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Robust Adaptive Control for an Electromechanical System with Backlash

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Abstract: In this paper, an adaptive super-twisting control for driving angular position of a servomotor system with backlash dynamic is proposed. In order to implement the proposed controller, information about angular velocity is estimated by means of a robust differentiator. In spite of being based on a simplified model of the system, the proposed scheme increases robustness against unmodeled dynamics as backlash, as not all the parameters of the system nor the bounds of the perturbations are required to be known. Experimental results considering a wide backlash angle near to 2π, illustrate the feasibility and performance of the proposed control methodology.

Keywords: Robust control, Adaptive control, Servomotor, Backlash nonlinearity.

1. INTRODUCTION

Backlash can be defined as the play between adjacent movable parts, which is present in many mechanical systems, typically those with gears, e.g. the drive train in cars, rolling mills, printing presses and industrial robots. Backlash or backlash-like characteristic, is also common in control systems such as servomechanisms, electronic relay circuits and electromagnetic devices with hysteresis.

Control of systems with backlash nonlinearity is an important and challenging area of control system research. It imposes serious limits to performance, rendering inaccuracies in the position and velocity of a machine and undesirable delays and oscillations, which can even lead to instability.

With respect to backlash in servomechanisms, the motor losses contact with the load for a time instant. This may happen when a disturbance acts on the load, or when the motor applies a corrective action in the opposite sense regarding the load position. Furthermore, at time that backslash gaps open, the movement of the load is free. Therefore, the torque generated by the motor does not acts over the load.

In order to tackle this control problem, several approaches have been designed. For instance, a survey of this problem, including classical PI, linear controllers, non-linear and adaptive control approaches, is presented in (Nordina and Gutman, 2002). Despite basic backslash models were incorporated in order to improve the performance, the backslash nonlinearity is not completely compensated. Moreover, the control design requires an important analytical effort.

A different approach is the use of neural networks and fuzzy logic, as developed in (Guo et al., 2009) and (Su et al., 2003); where a feedback control is designed and compensated by means of approximations of backslash given by neural networks or fuzzy logic. However, this approach demands intensive calculations.

On the other hand, a smooth inverse of backlash was developed to compensate the nonlinearity through a backstepping approach in (Zhou et al., 2007), where the derivation of the control input was used to get the controller. Nonetheless, this is not always possible.

More recently, in (Guo et al., 2004) an adaptive robust control for nonlinear system with unknown input backlash is presented. Nevertheless, this methodology is based in a linear model of the backslash. Moreover, in (Dong and Mo, 2013), an adaptive PID controller is designed for a motor system with backlash which eliminates the vibration caused by the backslash. However, this work is based on the backlash model and analyses the system properties according to engineering experience.

Sliding mode control is used in many applications; in nonlinear plants, it enables high gain accuracy tracking and insensitivity to disturbances and plant parameter variations. The main drawback of sliding mode techniques is the chattering effect, which can damage the actuator due to the high frequency commutation. A sliding mode controller is the super-twisting control algorithm, this controller is designed to converge in a finite-time and ensures the robustness of the system under uncertainties. However, this controller needs to know the bounds of uncertainties and perturbations present on the system. On the other hand, Adaptive Super Twisting approach (see

Shtessel et al. (2012)) represents an alternative to cope with uncertainties as it is not necessary to know their bounds.

While in the literature, to the best of the authors knowledge, it is considered a narrow backlash angle (Nordina and Gutman, 2002; Merzouki et al., 2007); in this work, a backlash angle near to 2π will be considered, i.e. a one tooth gear mechanism.

This paper addresses the angular position control of a DC Servomotor System with Backlash Nonlinearity. With the aim of solving the control problem, an Adaptive Super Twisting Control Algorithm is proposed. Furthermore, in order to implement the proposed controller, necessary information about angular velocity is estimated through a Robust Differentiator. Due to its robustness properties, the proposed control scheme is able enough to compensate parametric uncertainties and unmodeled dynamics as backlash. Experimental results illustrate the performance of the proposed scheme.

The layout of this paper is as follows: Section 2 deals with the problem statement and a system description. In section 3, an Adaptive Super Twisting Controller is derived with the aim of providing robustness under parametric uncertainties and unmodeled dynamics. Furthermore, in order to implement the proposed controller, angular speed is estimated by a Robust Differentiator presented in section 4. Experimental results given in section 5 illustrate the effectiveness of the proposed scheme. Finally, conclusions are drawn.

