Memorias del XVI Congreso Latinoamericano
de Control Automático, CLCA 2014
Octubre 14-17, 2014. Cancún, Quintana Roo, México

Leader-oriented Formation Flight Control for Multi-agent Systems ⋆

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Abstract: : This paper is focused on the formation flight control for multi-agent systems. The proposed control strategy is decentralized and allows a set of unmanned aerial vehicles to track a predetermined trajectory while they achieve a time-varying formation with respect to a leader. Formation graphs are used to represent interactions between agents; in particular, we use a leader-centered graph. The aerial mobile agents used in this paper are quad-rotor helicopters. The theoretical results are verified by numerical simulation.

Keywords: Time-varying Formation, Multi-agent Systems, Quad-rotor Helicopters, Formation Graphs.

1. INTRODUCTION

A multi-agent system can be defined as a system composed of a set of mobile robots that interact with each other, working together to achieve complex tasks that are difficult to perform by individual robots. The main applications of multi-agent systems include object manipulation, payload transport, search and rescue missions, surveillance and motion coordination Arai et al. (2002), Cao et al. (1997). A problem to be solved in the motion coordination is the displacement in formation, where a set of agents go from one place to another by keeping a desired formation between them.

Most works devoted to multi-agent systems are focused on the use of ground vehicles, but in recent years, researchers have paid special attention to the multi-agent systems formed by aerial vehicles, mainly by quad-rotor helicopters. The great interest in the use of quad-rotor helicopters comes from the fact that they are easy to control compared to other aerial vehicles and the ability to perform aggressive maneuvers. The main advantages of the quad-rotor helicopters are that they can take off and land vertically and hover, so they can be used in areas with limited space, but have the disadvantage that their flight time is very short.

In Nathan et al. (2011) a review of some control techniques and applications of multi-agent systems using quad-rotor helicopters are shown. Among control goals, formation, collision avoidance and path planning control are mentioned; the applications include indoor navigation and visual guidance. Almurib et al. (2011) presents a path planning control applied to search and rescue operations. In the literature some related works for formation control with quad-rotor helicopters can be found. For instance, Pilz et al. (2009) and Pilz and Werner (2010) present the design of a robust control, García-Delgado et al. (2012) proposes control laws based on potential functions and nested saturations, while Guerrero et al. (2012) develops a nonlinear control based on separated saturations, Abbas and Wu (2013) designs a sliding mode control and Fierro et al. (2001) presents a formation control based on input-output feedback linearization. Finally, Mellinger et al. (2012a) and Mellinger et al. (2012b) study the problem of trajectory generation. Mellinger et al. (2012a) provides a controller that allows aggressive maneuvers and Mellinger et al. (2012b) presents collision avoidance for a group of heterogeneous quad-rotor helicopters.

This paper boards the problem of the time-varying formation using aerial vehicles, in particular, quad-rotor helicopters. Some examples of time-varying formation considering ground vehicles can be found in previous works González-Sierra et al. (2013), Peñaloza-Mendoza et al. (2011), Rendón-Benitez et al. (2012), Santiaguillo-Salinas and Aranda-Bricaire (2013). The goal of this paper is to design a decentralized control strategy that achieves the trajectory tracking with time-varying formation for a set of quad-rotor helicopters. The control strategy is decentralized since the agents have no global knowledge of the goal to achieve, knowing only the positions, velocities and accelerations of a subset of agents of the system. The time-varying formation allows the trajectory tracking with formations oriented to the yaw angle of the leader robot. For the time-varying formation we use time-varying position vectors, these vectors are given by some static predefined desired formation which is transformed by a rotation matrix that depends on the yaw angle of the leader robot.

To represent the interaction between agents of a system, we will use graph theory, where each agent is represent by a vertex and the sharing of information between them is represent by an edge. In particular, we will use a leader-followers interaction represented by a leader-centered graph.

⋆ This work was partially supported by CONACyT, México, through scholarship holder No. 243226.

