Memorias del XVI Congreso Latinoamericano
de Control Automático, CLCA 2014
Octubre 14-17, 2014. Cancún, Quintana Roo, México

Operation of the Power Distribution System via Sensitivity Analysis

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Abstract: This paper presents an operation study of the power distribution system via sensitivity analysis (SA). This study is based on first-order SA applied in the load flow (PF) solution. Unlike the PF algorithms, this technique does not require an iterative process, which results in a faster methodology with high accuracy. Starting from a known solution of a PF, considered as the base case, the new operating point is calculated directly after making a perturbation in the loads. The methodology is applied to 34, 70,126 and 476-bus distribution power systems. Test results demonstrate the efficiency of the approach.

Keywords: Sensitivity analysis, load flow, distribution power systems.

INTRODUCTION

The electricity sector has undergone in recent years major changes in search of a more competitive and integrated environment, in addition to the great challenge is the implementation of smart grids. Smart grids will bring progress toward new technologies that will enable a better management system promoting efficiencies.

Allied to these changes the electrical power systems has experienced a large increase in demand. The global consumption of electricity will increase by 84% in the corresponding period between 2008 and 2035 (Ieo, 2011). In Brazil´s Decennial Plan for Expansion of Energy 2020 pro-vides for an increase of 4.6% per year of electricity consumption in the country in the period 2010-2020 (Ieo, 2011).

To this end, electrical power systems re-quire rapid analysis tools to assist agents in making decisions. The Sensitivity Analysis (SA) can be used in this context to promote gains in operating systems.

The SA is an integral part of many methodologies. The status of a solution cannot be understood without such information. SA can be defined as a technique for assessing impacts associated with changes in the values of the input variables with respect to the parameters of the system output. In mathematical programming, sensitivity techniques have been used to obtain optimality conditions, dual results, solution algorithms, rate of convergence and acceleration convergence for iterative algorithms, in addition to its most obvious and immediate application that is to estimate solutions considering different input parameters.

The SA is of great importance in studies of the electrical power system operation because it helps in understanding the cause-effect relationship between the system parameters and can be used in some applications in real-time operation due to obtaining a direct relationship between control variables and controlled, important in the context of smart grids (Gungor et al., 2011).

In electrical power system operation have two types of relations predominant of sensitivity: (a) the sensitivity of an electric variable with respect to another variable of the system, and (b) the sensitivity of a variable in relation to the objective function system (Peschon et al., 1968).

The SA technique developed in this paper is focused on the first type of sensitivity and can be used to evaluate the behavior of the variables and parameters of the system as examples: voltage limits, active power losses, power flows in branches, etc. Thus, it is possible to calculate the sensitivity of one electrical variable, such as voltage V_i at node i, with respect to the injection of active power P_j at node j, as an example.

The methodology presented in this paper consists of apply a perturbation in the operation point obtained by Power Flow (PF) problem (Monticelli, 1983), considering this solution as the base case, and by SA estimate the new operating point for the network after perturbation occur in the input variables. The methodology can be divided into two parts: the first part is related to the implementation of the PF for the solution of the base case and the second part in the implementation of first-order sensitivity analysis on the PF solution obtained in the first part.

The obtaining of the solution of PF consists in solving a system of nonlinear equations using numerical methods. The methods have evolved following the technical advances that allow, through the use of computer, prepare sophisticated algorithms for solving the problem. One of the most widely used methods for solving the problem of PF for transmission systems was proposed by (Tinney e Hart, 1967), which uses the method of Newton-Raphson (NR) in the solution process. Due to the differences of the transmission and distribution systems, such as X/R ratio and arrangement of lines, the method NR loses efficiency when applied to distribution systems compared to algorithms developed exclusively for transmission systems (Koto et al., 2012).

Regarding the technical SA, they were used for the first time in electrical power system in the late 70's (Peschon et al., 1968). In (Belati Costa., 2008) a methodology using SA sensitivity of second-order factors in order to determine new solutions to the problem of Optimal Power Flow was developed. In (Belati et al., 2005) was presented an approach to the problem of allocation of power losses active in a transmission system. In (Da Rosa e Belati, 2013), technical losses analysis was presented using sensitivity analysis of the first-order. Yao and Huang proposed which uses a study using a SA to solve the PF problem in real time (Huang e Yao, 2012).