2. SYSTEM DESCRIPTION

In this paper, a DC servo motor system with a couple of gears with backlash is considered. A schematic view of this system can be seen in the Figure 1. Electrical and mechanical dynamics of a DC motor can be described by following equations

$$\frac{di}{dt} = \frac{1}{L}\left(-Ri + V - K_e\frac{d\theta}{dt}\right), \tag{1}$$

$$\frac{d\omega}{dt} = \frac{1}{J}\left(K_m i - B\frac{d\theta}{dt}\right), \tag{2}$$

where i represent motor current and V the input voltage. θ and ω, denotes the angular position and speed of the rotor. K_e denotes the back electromechanical torque. L correspond to motor armature inductance, B stands for viscous friction and R the resistance of armature winding, while J represent the inertia moment of the moving parts. K_m describes the coefficient of electromechanical torque. Since $L << R$, the motor inductance can be neglected.

Thus equations (1) and (2) can be written in state space form as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x)x_2 + g(x)u + \Delta(x,u,t), \\ f(x) &= -\frac{BR + K_e K_m}{RJ} < 0\,,\; g(x) = \frac{K_m}{RJ} > 0 \end{aligned} \tag{3}$$

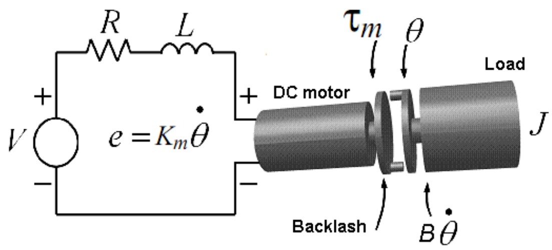


Fig. 1. Electromechanical system diagram.

where $x = (x_1, x_2)^T$ correspond to $(\theta, \dot{\theta})^T$, respectively. u denotes the motor input voltage. $\Delta(x,u,t)$ represent the unmodeled dynamics including external disturbance lumped together.

3. ADAPTIVE SUPER-TWISTING ALGORITHM

In this section, the synthesis of a control law to track a desired angular reference (θ_d), is addressed. The proposed controller is based on an adaptive super-twisting control algorithm, which has been presented in (Shtessel et al., 2012). The main advantage of such algorithm is that, it combines the advantage of chattering reduction and the robustness of high order sliding mode approach. The controller designed ensure its convergence in a finite-time and the robustness of the system under disturbances, where the bounds of the disturbances are not required to be known.

Now, consider the super-twisting control algorithm (see Levant (2003)), which is given by

$$\begin{aligned} u &= -K_1|s|^{1/2}\mathrm{sign}(s) + \upsilon, \\ \dot{\upsilon} &= -K_2\mathrm{sign}(s), \end{aligned} \tag{4}$$

where u represents the control signal, K_1, K_2 are the control gains and s is a sliding variable.

According to adaptive super twisting algorithm (ASTA) approach, the gains K_1 and K_2 are chosen such that they are functions of the sliding surface dynamics as follows

$$K_1 = K_1(t, s, \dot{s}), \quad K_2 = K_2(t, s, \dot{s}). \tag{5}$$

Then, in order to design an adaptive super-twisting control for the uncertain nonlinear system

$$\dot{x} = f(x,t) + g(x,t)u, \tag{6}$$

where $x \in \Re^n$ is the state, $u \in \Re$ the control input, $f(x,t) \in \Re^n$ is a continuous function.

With the objective to satisfy the control purposes, we introduce the following assumptions

Assumption A1. The sliding variable $s = s(x,t) \in \Re$ is designed so that the desired compensated dynamics of the system (6) are achieved in the sliding mode $s = s(x,t) = 0$.

Assumption A2. The relative degree of the system (6) is equal to 1 with respect to the sliding variable s, and the internal dynamics of s are stable.

Then, the dynamics of the sliding variable s are given by

$$\dot{s} = a(x,t) + b(x,t)u. \tag{7}$$

where $a(x,t) = \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} f(x,t)$ and $b(x,t) = \frac{\partial s}{\partial x} g(x)$.