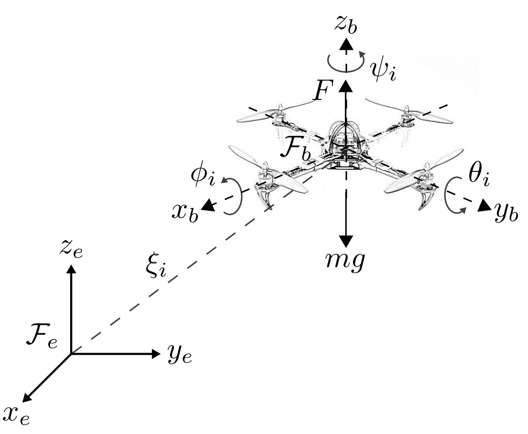

Fig. 1. Quad-rotor helicopter scheme.

The paper is organized as follows: Section 2 presents the dynamical model of quad-rotor helicopters, the basic graph theory and the time-varying position vector necessary to design the control strategy. The problem statement is presented in Section 3. The control strategy and the convergence analysis of the closed loop system are presented in Section 4. Section 5 shows the results of a numerical simulation. Finally, some concluding remarks are presented in Section 6.

2. PRELIMINARIES

2.1 Dynamical Model of Quad-rotor Helicopters

Let $N = \{R_1, \ldots, R_n\}$ a set of quad-rotor helicopters moving on the space. According to the Fig. 1, $\xi_i = [x_i, y_i, z_i]^T$ represents the position of the center of mass and $\eta_i = [\phi_i, \theta_i, \psi_i]^T$ represents the Euler angles (roll, pitch and yaw angles, respectively) that describe the orientation of the i-th quad-rotor helicopter, with $i = 1, \ldots, n$. Using the Euler-Lagrange approach and based on García et at. (2013) the dynamical model of each quad-rotor helicopter, assuming that all quad-rotor helicopters have the same mass, is given by

$$\ddot{\xi}_i = \frac{F_i}{m}\begin{pmatrix} \cos\phi_i \sin\theta_i \cos\psi_i + \sin\phi_i \sin\psi_i \\ \cos\phi_i \sin\theta_i \sin\psi_i - \sin\phi_i \cos\psi_i \\ \cos\phi_i \cos\theta_i \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} \quad (1)$$

$$\ddot{\eta}_i = \mathbb{J}^{-1}\left(\tau_i - C(\eta_i, \dot{\eta}_i)\dot{\eta}_i\right) \quad (2)$$

where $\mathbb{J}$ is the inertia matrix and $C(\eta_i, \dot{\eta}_i)$ is referred to as the Coriolis term. In order to simplify (2), let us take

$$\tilde{\tau}_i = \mathbb{J}^{-1}\left(\tau_i - C(\eta_i, \dot{\eta}_i)\dot{\eta}_i\right) \quad (3)$$

to obtain

$$\ddot{\eta}_i = \tilde{\tau}_i \quad (4)$$

with $\tilde{\tau}_i = [\tilde{\tau}_{\phi_i}, \tilde{\tau}_{\theta_i}, \tilde{\tau}_{\psi_i}]^T$.

We linearize (1) about $\xi_i = \xi_{i_d}(t)$, $\dot{\xi}_i = 0$, $\phi_i = \theta_i = 0$, $\psi_i = \psi_{i_d}(t)$, $\dot{\phi}_i, \dot{\theta}_i, \dot{\psi}_i = 0$ with nominal inputs given by $F_i = mg$ and $\tilde{\tau}_i = 0$ like in Mahony et al. (2012) to obtain

$$\ddot{\xi}_i = g\begin{pmatrix} \sin\psi_{i_d} & \cos\psi_{i_d} & 0 \\ -\cos\psi_{i_d} & \sin\psi_{i_d} & 0 \\ 0 & 0 & \dfrac{1}{mg} \end{pmatrix}\begin{pmatrix} \phi_{i_d} \\ \theta_{i_d} \\ F_i \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} \quad (5)$$