The main objective of this paper is to present a methodology for application in networks of distribution of electricity that can be used when it is requested multiple calculations of PF in order to optimize the calculations in ways that promote gain of computational time. This paper is organized as follows: in section 2 general aspects of NR; SA technique and steps of the methodology are presented; in section 3 tests and studies are conducted to demonstrate the applicability of the methodology; and finally in section 4 conclusions of the work are presented.

DESCRIPTION OF THE METHODOLOGY

The methodology can be divided into two parts: the first consists of solving the Power Flow (PF)using the Newton-Raphson method to the Base Case (BC); the second parts in the application of sensitivity analysis on the PF solution obtained in the first part.

A. Power Flow

Injections of active and reactive power are obtained from imposing the law of Kirchhoff currents at each bus and can be calculated in polar form by Eq (1) and Eq (2), respectively.

$$P_k(V,\theta) = V_k \sum_{m\in k} V_m (G_{km} cos\theta_{km} + B_{km} sen\theta_{km}) \quad (1)$$

$$Q_k(V,\theta) = V_k \sum_{m\in k} V_m (G_{km} sen\theta_{km} - B_{km} cos\theta_{km}) \quad (2)$$

Where:

G_{km} – real element of the matrix Y_{BUS} associated with the bus k and m;
B_{km} – imaginary element of the matrix Y_{BUS} associated with the bus k and m;
$m \in k$ – the set of all m bus having connection with the bus k.

The solution of the problem of PF is obtained with solution the equations of balance of active and reactive power given, respectively, by Eq. (3) and Eq. (4).

$$\Delta P_k = P_k^{esp} - P_k^{calc}(V,\theta) = 0 \quad (3)$$

$$\Delta Q_k = Q_k^{esp} - Q_k^{calc}(V,\theta) = 0 \quad (4)$$

Where *esp* superscript represents the values specified of power injections at load bus that are considered constant (constant power load model) and *calc* superscript is the calculated values of power injections obtained from the vector of state variables(V, θ) and system parameters.

To solve the problem of PF was developed the method of NR (Monticelli, 1983). Although the NR method is not the most used in distribution systems it can be applied when sparsity techniques are used [6]. Another reason to the use NR method is the use of data from the last interaction of the Jacobian matrix in the sensitivity analysis, wich promotes computational gains.

B. Sensitivity

The SA is of great importance in studies of electrical power system operation. It helps in understanding the relationship existing between cause and effect system parameters and can be used in applications in real time.

Consider two types of variables: operating variables denoted by the vector $\boldsymbol{u}$; and controlled variables denoted by the vector $\boldsymbol{x}$ in this study, we have:

$\boldsymbol{x}$ – vector of state variables of the problem (V, θ);
$\boldsymbol{u}$ – perturbation vector of the active and reactive power injections at load buses (P_k^{esp}, Q_k^{esp}).

The flow equations of active and reactive power, Eq. (3) and Eq. (4) can be written compactly as:

$$g(x,u) = 0 \quad (5)$$

Suppose these $x = x^*$ is the solution to the vector control specified $u = u^*$ that satisfies Eq. (5), then:

$$g(x^*,u^*) = 0 \quad (6)$$

Knowing that a change Δu in u^*, cause a change Δx in x^*, applies a Taylor series expansion in equation (6), obtaining:

$$g(x^* + \Delta x, u^* + \Delta u) = g(x^*,u^*) + S_x\Delta x + S_u\Delta u \quad (7)$$

$$S_x = J = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} \end{bmatrix} \tag{8}$$

The S_u matrix is obtained as follows:

$$S_u = \begin{bmatrix} \frac{\partial \Delta P}{\partial P} & \frac{\partial \Delta P}{\partial Q} \\ \frac{\partial \Delta Q}{\partial P} & \frac{\partial \Delta Q}{\partial Q} \end{bmatrix} \tag{9}$$

The S_u matrix results in an identity matrix, when considering the load model with injections of constant power, which is the case adopted in this paper.