Assumption A3. The function $b(x,t) \in \Re$ is unknown and different from zero $\forall x$ and $t \in [0, \infty)$. Furthermore, $b(x,t) = b_0(x,t) + \Delta b(x,t)$, where $b_0(x,t)$ is the nominal part of $b(x,t)$ which is known, and there exists γ_1 an unknown positive constant such that $\Delta b(x,t)$ satisfies

$$\left| \frac{\Delta b(x,t)}{b_0(x,t)} \right| \leq \gamma_1.$$

Assumption A4. There exist δ_1, δ_2 unknown positive constants such that the function $a(x,t)$ and its derivative are bounded

$$|a(x,t)| \leq \delta_1 |s|^{1/2}, \; |\dot{a}(x,t)| \leq \delta_2. \tag{8}$$

The objective of ASTA approach is to design a continuous control without overestimating the gain, to drive the sliding variable s and its derivative $\dot{s}$ to zero in finite time, under bounded additive and multiplicative disturbances with unknown bounds γ_1, δ_1 and δ_2.

Then, the closed loop system (7) becomes

$$\begin{aligned} \dot{s} &= a(x,t) - K_1 b(x,t)|s|^{1/2}\text{sign}(s) + b(x,t)\upsilon, \\ \dot{\upsilon} &= -K_2 \text{sign}(s), \end{aligned} \tag{9}$$

Now, consider the following change of variable

$$\varsigma = (\varsigma_1, \varsigma_2)^T = (|s|^{1/2}\, sign(s), b(x,t)\upsilon + a(x,t))^T. \tag{10}$$

Then, the system (9) can be written as

$$\dot{\varsigma} = \mathbf{A}(\varsigma_1)\varsigma + g(\varsigma_1)\bar{\varrho}(x,t), \tag{11}$$

where

$$\mathbf{A}(\varsigma_1) = \frac{1}{2|\varsigma_1|} \begin{pmatrix} -2b(x,t)K_1 & 1 \\ -2b(x,t)K_2 & 0 \end{pmatrix}, \qquad g(\varsigma_1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

and $\bar{\varrho}(x,t) = \dot{b}(x,t)\upsilon + \dot{a}(x,t) = 2\varrho(x,t)\frac{\varsigma_1}{|\varsigma_1|}$.

To prove the closed loop stability of the system the following assumption is given

Assumption A5. $\dot{b}(x,t)\upsilon$ is bounded with unknown boundary δ_3, *i.e.* $|\,\dot{b}(x,t)\upsilon\,| < \delta_3$.

Then, system (11) can be rewritten as follows

$$\dot{\varsigma} = \bar{\mathbf{A}}(\varsigma_1)\varsigma \quad \bar{\mathbf{A}}(\varsigma_1) = \frac{1}{2|\varsigma_1|} \begin{pmatrix} -2b(x,t)K_1 & 1 \\ -2b(x,t)K_2 + 2\varrho(x,t) & 0 \end{pmatrix}, \tag{12}$$

where $|\varsigma_1| = |s|^{\frac{1}{2}}$, it is appealing to consider the quadratic function

$$V_0 = \varsigma^T \mathbf{P} \varsigma, \tag{13}$$

where $\mathbf{P}$ is a constant, symmetric and positive matrix, as a strict Lyapunov candidate function for (4). Taking its derivative along the trajectories of (12), we have

$$\dot{V}_0 = -|s|^{-\frac{1}{2}} \varsigma^T \mathbf{Q} \varsigma, \tag{14}$$

almost everywhere, where $\mathbf{P}$ and $\mathbf{Q}$ are related by the Algebraic Lyapunov Equation

$$\bar{\mathbf{A}}^T \mathbf{P} + \mathbf{P}\bar{\mathbf{A}} = -\mathbf{Q}. \tag{15}$$

Since $\bar{\mathbf{A}}$ is Hurwitz if $b(x,t)K_1 > 0$, $2b(x,t)K_2 + 2\varrho(x,t) > 0$, for every $\mathbf{Q} = \mathbf{Q}^T > 0$, there exist a unique solution $\mathbf{P} = \mathbf{P}^T > 0$ for (15), so that V_0 is a strict Lyapunov function.

