Memorias del XVI Congreso Latinoamericano de Control Automático, CLCA 2014
Octubre 14-17, 2014. Cancún, Quintana Roo, México

Observer based Adaptive Super Twisting Control for an Aerodynamical System

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Abstract: In this paper, an Adaptive Super Twisting Control Algorithm (ASTA) is designed to drive the angular tracking of a Two-Rotor Aerodynamical System. Due to the fixed angle of attack of the rotor blades, the system is controlled by varying angular rotor speeds, which introduces highly nonlinear coupling dynamics. With the aim of implementing the ASTA control and taking into account the difficulties for measuring some of its states, a Super Twisting Observer (STO) is used to estimate the unmeasured dynamics and external disturbance, avoiding overestimate the gain. Based on a reduced model of the system, this scheme increases robustness against external disturbance and unmodeled dynamics. Experimental results illustrate the performance of the proposed control scheme, under parametric uncertainties and external disturbance.

Keywords: Sliding-mode control, Flight control, Adaptive control, Robust control, MIMO.

1. INTRODUCTION

Helicopter control has been a difficult task in nonlinear systems control theory due to the inherent complexity, high nonlinearity, cross-coupling dynamics and furthermore it is subjected to external disturbances as wind flows and variations in payload.

Contrary to conventional real-size helicopters (Prouty, 1986), in small setups it is easier to have a propeller with fixed blade angle and to adjust the aerodynamic force by manipulating the propeller speed, instead of having a more complex mechanism as the swashplate. In the *Two-Rotor Aerodynamical System (TRAS)* setup (see Fig.1) the blades of the rotors have a fixed angle of attack, and thus the voltages that regulates the speeds of the rotors are the only control inputs. Nonetheless, the fixed angle of attack of the rotor blades on the TRAS platform adds an extra coupling caused by the reaction of the force necessary to change the propeller speed, instead of removing any of the essential couplings present on a conventional helicopter that need to be illustrated (see Mullhaupt et al. (1997) for further details). Additionally, it is considered a non minimum phase system with unstable zero dynamics.

Control of 2-DOF helicopters has been investigated under algorithms ranging from linear robust control to nonlinear control domains (Wen and Lu, 2008; Ahmed et al., 2009). However, even though feedback linearization is an attractive approach, all the system parameters have to be known. Besides, feedback linearization needs to be complemented with adaptive or robust control in case of uncertainty, where stability becomes a primary target as uncertainty increases. Furthermore, many mathematical helicopter models does not describe satisfactorily inter-axis coupling as gyroscopic effects, whose influence cause a disturbance usually avoided as it leads to a dependence of the system on rotary frequency. Although accurate models can be found, e.g. (Rahideh et al., 2008), whole system modeling represents a challenging task.

This paper addresses the attitude control of a two-rotor aerodynamical system setup. With the aim of solving the attitude control problem, an Adaptive Super Twisting Control Algorithm is proposed.Furthermore, in order to implement the proposed controller, necessary information about unmeasurable states, as well as parametric uncertainties and external disturbances is estimated through Super Twisting Observers (Fridman et al., 2007).

The layout of this paper is as follows: Section 2 deals with the problem statement and a system description containing a mathematical model of a 2-DOF helicopter. In section 3, the design of an Adaptive Super-Twisting Control is addressed. Angular speeds and external disturbance are estimated by means of Super Twisting Observers designed in section 4. Experimental results given in section 5 illustrate the effectiveness of the proposed scheme. Finally, conclusions are drawn.

2. SYSTEM DESCRIPTION

The *Two-Rotor Aerodynamical System (TRAS)* consists of a beam pivoted on its base in such a way that it can rotate freely both in the horizontal (azimuth) and vertical (pitch) planes. At both ends of the beam there are rotors driven by DC motors. A counterbalance arm with a mass at its end is fixed to the beam at the pivot.