By defining auxiliary control variables $r_i = [r_{ix}, r_{iy}, r_{iz}]^T$ it is possible to establish a strategy for controlling the position ξ_i by

$$\begin{pmatrix} \phi_{i_d} \\ \theta_{i_d} \\ F_i \end{pmatrix} = \frac{1}{g}\begin{pmatrix} \sin\psi_{i_d} & -\cos\psi_{i_d} & 0 \\ \cos\psi_{i_d} & \sin\psi_{i_d} & 0 \\ 0 & 0 & mg \end{pmatrix}\left[\begin{pmatrix} r_{ix} \\ r_{iy} \\ r_{iz} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}\right] \quad (6)$$

The closed-loop system (5)-(6) produces

$$\ddot{\xi}_i = r_i \quad (7)$$

2.2 Basic Graph Theory

Interaction between agents of a system can be represent by graph theory. Some basic concepts of graph theory is present in this section; for more information Desai (2002), Lafferriere et al. (2004) can be consulted.

Definition 1. (Formation Graph). Let $N = \{R_1, \ldots, R_n\}$ be a set of aerial agents and N_i be the set of agents that have a communication link with the i-th agent. A formation graph $G = \{V, E, C\}$ consists of

- A set of vertices $V = \{R_1, \ldots, R_n\}$ corresponding to the n agents of the system.
- A set of edges $E = \{(R_j R_i) \in V \times V\}$ with $i \neq j$ containing pairs of vertices which represent the communication between agents, $(R_j R_i) \in E$ iff $R_j \in N_i$.
- A set of labels $C = \{c_{ji} = R_i - R_j\}$ with $(R_j R_i) \in E, c_{ji} \in \mathbb{R}^3, \forall i \neq j, j \in N_i$, i.e. c_{ji} is a vector specifying a desired relative position between the agents R_j and R_i.

A well-defined formation graph satisfies the following conditions

- Be connected.
- If there are edges $R_j R_{m_1}$,$R_{m_1} R_{m_2}$,$\ldots$,$R_{m_r} R_j \in E$, then the corresponding labels must satisfy
$$c_{jm_1} + c_{m_1 m_2} + c_{m_2 m_3} + \ldots + c_{m_r j} = 0.$$
- Specifically, if $R_j R_i, R_i R_j \in E$ with $i \neq j$, then $c_{ji} = -c_{ij}$.

A formation graph G in which the agent i interacts only with agent n that serves as a leader, responsible for guiding the other agents in the system, is called directed leader-centered graph (see Fig. 2). In this particular case, $N_i = \{R_n\}$, $i = 1, \ldots, n-1$ and $N_n = \emptyset$.

Definition 2. (Laplacian). Let a formation graph G, the Laplacian associated whit G is given by

$$\mathcal{L}(G) = \Delta - \mathcal{A}_d \quad (8)$$

where Δ is the degree matrix defined by

$$\Delta = \text{diag}\{g_1, \ldots, g_n\} \quad (9)$$

where g_i is the number of edges that are directed to the vertex R_i, $i = 1, \ldots, n$, i.e. $g_i = \text{card}\{N_i\}$ and $\mathcal{A}_d$ is the adjacency matrix of G defined by

$$a_{ij} = \begin{cases} 1, & \text{if } (R_j, R_i) \in E \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

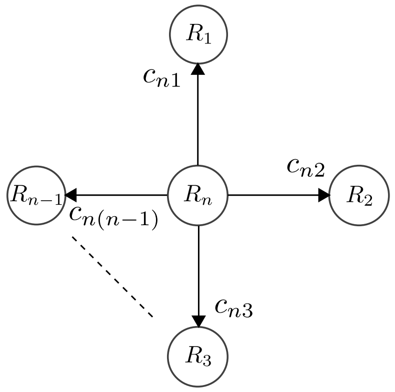

Fig. 2. Directed leader-centered graph.

where $R_i, R_j \in V$.