Remark 1. The stability of the equilibrium $\varsigma = 0$ of (12) is completely determined by the stability of the matrix $\bar{\mathbf{A}}$. However, classical versions of Lyapunov's theorem (Filippov, 1988) cannot be used since they require a continuously differentiable, or at least locally Lipschitz continuous Lyapunov function, though V_0 (13) is continuous but not locally Lipschitz. Nonetheless, as it is explained in Theorem 1 in (Moreno and Osorio, 2012), it is possible to show the convergence properties by means of Zubov's theorem (Pozniak, 2008), that requires only continuous Lyapunov functions. This argument is valid in all the proofs of the present paper, so that no further discussion of these issues will be required.

From Assumption A4 and A5, it follows that

$$0 < \varrho(x,t) < \delta_2 + \delta_3 = \delta_4.$$

Notice that, while ς_1 and ς_2 converge to 0 in finite time, it follows that s and $\dot{s}$ converge to 0 in finite time, too.

The control design based on ASTA approach is formulated in the following theorem.

Theorem 1. (Shtessel et al., 2012) Consider the system (6) in closed-loop with the control (4), expressed in terms of the sliding variable dynamics (7). Furthermore, the assumptions $A1 - A5$ for unknown gains $\gamma_1, \delta_1, \delta_2 > 0$ are satisfied. Then, for given initial conditions $x(0)$ and $s(0)$, there exists a finite time $t_F > 0$ and a parameter ι, as soon as the condition

$$K_1 > \frac{(\lambda + 4\epsilon_*)^2 + 4\delta_4^2 + 4\delta_4(\lambda - 4\epsilon_*^2)}{16\epsilon_*\lambda},$$

holds, if $|s(0)| > \iota$, so that a real 2-sliding mode, *i.e.* $|s| \leq \eta_1$ and $|\dot{s}| \leq \eta_2$, is established $\forall t \geq t_F$, under the action of Adaptive Super-Twisting Control Algorithm (4) with the adaptive gains

$$\begin{aligned} \dot{K}_1 &= \begin{cases} \omega_1 \sqrt{\dfrac{\gamma_1}{2}} \text{sign}(|s| - \iota), & \text{if } K_1 > K_*, \\ K_*, & \text{if } K_1 \leq K_*, \end{cases} \\ K_2 &= 2\epsilon_* K_1, \end{aligned} \tag{16}$$

where $\epsilon_*, \lambda, \gamma_1, \omega_1, \iota$ are arbitrary positive constants, K_* a small positive value, $\eta_1 \geq \iota$ and $\eta_2 > 0$. ⋄

Notice that, according to system (3), the sliding surface for the control (4)-(5) is defined as

$$s = \left(x_2 - \dot{\theta}_d(t) \right) + \lambda_1 \left(x_1 - \theta_d(t) \right), \tag{17}$$

whose time derivatives are given by

$$\dot{s} = \dot{x}_2 - \ddot{\theta}_d(t) + \lambda_1 \left(x_2 - \dot{\theta}_d(t) \right) + b_1 v_1 = a_1 + b_1 v_1 \tag{18}$$

where $\theta_d(t)$ denotes the desired angular trajectory, v_1 the control inputs defined according to (4)-(16).

However, to implement the proposed controller, it is necessary to know the values of (x_1, x_2). Then, to overcome this difficulty, the estimation of unmeasurable terms will be addressed in next section.

4. HIGH-ORDER SLIDING MODE DIFFERENTIATOR

In this section, some results are introduced in order to build a differentiator for computing the real-time derivative of output function with finite-time convergence.

Let $f(t) \in [0, \infty)$, consisting of a bounded Lebesgue-measurable noise with unknown features and $f_0(t)$ an unknown basic signal, whose k-th derivative has a known Lipschitz constant $\tilde{L} > 0$. Thus, the problem of finding real-time robust estimations of $f_0^{(i)}(t)$, for $i = 0, ..., k$; being exact in the absence of measurement noises, is known to be solved by the robust exact differentiator (see Levant (2003) for more details.), which is given by