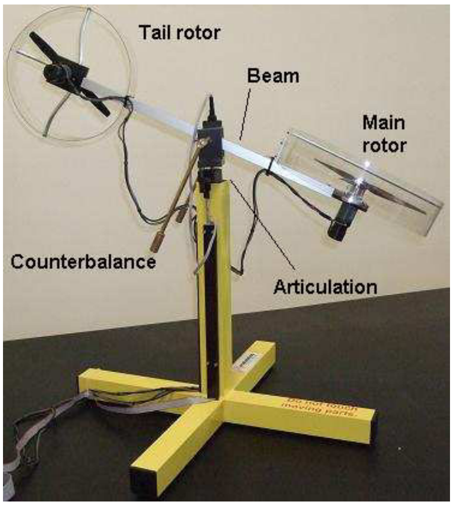

Fig. 1. The two-rotor aerodynamical system.

A reduced mathematical model is obtained when the angle of attack is small (see Sun et al. (2009)). Furthermore, when pitch and azimuth angles are decoupled, and taking into account the dynamics of the rotors, the state space representation of the 2-DOF helicopter model can be written as

$$\begin{aligned}\Sigma_1 &: \begin{cases} \dot{x}_1 = F_v + k_{Fv} x_2, \\ \dot{x}_2 = \dfrac{1}{I_v}(u_v - \dfrac{1}{k_{Hv}} x_2), \end{cases} \\ \Sigma_2 &: \begin{cases} \dot{x}_3 = F_h + k_{Fh} x_4, \\ \dot{x}_4 = \dfrac{1}{I_h}(u_h - \dfrac{1}{k_{Hh}} x_4), \end{cases}\end{aligned} \tag{1}$$

where $X = [x_1, x_2, x_3, x_4]^T = [\theta, \omega_v, \psi, \omega_h]^T$ represents the state vector of the whole system. Taking into account the nature of the system, it will be partitioned in two subsystems as follows: the first subsystem $\Sigma_1(\theta, \omega_v)$ consists of the big propeller driving the rotation around horizontal axis. The second subsystem $\Sigma_2(\psi, \omega_h)$ consists of the small propeller which drives the rotation around vertical axis. u_h, u_v, denote the input voltage supplied to the tail and main rotors respectively. Moreover, F_v, F_h are bounded but unknown terms, containing information about the angular speed. I_v, I_h, represent the moment of inertia of the rotors, while k_{Hh}, k_{Hv} represent the velocity coefficients and k_{Fv}, k_{Fh} the thrust coeffcients.

Assumption 1. The moments of inertia of the rotors are neglected with respect to the moments of inertia of the helicopter (see Lopez-Martinez et al. (2007)).

According to **assumption 1**, the dynamics of subsystems (Σ_1, Σ_2) can be separated into two time scales (Kokotovic and Khalil, 1986), as follows:
·*Slow dynamic* refers to the dynamics of the helicopter, *i.e.* pitch and azimuth dynamics.
·*Fast dynamic* represents the actuator dynamics, *i.e.* motor-propeller groups.

Then, the 2-DOF helicopter model (1) can be represented by the following MIMO singular perturbed interconnected system

$$\dot{\chi}_i = F_i + \mathtt{h}_i \zeta_i, \tag{2a}$$

$$\mu_i \dot{\zeta}_i = \bar{\mathtt{h}}_i \zeta_i + u_i, \qquad i = 1, 2, \tag{2b}$$

where χ_i represents the state vector of the slow subsystems (2.a) such as $\chi_1 = \theta$, $\chi_2 = \psi$, while $\zeta_1 = \omega_v$ and $\zeta_2 = \omega_h$ correspond to the state vector of the fast subsystem (2.b) with $\mu_1 = I_v$, $\mu_2 = I_h$. Besides, $u_1 = u_v$, $u_2 = u_h$, $F_1 = F_v$, $F_2 = F_h$, $\mathtt{h}_1 = k_{Fv}$, $\mathtt{h}_2 = k_{Fh}$, $\bar{\mathtt{h}}_1 = -1/k_{Hv}$, and $\bar{\mathtt{h}}_2 = -1/k_{Hh}$.

Taking into account the magnitude of the moment of inertia of the rotors, whose experimental values satisfy $\mu_i << 1, i = 1, 2$; several methods can be applied to reduce the order of the model. For that, the following assumption is introduced:

Assumption 2. The generated thrust can be considered smooth, having as the dominant forces on the dynamics the aerodynamics effects instead of the inertial forces.