Combining equations (6) and (7) gives:

$$S_x \Delta x + S_u \Delta u = 0 \tag{10}$$

by rearranging we have Δx in (10) gives:

$$\Delta x = -S_x^{-1} S_u \Delta u = 0 \tag{11}$$

Su is an identity matrix and ${S_x}^{-1}$ is equal to J^{-1}, used in the last iteration of the method of resolution of the PF, the expression for the correction vector Δx isgiven by:

$$\Delta x = J^{-1} \Delta u \tag{12}$$

The Eq. (12) can be written in matrix form as (13), wherein NPQ is the number of load bus of the distribution system.

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_{(2NPQ)} \end{bmatrix} = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V} \end{bmatrix}^{-1} \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \\ \vdots \\ \Delta u_{(2NPQ)} \end{bmatrix} \tag{13}$$

In this equation it was considered that the distribution system is composed of the substation and the other nodes are load bus. As the vector u consists of the independent variables that are active and reactive injections on the buses, and the vector x the controlled variables that are voltage magnitudes and phase angles of the buses, the matrix system (13) can be rewritten as (14).

$$\begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \vdots \\ \Delta\theta_{(NPQ)} \\ \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_{(NPQ)} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P_1^{esp} \\ \Delta P_2^{esp} \\ \vdots \\ \Delta P_{NPQ}^{esp} \\ \Delta Q_1^{esp} \\ \Delta Q_2^{esp} \\ \vdots \\ \Delta Q_{NPQ}^{esp} \end{bmatrix} \tag{14}$$

The matrix system of Eq (14) has the right side vector the perturbation (injections of active and reactive power) multiplied by the inverse of the matrix J and the left side of the equality the offset vector of state variables.

Using the matrix system (14) new solutions for the state variables of the problem, x_{new}, can be obtained when perturbations are performed in the load bus as follows:

$$x_{new} = x^* + \Delta x \tag{15}$$

A. Algorithm

An algorithm proposed to solve the problem consists of the following steps represented in the flowchart of Fig. 1:

(i) Enter system data;
(ii) Obtain operational point by PF;
(iii) Perturb the power injections with Δu;
(iv) Use (13) to compute Δx;
(v) Use equation (15) to update x_{new};
(vi) If a new perturbation is desired go to item (iii);
(vii) Otherwise - End.

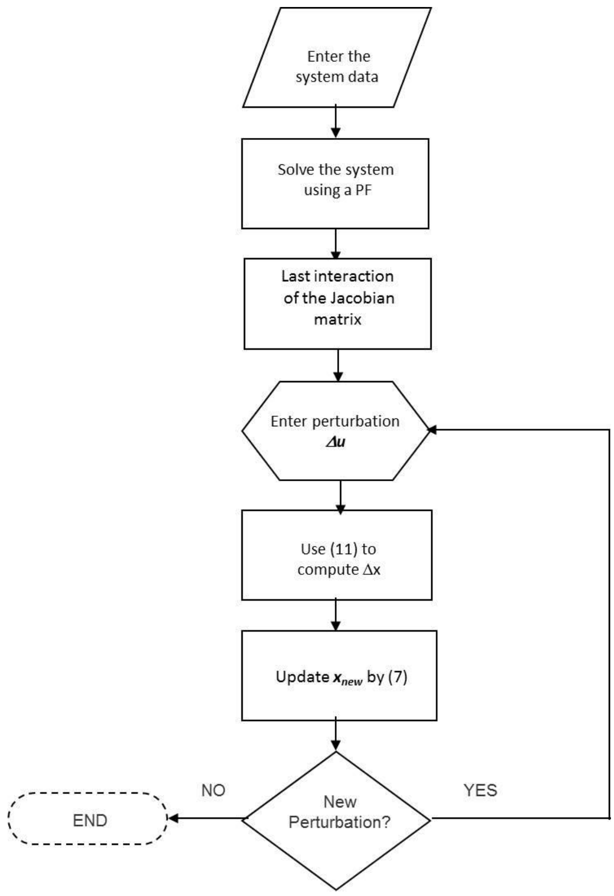

Fig. 1. SENS methodology flowchart.