$$
\begin{aligned}
\dot{z_0} &= \nu_0 \\
\nu_0 &= -\lambda_k \tilde{L}^{\frac{1}{k}} \left| z_0 - f(t) \right|^{\frac{k-1}{k}} sign\left(z_0 - f(t)\right) + z_1 \\
&\vdots \\
\dot{z}_j &= \nu_j \\
\nu_j &= -\lambda_{k-j} \tilde{L}^{\frac{1}{k-j}} \left| z_j - \nu_{j-1} \right|^{\frac{k-j-1}{k-j}} sign\left(z_j - \nu_{j-1}\right) \\
&\quad + z_{j+1} \\
\dot{z}_{k-1} &= -\lambda_1 \tilde{L} sign\left(z_{k-1} - \nu_{k-2}\right),
\end{aligned} \tag{19}
$$

for j=0,...,k-2; where $z_0, z_1, ..., z_j$ are estimates of the j-th derivatives of $f(t)$. In order to assure the initial differentiator convergence, one can take a voluntarily large constant parameter $\tilde{L}_0$ and switch it to the given variable value $\tilde{L}$ after the convergence (Levant and Livne, 2012).

Then, according to (19), the homogeneous differentiator for (3) is given by

$$
O : \begin{cases}
\dot{z}_0 = -\lambda_3 \tilde{L}^{\frac{1}{3}} \left| z_0 - \sigma \right|^{\frac{2}{3}} sign\left(z_0 - \sigma\right) + z_1 \\
\dot{z}_1 = -\lambda_2 \tilde{L}^{\frac{1}{2}} \left| z_1 - \nu_0 \right|^{\frac{1}{2}} sign\left(z_1 - \nu_0\right) + z_2 \\
\dot{z}_2 = -\lambda_1 \tilde{L} sign\left(z_2 - \nu_1\right),
\end{cases} \tag{20}
$$

where σ is the output measurable, $\hat{e}(t) = x - z$ is the estimation error and $Z = (z_0, z_1)^T$, is the estimated state vector.

Consider the system (3) in closed-loop with the adaptive super-twisting controller (4)-(5), using the estimates obtained by the differentiator (19). Then, the trajectories of the system (3) converge in finite-time to the reference signal $\theta_d(t)$.

Remark 2: Since the observer converges in finite-time, the control law and the observer can be designed separately, *i.e.*, the separation principle is satisfied. Thus, if the controller is known to stabilize the process then the stabilization of the system in closed-loop is assured whenever the differentiator dynamics are fast enough to provide an exact calculation of the modes s, $\dot{s}$.

Thus, a general control scheme is given in the Figure 2.

5. EXPERIMENTAL RESULTS

In this section, experimental results carried out on the Modular Servo System (MSS) platform (see Figure 3) are provided to illustrate the feasibility of the proposed methodology. The MSS experimental platform consist of a DC motor with several modules, arranged in a chain, mounted on a metal rail and coupled with small clutches. Modules as backlash and inertia load are attached to the chain (see Figure 3). The measurement system is based on RTDAC/PCI acquisition board equipped with A/D converters. The angle of the load is measured using an incremental encoder and thus the system has no inner feedback for dead zone compensation. The accuracy of angle measurement is 0.1%. Angular velocity of the DC motor is measured through a tachogenerator. The control signal is normalized to ± 1, corresponding to $\pm 12V$ (see Anon1 (2006) for further information).

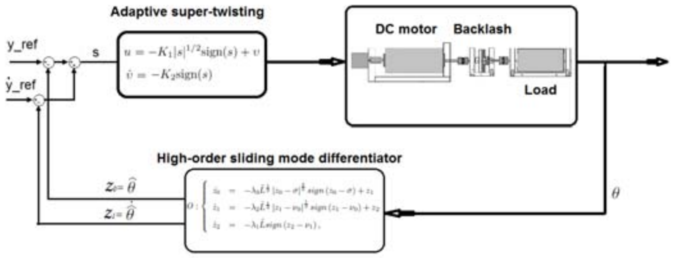


Fig. 2. Proposed control scheme.

Fig. 3. Experimental setup.

On the other hand, controller and observer algorithms were developed in the MATLAB/Simulink environment, while the associated executable code was automatically generated by the RTW/RTWI rapid prototyping environment, with a sampling time of $0.4kHz$ using Euler solver.