Under this assumption the rotor speeds are supposed to be constants. By applying the classic quasi-steady-state approach (Saksena et al., 1984), μ_i can be considered as zero in the fast dynamic subsystem (2.b). Then, it follows that $0 = \bar{\mathtt{h}}_i \zeta_i + u_i$, for $i = 1, 2$. Solving for ζ_i and substituting in (2.a), we obtain the *Slow dynamics*,

$$\Sigma_i : \{\dot{\chi}_i = F_i + b_i u_i, \quad i = 1, 2, \tag{3}$$

where $b_1 = \mathtt{h}_1 \bar{\mathtt{h}}_1, b_2 = \mathtt{h}_2 \bar{\mathtt{h}}_2$ and $F_i(\cdot)$ include dynamics, parametric uncertainties and external disturbances lumped together, for each subsystem. u_i represents the motor voltage input, while b_i are positive constants. Due to parameters variations and uncertainties, model (3) represents an approximation of the behavior of the whole system.

The angles (x_1, x_4) will be hereafter considered as the measurable outputs. Angular velocities (x_2, x_5), are assumed to be unmeasurable, and terms F_i, for $i = 1, 2$; are unknown. Thus, in order to implement the proposed control laws, angular velocities and terms $F_i, i = 1, 2$; will be estimated through super twisting observers.

3. ADAPTIVE SUPER-TWISTING ALGORITHM

In this section, the synthesis of a control law able to track a desired angular reference (θ_d, ψ_d), is addressed. The proposed controller is based on an adaptive super-twisting control algorithm, which has been presented in (Shtessel et al., 2012). The main advantage of such algorithm is that it combines the advantage of chattering reduction and the robustness of high order sliding mode approach. The controller designed ensure its convergence in a finite-time and the robustness of the system under uncertainties, where the bounds of the uncertainties are not required to be known.

Now, consider the super-twisting control algorithm (Levant, 2003), which is given by

$$\begin{aligned} u &= -K_1|s|^{1/2}\mathrm{sign}(s) + \upsilon, \\ \dot{\upsilon} &= -K_2\mathrm{sign}(s), \end{aligned} \tag{4}$$

where u represents the control signal, K_1, K_2 are the control gains and s is a sliding variable.

According to adaptive super-twisting algorithm (ASTA) approach, the gains K_1 and K_2 are chosen such that they are functions of the sliding surface dynamics as follows

$$K_1 = K_1(t, s, \dot{s}), \quad K_2 = K_2(t, s, \dot{s}). \tag{5}$$

Now, in order to design an adaptive super-twisting control for the uncertain nonlinear system

$$\dot{x} = f(x,t) + g(x,t)u, \tag{6}$$

where $x \in \Re^n$ is the state, $u \in \Re$ the control input, $f(x,t) \in \Re^n$ is a continuous function.

Next, we introduce the following assumptions

Assumption 3. The sliding variable $s = s(x,t) \in \Re$ is designed so that the desired compensated dynamics of the system (6) are achieved in the sliding mode $s = s(x,t) = 0$.

Assumption 4. The relative degree of the sliding variable s, is equal to 1 and the internal dynamics are stable.

Then, the dynamics of the sliding variable s are given by

$$\dot{s} = a(x,t) + b(x,t)u. \tag{7}$$

where $a(x,t) = \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} f(x,t)$ and $b(x,t) = \frac{\partial s}{\partial x} g(x)$.

Assumption 5. The function $b(x,t) \in \Re$ is unknown and different from zero $\forall x$ and $t \in [0, \infty)$. Furthermore, $b(x,t) = b_0(x,t) + \Delta b(x,t)$, where $b_0(x,t)$ is the nominal part of $b(x,t)$ which is known, and there exists γ_1 an unknown positive constant such that $\Delta b(x,t)$ satisfies

$$\left| \frac{\Delta b(x,t)}{b_0(x,t)} \right| \le \gamma_1.$$

Assumption 6. There exist δ_1, δ_2 unknown positive constants such that the function $a(x,t)$ and its derivative are bounded

$$|a(x,t)| \le \delta_1 |s|^{1/2}, \ |\dot{a}(x,t)| \le \delta_2. \tag{8}$$

The objective of ASTA approach is to design a continuous control without overestimating the gain, to drive the sliding variable s and its derivative $\dot{s}$ to zero in finite time, under bounded additive and multiplicative disturbances with unknown bounds γ_1, δ_1 and δ_2.