The methodology requires only one resolution of the problem of PF. In the PF solution the Jacobian matrix of the last iteration is stored to estimate new solutions after perturbation occur in the loads of the system. In Fig. 1 the flowchart of methodology is presented.

CASE STUDIES

The studies were carried out on the 34, 70, 126 and 476 bus of distribution systems. The data systems may be obtained respectively (Chis *et al.*, 1997), (Baran e Wu, 1989), (Yan, 2011) and (Gomes et al., 2005). Following the steps of the methodology, was initially obtained the status of the system, called base case with PF (NR method), adopting an accuracy of 10^{-5} pu for power balance equations. With the solution of the base case new solutions were estimated directly using the SA.

The studies were performed at platform Matlab® LG A520 with Intel ® Core ™ i5-2410M CPU@2.30 GHz, 4GB RAM, Windows 7 Home Premium Operating System Processor - 64 Bit.

To validate the methodology was performed several tests and compared with o PF. The results obtained by the methodology are identified as "Sens" and the Power Flow as “PF”.

C. Active Analysis of Losses

Starting with the base case, the active and reactive load bus powers were increased by 2, 4, 6%, to a maximum of 50% at all load bus simultaneously, keeping power factor constant. For each 2% perturbation, the technique of SA referred to as "Sens" in this work was applied and the new state of the system estimated. Figures 2, 3 and 4 show comparative graphs of the power loss obtained by the technique SA and the method of PF (NR method), and Figure 2 relates to bus code system 34, Figure 3 relates to System 70 bus and connected to the system 476 of Figure 4 bus.

For the system of 34 bus the values estimated by the "Sens" losses were very close to those obtained by the PF. In the 70-bus system, the values estimated for the smaller perturbation than 35% by the methodology "Sens" losses were approximately obtained by the PF. At 476 bus system, a system larger, the values estimated for the smaller perturbation than 20% by the methodology "Sens" losses were close to those obtained by the PF. Comparing the curves it appears that the methodology can be applied to studies on systems with active losses.

For each system, the base case solution was obtained via PF with a Newton-Raphson method for the power balance equations. In the studies performed, demand variations were supplied by the slack bus.

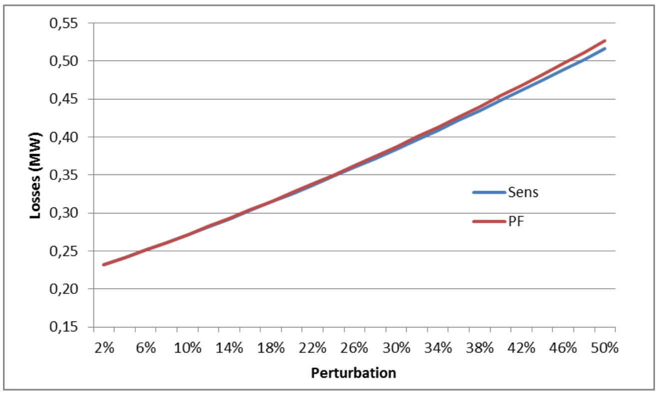

Figure 2. Comparison of active power losses of 34 bus system.

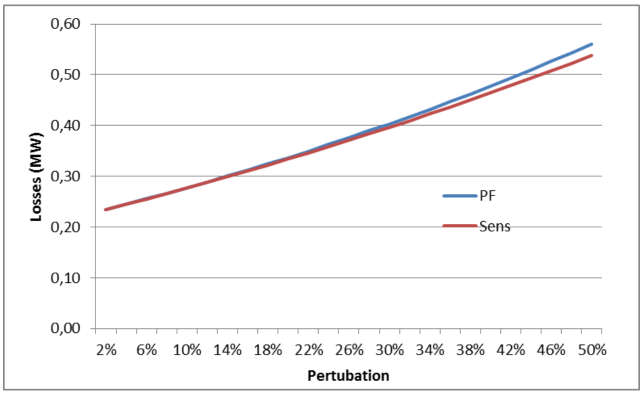

Figure 3: Comparison of active power losses of 70 bus system.