Parameters of the DC motor can be seen in the Table 1. Additionally, controller and observer parameters are displayed in the Table 2. Furthermore, in order to provide a comparative study, a PID controller and the Super Twisting Algorithm (STA, see Levant (2003)) were also considered. The experiment, which consist in two cases, will be described in sequel.

Parameter	Value	Unit
Rated voltage V	$[-24, 24]$	Volts
Rated current i	3.1	Ampere
Armature resistance R	2	Ohm
Rotor inertia J	63.41	oz-in^2
Torque back emf K_e	1	ms
Electromechanical torque constant K_m	13	ms

Table 1. DC Servomotor parameters.

ω_1	λ	ι	γ	ϵ_*	λ_1	λ_2	λ_3	$\tilde{L}$
0.1	0.3	0.1	0.1	0.01	10.5	10	0.02	100

Table 2. Controller and differentiator gains.

5.1 *E1. Nominal case.*

The control task consists in tracking a square signal of amplitude 40 rad and frequency of 0.1Hz. The angular response can be seen in the Figure 4, as can be seen, controllers have similar responses.

On the other hand, in the Figure 5 the corresponding control signals are shown. Additionally, in the Figure 8 adaptation of ASTA gains is presented.

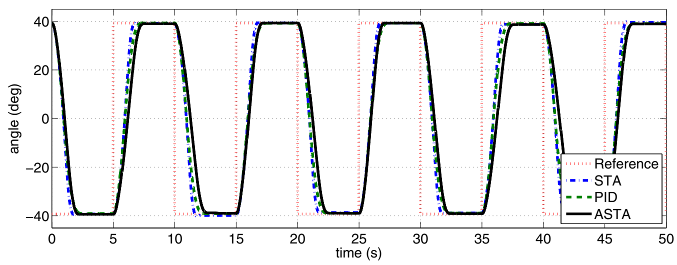


Fig. 4. E1. Servo nominal response.

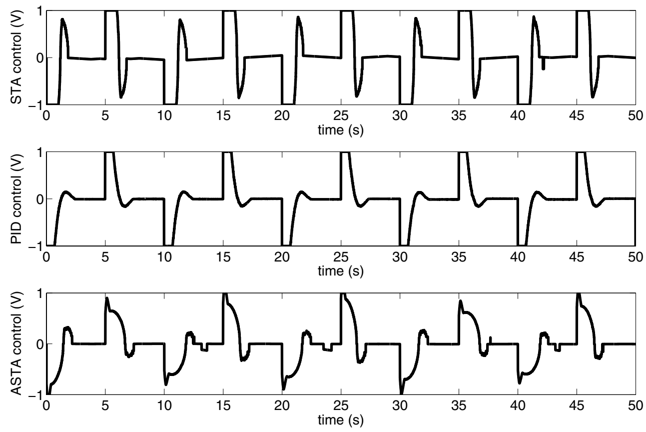


Fig. 5. E1. Control signal: (top) STA, (center) PID, (bottom) ASTA.

Moreover, in the Table 3, Mean Square Error (MSE), Integral Time Absolute Error (ITAE), Norm of the Error ($\|e\|$) and Norm of the Control Signal ($\|u\|$) illustrate the performance of the controllers. As can be seen, ASTA controller requires less control effort.

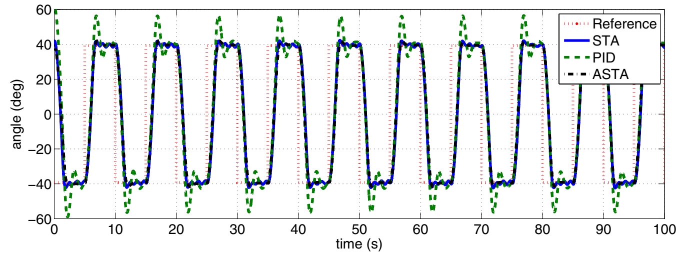
Control	MSE	ITAE	$\|e\|_2$	$\|u\|_2$
STA	1528.94	$3.52{\times}10^6$	3029.05	40.96
PID	1526.60	$3.51{\times}10^6$	3026.74	39.23
ASTA	1527.52	$3.51{\times}10^6$	3027.65	30.31

Table 3. Performance for nominal case.