Then, the closed loop system (7) becomes

$$\begin{aligned} \dot{s} &= a(x,t) - K_1 b(x,t)|s|^{1/2}\mathrm{sign}(s) + b(x,t)\upsilon, \\ \dot{\upsilon} &= -K_2\mathrm{sign}(s), \end{aligned} \tag{9}$$

Now, consider the following change of variable

$$\varsigma = (\varsigma_1, \varsigma_2)^T = (|s|^{1/2}\, sign(s), b(x,t)\upsilon + a(x,t))^T. \tag{10}$$

Then, the system (9) can be written as

$$\dot{\varsigma} = \tilde{A}(\varsigma_1)\varsigma + \tilde{g}(\varsigma_1)\bar{\varrho}(x,t), \tag{11}$$

where

$$\tilde{A}(\varsigma_1) = \frac{1}{2\,|\varsigma_1|} \begin{pmatrix} -2b(x,t)K_1 & 1 \\ -2b(x,t)K_2 & 0 \end{pmatrix}, \quad \tilde{g}(\varsigma_1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and $\bar{\varrho}(x,t) = \dot{b}(x,t)\upsilon + \dot{a}(x,t) = 2\varrho(x,t)\frac{\varsigma_1}{|\varsigma_1|}$.

To prove the closed loop stability of the system,

Assumption 7. $\dot{b}(x,t)\upsilon$ is bounded with unknown boundary δ_3, *i.e.* $|\, \dot{b}(x,t)\upsilon \,| < \delta_3$.

Then, system (11) can be rewritten as follows

$$\dot{\varsigma} = \bar{A}(\varsigma_1)\varsigma, \ \bar{A}(\varsigma_1) = \frac{-1}{|\varsigma_1|} \begin{pmatrix} b(x,t)K_1 & -\frac{1}{2} \\ b(x,t)K_2 - \varrho(x,t) & 0 \end{pmatrix} \tag{12}$$

where $|\varsigma_1| = |s|^{1/2}$, it is appealing to consider the quadratic function

$$V_0 = \varsigma^T \tilde{P} \varsigma, \tag{13}$$

where $\tilde{P}$ is a constant, symmetric and positive matrix, as a strict Lyapunov candidate function for (4). Taking its derivative along the trajectories of (12), we have

$$\dot{V}_0 = -\,|s|^{-1/2}\,\varsigma^T \tilde{Q} \varsigma, \tag{14}$$

almost everywhere, where $\tilde{P}$ and $\tilde{Q}$ are related by the Algebraic Lyapunov Equation

$$\bar{A}^T \tilde{P} + \tilde{P}\bar{A} = -\tilde{Q}. \tag{15}$$

Since $\bar{A}$ is Hurwitz if $b(x,t)K_1 > 0$, $2b(x,t)K_2 + 2\varrho(x,t) > 0$, for every $\tilde{Q} = \tilde{Q}^T > 0$, there exist a unique solution $\tilde{P} = \tilde{P}^T > 0$ for (15), so that V_0 is a strict Lyapunov function.

Remark 1. The stability of the equilibrium $\varsigma = 0$ of (12) is completely determined by the stability of the matrix $\bar{A}$. However, classical versions of Lyapunov 's theorem (Filippov, 1988) cannot be used since they require a continuously differentiable, or at least locally Lipschitz continuous Lyapunov function, though V_0 (13) is continuous but not locally Lipschitz. Nonetheless, as it is explained in Theorem 1 in (Moreno and Osorio, 2012), it is possible to show the convergence properties by means of Zubov 's theorem (Pozniak, 2008), that requires only continuous Lyapunov functions. This argument is valid in all the proofs of the present paper, so that no further discussion of these issues will be required.

From Assumption 6 and 7, it follows that

$$0 < \varrho(x,t) < \delta_2 + \delta_3 = \delta_4.$$

Notice that, while ς_1 and ς_2 converge to 0 in finite time, it follows that s and $\dot{s}$ converge to 0 in finite time, too.