2.3 Time-varying Position Vector

In order to maintain a formation (by the follower agents) oriented to the yaw angle of the leader agent while it is flying, we use a time-varying position vector given by

$$C_{ji}(t) = R(\psi_n)c_{ji} \tag{11}$$

where c_{ji} is a static position vector that is obtained with the desired geometric pattern and $R(\psi_n)$ is a rotation matrix given by

$$R(\psi_n) = \begin{bmatrix} \cos\psi_n & -\sin\psi_n & 0 \\ \sin\psi_n & \cos\psi_n & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

The first and second time-derivative of the time-varying position vector are given by

$$\dot{C}_{ji}(t) = \dot{R}(\psi_n)c_{ji} \tag{13}$$
$$\ddot{C}_{ji}(t) = \ddot{R}(\psi_n)c_{ji} \tag{14}$$

where

$$\dot{R}(\psi_n) = \begin{bmatrix} -\sin\psi_n & -\cos\psi_n & 0 \\ \cos\psi_n & -\sin\psi_n & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\psi}_n \tag{15}$$

and

$$\ddot{R}(\psi_n) = \begin{bmatrix} -\sin\psi_n & -\cos\psi_n & 0 \\ \cos\psi_n & -\sin\psi_n & 0 \\ 0 & 0 & 0 \end{bmatrix} \ddot{\psi}_n + \begin{bmatrix} -\cos\psi_n & \sin\psi_n & 0 \\ -\sin\psi_n & -\cos\psi_n & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\psi}_n^2 \tag{16}$$

are the first and second derivative of the rotation matrix, respectively.

3. PROBLEM STATEMENT

Let $m(t) = [m_x(t), m_y(t), m_z(t)]^T$ be a preestablished trajectory which is supposed to be twice continuously differentiable. The goal of this work is to design a decentralized control law $r_i = \left(\xi_i, \dot{\xi}_i, N_i\right), i = 1, \ldots, n$ such that

- It achieves asymptotic tracking of a prescribed trajectory by the leader agent, i.e.
$$\lim_{t\to\infty} (\xi_n(t) - m(t)) = 0$$
- It reaches a desired time-varying formation by the follower agents with respect to the leader (Time-varying Formation Control), i.e. for $i = 1, ..., n-1$
$$\lim_{t\to\infty} (\xi_i(t) - \xi_n(t) - C_{ni}(t)) = 0$$

The decentralized control law r_i will result in the desired orientations ϕ_{i_d} and θ_{i_d} given in (6) which will be provided as inputs to the orientation control.

4. CONTROL STRATEGY

For tracking, we propose an auxiliary control law for the leader defined by

$$r_n = -K_{n_{D\xi}}(\dot{\xi}_n - \dot{m}(t)) - K_{n_{P\xi}}(\xi_n - m(t)) + \ddot{m}(t) \tag{17}$$

where $m(t)$ is the desired trajectory, $\dot{m}(t)$ is the desired velocity, $\ddot{m}(t)$ is the desired acceleration,

$$K_{n_{D\xi}} = \begin{bmatrix} k_{n_{dx}} & 0 & 0 \\ 0 & k_{n_{dy}} & 0 \\ 0 & 0 & k_{n_{dz}} \end{bmatrix} \tag{18}$$

and

$$K_{n_{P\xi}} = \begin{bmatrix} k_{n_{px}} & 0 & 0 \\ 0 & k_{n_{py}} & 0 \\ 0 & 0 & k_{n_{pz}} \end{bmatrix} \tag{19}$$

are the control gains. For the orientation control of the leader we propose

$$\tilde{\tau}_n = -K_{n_{D\eta}}(\dot{\eta}_n - \dot{\eta}_{n_d}) - K_{n_{P\eta}}(\eta_n - \eta_{n_d}) + \ddot{\eta}_{n_d} \tag{20}$$

where $\eta_{n_d} = [\phi_{n_d}, \theta_{n_d}, \psi_{n_d}]^T$ with ϕ_{n_d} and θ_{n_d} defined in (6) and $\psi_{n_d} = \arctan\left(\frac{\dot{m}_y}{\dot{m}_x}\right)$,