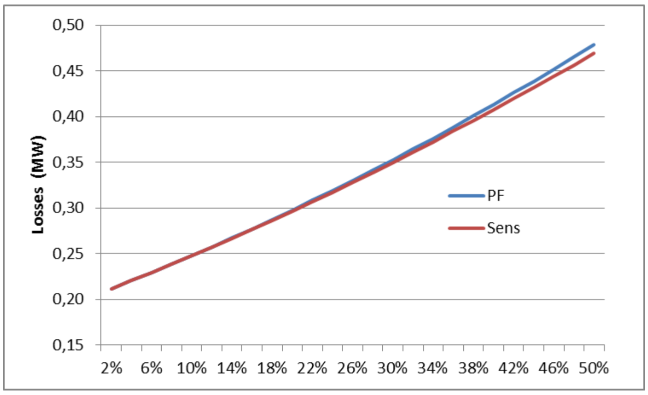

Figure 4. Comparison of active power losses of 476 bus System.

D. Analysis of Voltage Profiles

Using the same perturbation in the preceding item values of tensile modulus and phase angle of 214 bus to 476 bus system were obtained. Bus 214 was the one with the largest error compared to the solution of the PF. Table II presents these data, where the voltage values obtained by the methodology "Sens" with the exact values of the PF can be compared.

E. Computational Times

Earlier tests showed that the methodology "Sens" presents a good estimate for a wide range of perturbation. However, their use is justified due to the low computational cost when compared to PF.
Table I shows the time in *ms* (milliseconds) and the time saving in using the technique SA "Sens" after the solution of the given base case. The time spent to estimate the solution via SA were approximately 99% lower compared to the PF solved by the method of NR. The approximate times of the systems 34, 70, 126 and 476 can be checked in Table I.

TABLE II. COMPARISON OF TENSION BUS 214 OF 476 BUS SYSTEMS.

	Sens		LF	
Perturbation (%)	Voltage (pu)	Angle (º)	Voltage (pu)	Angle (º)
2	0,9449	-2,5523	0,9449	-2,5524
4	0,9437	-2,6053	0,9437	-2,6056
6	0,9426	-2,6583	0,9426	-2,6589
8	0,9415	-2,7112	0,9414	-2,7123
10	0,9403	-2,7642	0,9403	-2,7659
12	0,9392	-2,8172	0,9391	-2,8196
14	0,9380	-2,8702	0,9379	-2,8735
16	0,9369	-2,9231	0,9368	-2,9275
18	0,9357	-2,9761	0,9356	-2,9817
20	0,9346	-3,0291	0,9344	-3,0360
22	0,9334	-3,0820	0,9333	-3,0904
24	0,9323	-3,1350	0,9321	-3,1450
26	0,9311	-3,1880	0,9309	-3,1997
28	0,9300	-3,2409	0,9297	-3,2546
30	0,9289	-3,2500	0,9285	-3,3096
32	0,9277	-3,3469	0,9273	-3,3648
34	0,9266	-3,3999	0,9261	-3,3742
36	0,9254	-3,4067	0,9249	-3,4756
38	0,9243	-3,5058	0,9237	-3,5313
40	0,9231	-3,5588	0,9225	-3,5871
42	0,9220	-3,6117	0,9213	-3,6430
44	0,9208	-3,6647	0,9201	-3,6991
46	0,9197	-3,7177	0,9189	-3,7554
48	0,9186	-3,7706	0,9177	-3,8118
50	0,9174	-3,8236	0,9164	-3,8684

TABLE I. TIMES OF COMPUTER SYSTEMS OF DISTRIBUITION 34, 70, 126 AND 476 BUS SYSTEMS.

System	Time (ms) - PF	Time (ms) – Sens	Gain (%) Sens/PF
34 Bus	6,0	0,14547	97,58
70 Bus	27,1	0,07940	99,71
126 Buas	32,9	0,18690	99,43
476 Bus	454,0	0,99462	99,78

Based on the information in Table I can be affirming that the methodology shows an increase with computational studies that require several PF solutions, as presented in the next section.