The control design based on ASTA approach is formulated in the following theorem

Theorem 1. (Shtessel et al., 2012) *Consider the system (6) in closed-loop with the control (4), expressed in terms of the sliding variable dynamics (7). Furthermore, the assumptions 3 − 7 for unknown gains $\gamma_1, \delta_1, \delta_2 > 0$ are satisfied. Then, for given initial conditions $x(0)$ and $s(0)$, there exists a finite time $t_F > 0$ and a parameter ι, as soon as the condition*

$$K_1 > \frac{(\lambda + 4\epsilon_*)^2 + 4\delta_4^2 + 4\delta_4(\lambda - 4\epsilon_*^2)}{16\epsilon_*\lambda},$$

holds, if $|s(0)| > \iota$, so that a real 2-sliding mode, i.e. $|s| \leq \eta_1$ and $|\dot{s}| \leq \eta_2$, is established $\forall t \geq t_F$, under the action of Adaptive Super-Twisting Control Algorithm (4) with the adaptive gains

$$\dot{K}_1 = \begin{cases} \omega_1\sqrt{\frac{\gamma_1}{2}}\text{sign}(|s| - \iota), & \text{if } K_1 > K_*, \\ K_*, & \text{if } K_1 \leq K_*, \end{cases} \quad (16)$$
$$K_2 = 2\epsilon_* K_1,$$

where $\epsilon_*, \lambda, \gamma_1, \omega_1, \iota$ are arbitrary positive constants, K_* a small positive constant, $\eta_1 \geq \iota$ and $\eta_2 > 0$. ⋄

Notice that, according to subsystems (3), the sliding surface for the control (4)-(5) is defined as

$$\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} \chi_1 - \theta_d(t) \\ \chi_2 - \psi_d(t) \end{bmatrix}, \quad (17)$$

whose time derivatives are given by

$$\dot{\sigma} = \begin{pmatrix} F_1 - \dot{\theta}_d(t) + b_1 v_1 \\ F_2 - \dot{\psi}_d(t) + b_2 v_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 v_1 \\ a_2 + b_2 v_2 \end{pmatrix}, \quad (18)$$

where $(\theta_d(t), \psi_d(t))$ are the desired trajectories and (v_1, v_2) the control inputs defined according to (4)-(16).

However, to implement the proposed controller, it is necessary to know the values from the unknown dynamics of F_i, for i=1,2. Then, to overcome this difficulty, the estimation of unmeasured terms will be addressed in next section.

4. SUPER TWISTING OBSERVER

Let us consider a SISO nonlinear system in triangular observable form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= f(x) + g(x)u \end{aligned} \quad (19)$$

where $x = [x_1, \ldots, x_n]^T \in \Re^n$ is the state vector, $y = x_1 \in \Re$ is the output vector and $u \in \Re$ is the unknown input. $f(x)$ and $g(x)$ are bounded smooth scalar functions. It is assumed that the state of the system is uniformly bounded, i.e. for all $t > 0$, $| x_i(t) |< d_i$, and that for all $t > 0$:

$$\begin{gathered} | f(x) | < C_1, | \dot{f}(x) | <, \bar{C}_1 \\ | g(x) | < C_2, | \dot{g}(x) | < \bar{C}_2, \\ | u | < C_3, | \dot{u} | < \bar{C}_3, \end{gathered}$$

where C_i and $\bar{C}_i$ are some know positive scalars. Let us design the following observer proposed by (Floquet and Barbot, 2007)

$$\begin{aligned} \dot{\hat{x}}_1 &= \tilde{x}_2 + G_1 \mid e_1 \mid^{1/2} sign(e_1) \\ \dot{\tilde{x}}_2 &= g_1 sign(e_1) \\ \dot{\hat{x}}_2 &= E_1\left[\tilde{x}_3 + G_2 \mid e_2 \mid^{1/2} sign(e_2)\right] \\ &\vdots \\ \dot{\tilde{x}}_{n-1} &= E_{n-3} g_{n-2} sign(e_{n-2}) \\ \dot{\hat{x}}_{n-1} &= E_{n-2}\left[\tilde{x}_n + G_{n-1} \mid e_{n-1} \mid^{1/2} sign(e_{n-1})\right] \\ \dot{\tilde{x}}_n &= E_{n-2} g_{n-1} sign(e_{n-1}) \\ \dot{\hat{x}}_n &= E_{n-1}\left[\Lambda + G_n \mid e_n \mid^{1/2} sign(e_n)\right] \\ \dot{\tilde{\Lambda}} &= E_{n-1} g_n sign(e_n) \end{aligned} \quad (20)$$