$$K_{n_{D\eta}} = \begin{bmatrix} k_{n_{d\phi}} & 0 & 0 \\ 0 & k_{n_{d\theta}} & 0 \\ 0 & 0 & k_{n_{d\psi}} \end{bmatrix} \tag{21}$$

and

$$K_{n_{P\eta}} = \begin{bmatrix} k_{n_{p\phi}} & 0 & 0 \\ 0 & k_{n_{p\theta}} & 0 \\ 0 & 0 & k_{n_{p\psi}} \end{bmatrix} \tag{22}$$

are the control gains.

For time-varying formation, we propose an auxiliary control law for the follower given by

$$r_i = -K_{i_{D\xi}}\left(\dot{\xi}_i - \dot{\xi}_n - \dot{C}_{ni}(t)\right) - K_{i_{P\xi}}(\xi_i - \xi_n - C_{ni}(t)) + \ddot{\xi}_n + \ddot{C}_{ni}(t) \quad , \quad i = 1, \ldots, n-1 \tag{23}$$

where

$$K_{i_{D\xi}} = \begin{bmatrix} k_{i_{dx}} & 0 & 0 \\ 0 & k_{i_{dy}} & 0 \\ 0 & 0 & k_{i_{dz}} \end{bmatrix} \tag{24}$$

and

$$K_{i_{P\xi}} = \begin{bmatrix} k_{i_{px}} & 0 & 0 \\ 0 & k_{i_{py}} & 0 \\ 0 & 0 & k_{i_{pz}} \end{bmatrix} \tag{25}$$

are the control gains. For the orientation control of the followers we propose

$$\tilde{\tau}_i = -K_{i_{D\eta}} \left(\dot{\eta}_i - \dot{\eta}_{i_d}\right) - K_{i_{P\eta}} \left(\eta_i - \eta_{i_d}\right) + \ddot{\eta}_{i_d} \quad , \quad i = 1, \ldots, n-1 \tag{26}$$

where

$$K_{i_{D\eta}} = \begin{bmatrix} k_{i_{d\phi}} & 0 & 0 \\ 0 & k_{i_{d\theta}} & 0 \\ 0 & 0 & k_{i_{d\psi}} \end{bmatrix} \tag{27}$$

and

$$K_{i_{P\eta}} = \begin{bmatrix} k_{i_{p\phi}} & 0 & 0 \\ 0 & k_{i_{p\theta}} & 0 \\ 0 & 0 & k_{i_{p\psi}} \end{bmatrix} \tag{28}$$

are the control gains and $\psi_{i_d} = \psi_n$.

The main result of this paper is the following

Theorem 1. Consider the system (7) and the control laws (17)-(26). Suppose that $K_{n_{D\xi}}, K_{n_{P\xi}} > 0$ and $K_{i_{D\xi}}, K_{i_{P\xi}} > 0$, for $i = 1, \ldots, n-1$. Then in the locally closed-loop system (7)-(17)-(26), the leader R_n converge to the desired trajectory, i.e. $\lim_{t\to\infty} \left(\xi_n\left(t\right) - m\left(t\right)\right) = 0$, whereas the followers R_i, for $i = 1, 2, \ldots, n-1$ converge to the desired formation, i.e. $\lim_{t\to\infty} \left(\xi_i\left(t\right) - \xi_n\left(t\right) - C_{ni}(t)\right) = 0$.