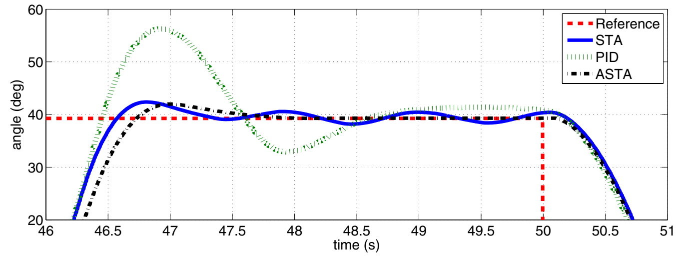
5.2 *E2. Backlash case.*

With the aim of testing the robustness of the proposed controller, a backlash module has been attached between the servo and the inertia load in the experimental platform.

Angular profiles for the second test are shown in the Figure 6, where it can be observed that the proposed controller shows a better response, while for PID and STA controls oscillations are present. Moreover, control signals are shown in the Figure 7. Behaviour of adaptive gains



(a) Angle complete view.



(b) Angle zoomed view.

Fig. 6. E2. Servo backlash response.

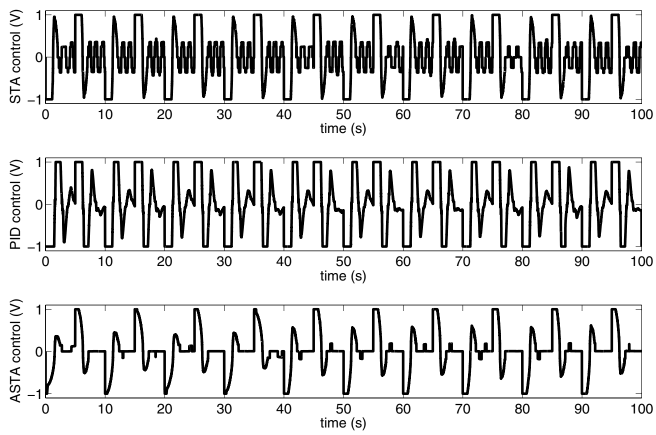


Fig. 7. E2. Control signal: (top) STA, (center) PID, (bottom) ASTA.

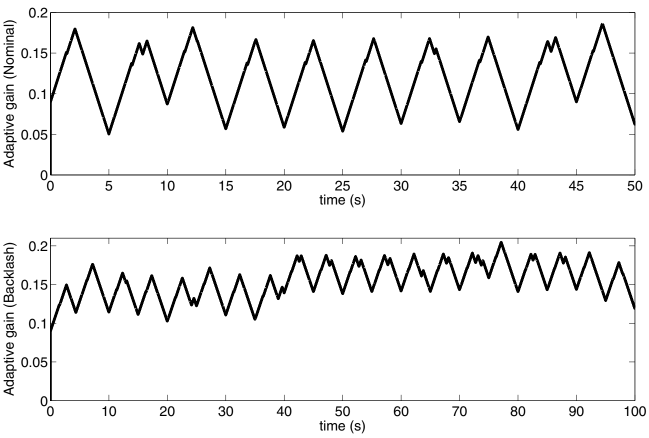


Fig. 8. Adaptive gains. (top) E1. Nominal case. (bottom) E2. Backlash case.

is shown in the Figure 8. Furthermore, in the Table 4 the performance of the controllers according to several indexes for backlash case is illustrated. From the indexes it can be observed, the proposed controller held the best tracking performance and required less control effort among the tested controls.

Control	MSE	ITAE	$\|e\|_2$	$\|u\|_2$
STA	991.54	1.58×10^7	4453.29	85.12
PID	985.90	1.78×10^7	4440.62	97.06
ASTA	958.94	1.54×10^7	4379.48	66.91

Table 4. Performance for backlash case.

6. CONCLUSIONS

An adaptive super-twisting control for driving the angular position of a direct current servomotor system with backslash nonlinearity has been designed. With the purpose of implementing the proposed controller, a robust differentiator was designed for estimating angular velocity. The proposed control scheme has been compared against a PID and the Super Twisting Algorithm, demonstrating its advantages for dealing with hard nonlinear dynamics as backlash. Furthermore, among the tested controllers, the proposed scheme required less control effort and held the best tracking performance. Experimental results demonstrated the robustness and efficiency of the proposed control methodology.

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