where, $e_i = \tilde{x}_i - \hat{x}_i$ for $i = 1, \ldots, n$ with $\tilde{x}_1 = x_1$ and $[\tilde{x}, \tilde{\Lambda}]^T = [\tilde{x}_1, \tilde{2}, \ldots, \tilde{n}, \tilde{\Lambda}]^T$ is the output of the observer. For $i = 1, \ldots, n-1$, the scalar functions E_i are defined as

$$E_i = \begin{cases} 1, & \text{if } \mid e_j \mid = \mid \tilde{x}_j - \hat{x}_j \mid, \leq \epsilon_0 \\ 0, & \text{if } \mid e_j \mid > \epsilon_0, \end{cases} \quad (21)$$

where ϵ_0 is a small positive constant. The observer gains G_i and g_i are positive scalars.

Proposition 1. Consider the system (1) can be written as in (3), and consider that Assumptions 1-2 are satisfied for each subsystem. Then, under these assumptions, system (3) in closed-loop with the adaptive super-twisting controller (4)-(5), using the estimates obtained by the differentiator (20), guarantees that the trajectories of the system (1) converge in finite-time to the reference signal (θ^*, ψ^*).

Remark 2: Since the observer converges in finite-time, the control law and the observer can be designed separately, *i.e.*, the separation principle is satisfied. Thus if the controller is known to stabilize the process then the stabilization of the system in closed-loop is assured whenever the super twisting observer dynamics are fast enough to provide an exact evaluation of the modes σ, $\dot{\sigma}$.

5. EXPERIMENTAL RESULTS

In this section, experimental results carried out on the TRAS platform (Fig. 1) are provided to illustrate the feasibility of the proposed methodology.

Control scheme algorithms were developed in the MATLAB/Simulink environment, while the associated executable code was automatically generated by the RTW environment, with a sampling time of $0.01s$ (for more information see Anon1 (2006)). Controller and observer parameters are displayed in the Tables 1-2. Furthermore, a Cross PID was also considered for a comparative study.

5.1 Nominal case (E1).

For nominal case, the reference to be tracked consists of a sinusoidal signal of amplitude 0.2 rad and frequency of

	ω_i	λ_i	ι_i	γ_i	ϵ_{*i}
Pitch	0.1	1	0.1	0.1	0.1
Azimuth	0.1	3	0.1	0.1	0.1

Table 1. ASTA control parameters.

	G_{i1}	G_{i2}	G_{i3}	g_{i1}	g_{i2}	g_{i3}
Pitch	50	450	750	4000	10	10
Azimuth	50	150	500	1000	10	10

Table 2. STO parameters.

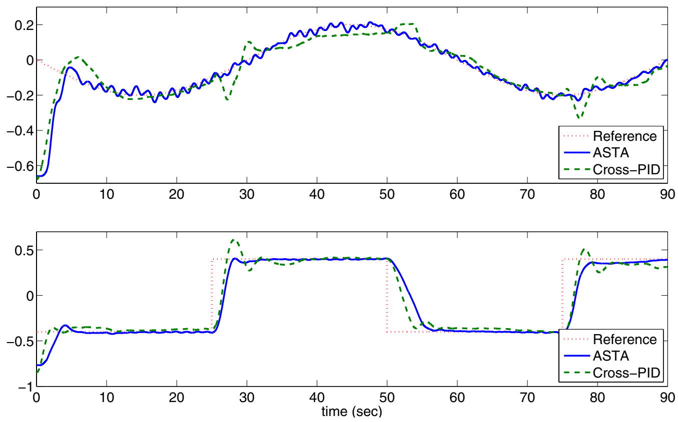

Fig. 2. E1 - Responses. (Top) Pitch, (Bottom) Azimuth.