Proof. The closed-loop system (7)-(17)-(26) is given by

$$\ddot{\xi} = A\dot{\xi} + B\xi + C \tag{29}$$

where $\xi = \left[\xi_1, \xi_2, \ldots, \xi_n\right]^T$,

$$A = -K_{D\xi} \left(\left(\mathcal{L}\left(G\right) + \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \right) \otimes I_3 \right) \tag{30}$$

$$B = -K_{P\xi} \left(\left(\mathcal{L}\left(G\right) + \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \right) \otimes I_3 \right) \tag{31}$$

with

$$K_{D\xi} = \begin{bmatrix} K_{1_{D\xi}} & 0 & \cdots & 0 \\ 0 & K_{2_{D\xi}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_{n_{D\xi}} \end{bmatrix} \tag{32}$$

$$K_{P\xi} = \begin{bmatrix} K_{1_{P\xi}} & 0 & \cdots & 0 \\ 0 & K_{2_{P\xi}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_{n_{P\xi}} \end{bmatrix} \tag{33}$$

and

$$C = \begin{bmatrix} \ddot{\xi}_n + \ddot{C}_{n1}(t) + K_{1_{D\xi}} \dot{C}_{n1}(t) + K_{1_{P\xi} C_{n1}(t)} \\ \ddot{\xi}_n + \ddot{C}_{n2}(t) + K_{2_{D\xi}} \dot{C}_{n2}(t) + K_{2_{P\xi} C_{n2}(t)} \\ \vdots \\ \ddot{m}(t) + K_{n_{D\xi}} \dot{m}(t) + K_{n_{D\xi}} m(t) \end{bmatrix} \tag{34}$$

Now define the systems errors as

$$e_n = \xi_n - m(t) \tag{35}$$

$$e_i = \xi_i - \xi_n - C_{ni}(t) \quad , \quad i = 1, 2, \ldots, n-1 \tag{36}$$

in matrix form we have

$$e = P\xi - \tilde{C} \tag{37}$$

where $e = \left[e_1, e_2, \ldots, e_n\right]^T$,

$$P = \left(\mathcal{L}\left(G\right) + \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \right) \otimes I_3 \tag{38}$$

and $\tilde{C} = \left[C_{n1}(t), C_{n2}(t), \ldots, m(t)\right]^T$.

The dynamics of the error coordinates are given by

$$\ddot{e} = Q\dot{e} + Se \tag{39}$$

with

$$Q = \begin{bmatrix} -K_{1_{D\xi}} & 0 & \cdots & 0 \\ 0 & -K_{2_{D\xi}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -K_{n_{D\xi}} \end{bmatrix} \tag{40}$$

$$S = \begin{bmatrix} -K_{1_{P\xi}} & 0 & \cdots & 0 \\ 0 & -K_{2_{P\xi}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -K_{n_{P\xi}} \end{bmatrix} \tag{41}$$

Defining

$$L = \left[e_{1_x} \dot{e}_{1_x} e_{1_y} \dot{e}_{1_y} e_{1_z} \dot{e}_{1_z} \cdots e_{n_x} \dot{e}_{n_x} e_{n_y} \dot{e}_{n_y} e_{n_z} \dot{e}_{n_z}\right]^T \tag{42}$$

we can rewrite the error dynamics as

$$\dot{L} = ML \tag{43}$$

where

$$M = \begin{bmatrix} N_{1_x} & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & N_{1_y} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & N_{1_z} & \cdots & 0 & 0 & 0 \\ & \vdots & & \ddots & & \vdots & \\ 0 & 0 & 0 & \cdots & N_{n_x} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & N_{n_y} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & N_{n_z} \end{bmatrix} \tag{44}$$

with

$$N_{i_j} = \begin{bmatrix} 0 & 1 \\ -k_{i_{pj}} & -k_{i_{dj}} \end{bmatrix} \tag{45}$$

for $i = 1, 2, \ldots, n$ and $j = x, y, z$. The matrix M is block diagonal so the analysis is reduced to the study of N_{i_j} which are simple two-dimensional matrices. So for each control law with $k_{i_{pj}}, k_{i_{dj}} > 0$, $i = 1, 2, \ldots, n$, $j = x, y, z$ the total dynamics is asymptotically stable.

We must remember that this result is local, because it depends on a linearization.

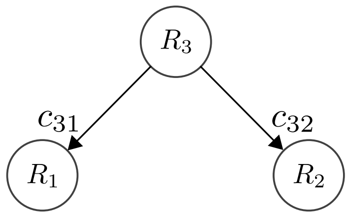

Fig. 3. Formation graph for the simulation.

