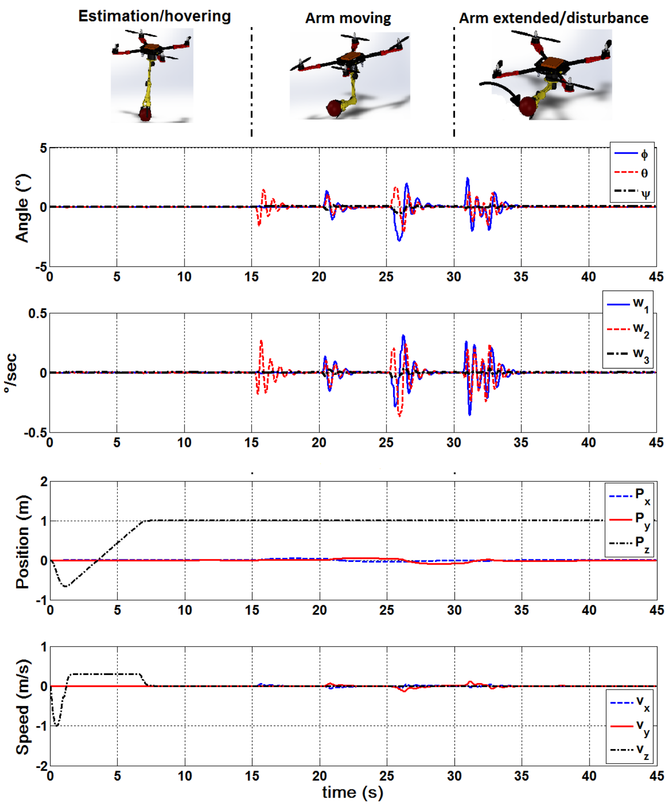

Fig. 3. Angular and linear position and velocity during the simulation.

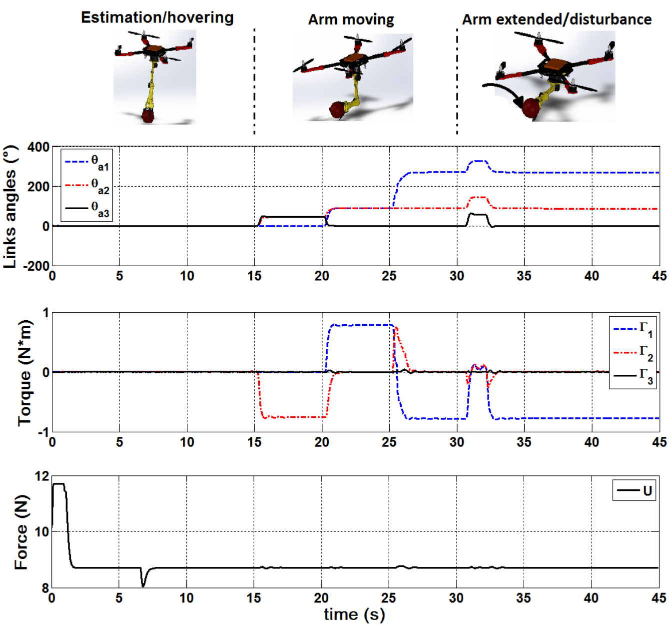

Fig. 4. Angular positions of the links of the manipulator, control torques and thrust signals during the simulation.

allowing the design of a feed-forward term. Since input constraints exist in the actuators, the control law takes into count the actuators saturations. Simulation results show the effectiveness of the proposed control law face to the continuous disturbances coming from the manipulator, even when it is carrying a load mass. Real-time implementation will be pursued as a further work.

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