Angle	Control	MSE	IAE	ITAE	$\|u\|_2$
Pitch	ASTA	0.009	262.33	4267.98	24.25
	Cross-PID	0.009	446.33	14126.22	43.56
Azimuth	ASTA	0.04	908.84	41075.95	27.33
	Cross-PID	0.03	760.66	36919.34	17.17

Table 3. Nominal case performance.

1/60Hz for pitch angle. On the other hand, for azimuth angle a square reference of amplitude 0.4 rad and a frequency of 1/50Hz has been chosen.

Figure 2 shows plots of the angular responses, it can be seen that both controllers rejected cross-coupling, having a better response the proposed scheme. In the Figure 3, can be seen the control signals applied to rotors by the ASTA and Cross-PID controllers. Control signals are normalized and takes values between the range [-1, +1] corresponding to the input voltage range [-24V, +24V]. Furthermore, the control signals have been saturated in the interval [0.2, 1] for pitch control and [-0.3, 0.3] for azimuth control, by recommendation of the manufacturer to avoid any damage into the platform. Additionally, adaptation of super-twisting control gain is presented in the Figure 4. A performance comparison between both controllers according to several indexes can be seen in the Table 3, last column on the right illustrate the control effort.

5.2 Extra mass case (E2).

With the aim of testing the robustness of the proposed scheme, an extra mass of 25% has been attached to the main rotor at 20 seconds after the beginning of the experiments.

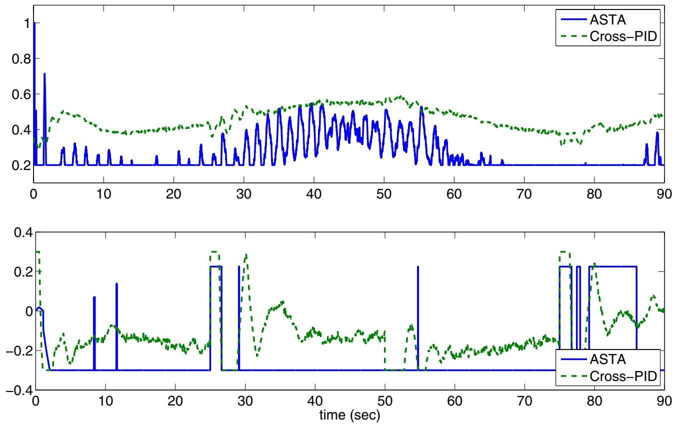

Fig. 3. E1 - Control signals. (Top) Pitch control, (Bottom) Azimuth control.

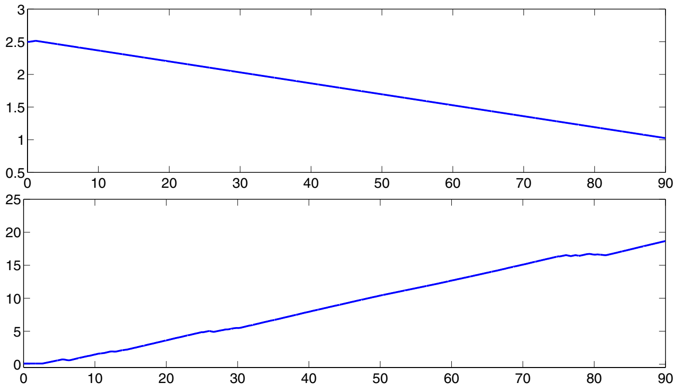

Fig. 4. E1 - Adaptive gains. (Top) Pitch gain. (Bottom) Azimuth gain.