5. NUMERICAL SIMULATION

The results of a numerical simulation using the control strategy given by (17)-(20) and (23)-(26) are shown below. For the simulation, we considered 3 quad-rotor helicopters and we assume that the linear and rotational velocities of each quad-rotor helicopter can be measured using sensors, while the accelerations are calculated mathematically.

The formation graph employed in the simulation is shown in Fig. 3. The parameters used in the simulation are $k_{i_{dx}}$, $k_{i_{dy}}$, $k_{i_{dz}} = 1$, $k_{i_{px}}$, $k_{i_{py}}$, $k_{i_{pz}} = 1$, $k_{i_{d\phi}}$, $k_{i_{d\theta}}$, $k_{i_{d\psi}} = 3$, $k_{i_{p\phi}}$, $k_{i_{p\theta}}$, $k_{i_{p\psi}} = 2$, $i = 1, 2, 3$, $m = 0.42$ Kg and 9.81m/s^2. The control gains are chosen such that the rotational dynamics converges faster than translational dynamics.

The desired trajectory in the plane XY is a Lemniscate of Gerono with a constant altitude of 1.5 m given by

$$m(t) = \left[3\cos\left(\frac{2\pi t}{T}\right), 1.5\sin\left(\frac{4\pi t}{T}\right), 1.5\right]$$

with a period of $T = 50$ s. The static position vectors are given by

$$c_{32} = [-1.4\sin(\pi/4), -1.4\cos(\pi/4), 0]$$
$$c_{31} = [-1.4\sin(\pi/4), 1.4\cos(\pi/4), 0]$$

Fig. 4 displays the trajectories of the agents in the space. The initial position of the agents are indicated with a circle “o” and the end position with an “x”. Fig. 5 show the time-varying formation of the agents during the simulation. The positions of the agents are marked every 4 s. Is observed how the leader follows the desired trajectory while the followers achieve a time-varying formation. Specifically, the quad rotor helicopters keep a triangular formation which is always oriented to the yaw angle of the leader.

Figs. 6, 7 and 8 show the position and the orientation errors of the agents. Such errors converge to zero.

6. CONCLUSIONS

The paper presents a time-varying formation control for multi-agent systems, where the agents are quad-rotor helicopters. We proposed a decentralized control strategy which ensures that the leader converge to a desired flight trajectory while the followers achieve a time-varying formation. The control strategy is designed based on the

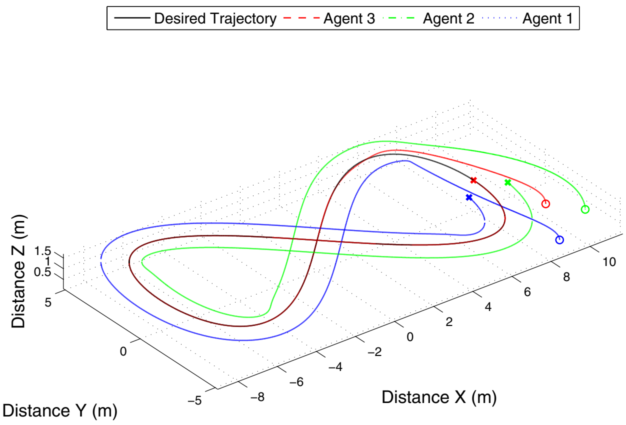

Fig. 4. Trajectories of the agents in the space.

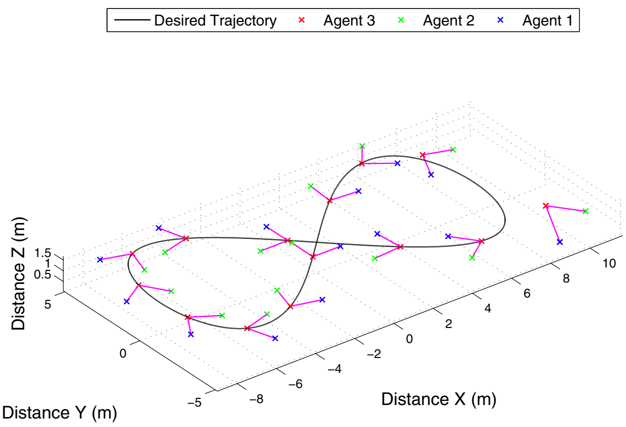

Fig. 5. Formation of the agents during the simulation.