F. Studies of active power losses in distribution systems.

Having demonstrated the applicability of the method in distribution systems, a study was carried out to obtain the rate of active loss in the 126 bus system, as shown in Table II.

The study consists of performing an increase of 100kW at all load bus individual shape and the methodology to calculate the new system losses. The objective of this analysis is to verify the bus causing major losses in the system. Fig. 5 shows the variations of active power losses of the systems in relation to the increase of load on each bus. As for the CPU time, the methodology has spent approximately 0.24 s to perform all calculations. The same calculation using the PF would take about 4 seconds.

Fig. 5 shows the variations of losses to the system 126 bus. The bus 122, 123 and 124 presented higher increase in active losses, 10.6 kW. The bus that showed lower increase was the bus 2 which rose in active losses of 1.1 kW.

CONCLUSIONS

The PF is an important tool used in the planning and operation of electrical power system. The objective of the implementation of the PF is to get the angles and voltage magnitudes to the bus of the network. This article reports a study on the functioning of a power distribution system with SA. The methodology consists of applying a perturbation in the operating point obtained in the solution of the PF, regarded as the solution to the base case, and by SA estimate the new operating point. In the study, perturbations were introduced into the load bus up to a limit of 50 % in 34, 70, 126 and 476 bus systems. The gain in time for processing methodology is approximately 99% compared to the PF. The methodology was also applied to obtain a relative sensitivity of the load bus are in relation to the active losses. The results showed that SA methodology provides a gain in processing time and can be applied to estimate good precision solutions for the distribution systems.

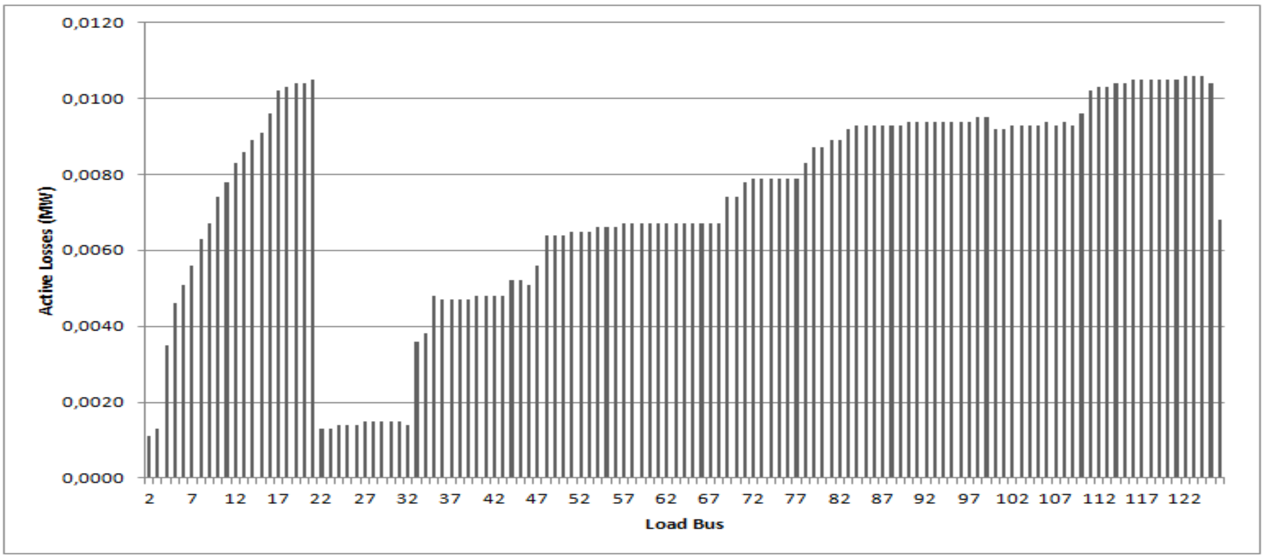

Figure. 5. Variation of active power losses for the system of 126 bus with an increase of 100kW at load buses.

ACKNOWLEDGMENT

This work is supported by Coordenação de aperfeiçoamento de pessoal de nível superior (Capes) and Universidade Federal do ABC.

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