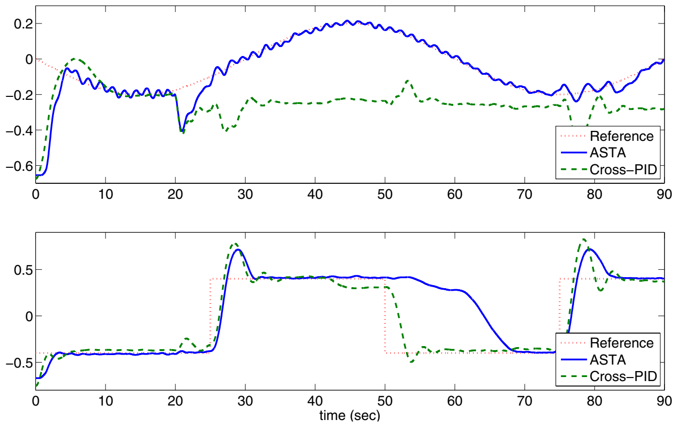

Fig. 5. E2 - Responses. (Top) Pitch, (Bottom) Azimuth.

Angular plots under for extra mass are shown in Figure 5. ASTA scheme kept good tracking along the experiment. However, as ASTA azimuth control signal reached the saturation level, reference tracking was slower. On the other hand, Cross-PID was unable to handle mass increment. Extra mass increase power demand, as can be seen in Figure 6. Adaptive gains help to reject the perturbation applied, Figure 7 shows their behavior. Table 4 illustrates several performance indexes for extra mass case.

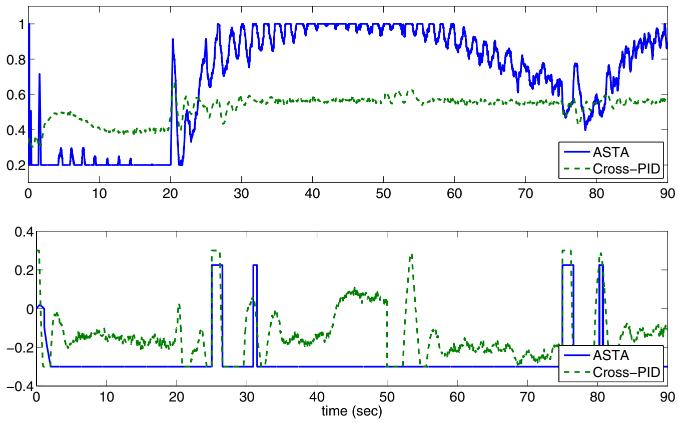

Fig. 6. E2 - Control signals. (Top) Pitch control, (Bottom) Azimuth control.

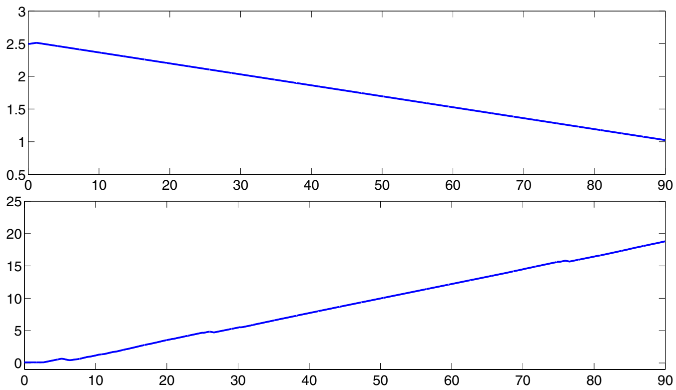

Fig. 7. E2 - Adaptive gains. (Top) Pitch gain, (Bottom) Azimuth gain.

Angle	Control	MSE	IAE	ITAE	$\|u\|_2$
Pitch	ASTA	0.011	363.85	8324.40	69.12
	Cross-PID	0.067	1947.91	89474.44	50.11
Azimuth	ASTA	0.127	1802.89	92926.46	27.53
	Cross-PID	0.031	842.70	40080.06	18.81

Table 4. Extra mass case performance.

6. CONCLUSIONS

An adaptive super-twisting algorithm for a two degrees of freedom helicopter platform has been designed. With the aim of implementing the proposed controller, a super twisting observer was designed for estimating the unmeasurable states as well as external disturbances. The proposed scheme has been compared with a Cross PID controller, demonstrating a better performance when facing external disturbances. Experimental results illustrated the robustness and the efficiency of the proposed methodology.

ACKNOWLEDGEMENTS

This work was partially supported by the Mexican CONACYT (Ciencia Basica) under grant no. 105799 and PAICYT-Universidad Autonoma de Nuevo Leon.

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