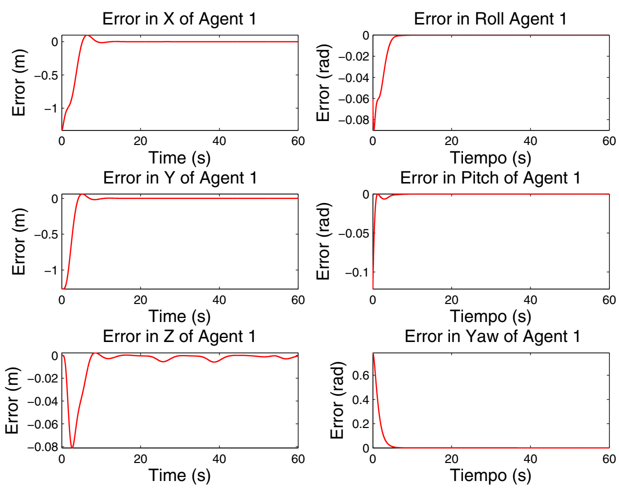

Fig. 6. Errors of agent 1.

linearized dynamical model of the quad-rotor and applied to the nonlinear model of the quad-rotor without linearization. As show in the simulation, the goal is achieved and the system errors converge to zero. As future work, we intend to verify the results obtained experimentally as well as propose a nonlinear control to get a more general result.

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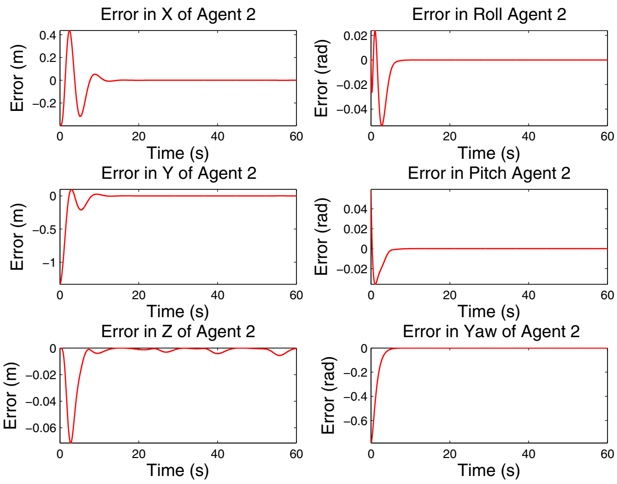

Fig. 7. Errors of agent 2.

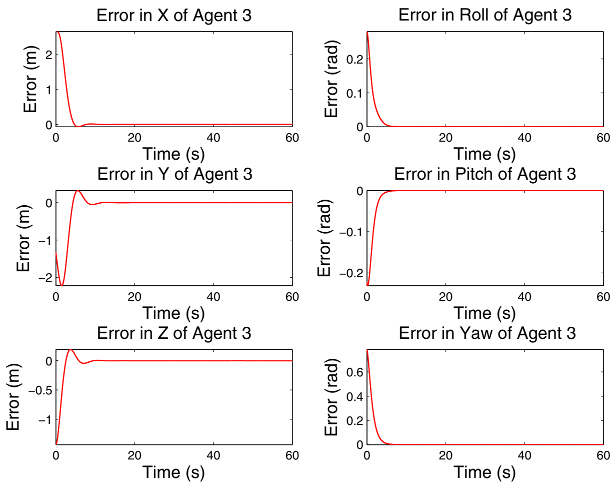

Fig. 8. Errors of agent 3.

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