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An SP-SD-type Global Adaptive Tracking Controller for Robot Manipulators with Bounded Inputs

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Abstract: In this work, an SP-SD-type adaptive scheme for the global tracking control of robot manipulators with constrained inputs is proposed. Compared with adaptive controllers previously developed in a bounded-input context, the proposed approach guarantees the tracking objective globally, avoiding discontinuities throughout the scheme, preventing the inputs to reach their natural saturation bounds, and imposing no saturation-avoidance restriction on the control gains. Experimental results show the efficiency of the proposed scheme.

Keywords: Adaptive control, global tracking, bounded inputs, robot manipulators, saturation.

1. INTRODUCTION

Motion control of robot manipulators with constrained inputs has proven to be a challenging task. In addition to the nonautonomous nature of the closed loop, the designer has to deal with the analytical complications and practical limitations entailed by the input restrictions. For instance, since the desired motion implies a specific form (time variation) of each of the reaction and inherent (generalized) forces involved in the system dynamics, shaped through the external driving devices, only trajectories ensuring that the mixed effect of such forces stay within the input physical ranges can be tracked. Any attempt to track a trajectory that does not fulfill such a characteristic would not only fail to accomplish the control goal but would also force the actuators to go beyond their natural capabilities giving rise to unexpected closed-loop performances.

In order to prevent such input-saturation-induced drawbacks, several bounded tracking control schemes have been proposed under various analytical frameworks. For instance, assuming that exact system parameter values and accurate position measurements are available, an output-feedback scheme has been developed in (Loría and Nijmeijer, 1998). This approach involves a reproduction of the various (reaction and inherent) forces that are present in the system dynamics, calculated involving the current values of the actual position variables but the desired velocity and acceleration time-variations. A suitable desired trajectory $q_d(t)$ keeps such shaping/compensation terms appropriately bounded and these in turn render $q_d(t)$ a solution of the closed-loop system. In order to accomplish the asymptotic tracking goal, correction terms on the position error and on an auxiliary state vector variable that approximates the velocity error —computed through an auxiliary dynamics— are included in the control law. Their associated control gains are applied to sigmoidal functions —specifically, the hyperbolic tangent— of the referred error variables, giving rise to bounded nonlinear P and D type terms. In a frictionless setting, such an algorithm was proven to semi-globally stabilize the closed loop. An extended version of the controller of (Loría and Nijmeijer, 1998) was further presented in (Santibáñez and Kelly, 2001) by additionally including *desired* viscous friction forces. Through such an additional consideration, global tracking is achieved for appropriate desired trajectories. An alternative output-feedback tracking scheme that keeps a *Computed-Torque*-like structure was proposed in (Dixon *et al.*, 1999). It considers the same form of the gravity, viscous friction, and Coriolis and centrifugal computed force vectors used in (Santibáñez and Kelly, 2001), but a special form of inertial (complemented) force vector where bounded nonlinear P and D terms analog to those of (Loría and Nijmeijer, 1998) are included. Semi-global tracking is concluded in (Dixon *et al.*, 1999). After the previously mentioned output-feedback approaches, two state-feedback schemes were proposed in (Aguiñaga-Ruiz *et al.*, 2009). While the first of such approaches keeps an SP-SD+ structure, with separated saturating proportional (SP) and saturating derivative (SD) error correction terms, the second one has an SPD+ form where both the P and D error correction terms are included within a single saturation function. Shaping/compensation terms analog

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to those of (Santibáñez and Kelly, 2001) are included, and a generalized type of saturation functions is involved. Both algorithms achieve global tracking for appropriate desired trajectories. Furthermore, an output-feedback extension of the SP-SD+ approach of (Aguiñaga-Ruiz *et al.*, 2009) was developed in (Zavala-Río *et al.*, 2011), where global tracking is achieved avoiding velocity measurements.

Because of the considered shaping/compensation terms, which involve the expressions of the system dynamics, the implementation of the above mentioned saturating schemes becomes difficult when the system parameters are uncertain. In view of such an additional restriction, a state-feedback bounded adaptive tracking controller was alternatively presented in (Dixon *et al.*, 1999). Such an adaptive approach considers SP and SD type correction terms similarly structured than those defined in (Loría and Nijmeijer, 1998) and (Santibáñez and Kelly, 2001) but involving online measurements of both the positions and the velocities. Additionally, adaptive *desired compensation* terms of the system dynamics involving parameter estimators are included. The adaptation algorithm is defined in terms of a discontinuous auxiliary dynamics by means of which the parameter estimators are prevented to take values beyond some pre-specified limits, which consequently keeps the adaptive compensation terms bounded. The tracking goal was proven to be accomplished for adequate desired trajectories, with a region of attraction that can be enlarged through the control gain values. It is worth pointing out that by the way the SP and SD terms are defined in (Dixon *et al.*, 1999), the bound of the control signal at every link happens to be defined in terms of the sum of the P and D control gains (and of an additional term involving the bounds of the parameter estimators). This limits the choice of such gains if the natural actuator bounds (or arbitrary input bounds) are aimed to be prevented. This, in turn, constrains the closed-loop region of attraction. Furthermore, as far as the authors are aware, a bounded adaptive tracking scheme guaranteeing the motion control objective globally, preventing input saturation, and avoiding discontinuities throughout the scheme, is still missing in the literature. Even though such achievements have been recently succeeded in a regulation context (López-Araujo *et al.*, 2013), the development of a tracking controller with analog characteristics remains an open problem requiring a more general and complex formulation within a more elaborated analytical framework.

In this work, we propose an SP-SD-type scheme for the global adaptive tracking control of robot manipulators with saturating inputs under parameter uncertainty. With respect to the bounded adaptive controllers previously developed, the proposed approach guarantees the adaptive tracking objective for any initial condition (globally), avoiding discontinuities throughout the scheme, preventing the inputs to attain their natural saturation bounds, and imposing no saturation avoidance restriction on the control gains. Additionally, the algorithm proposed in this work is not restricted to the use of a specific saturation function to achieve the required boundedness, but can rather involve any one within a set of bounded passive functions that include the hyperbolic tangent as a particular case. The efficiency of the proposed adaptive scheme is corroborated through experimental results.

2. PRELIMINARIES

Let $X \in \mathbb{R}^{m\times n}$ and $y \in \mathbb{R}^n$. Throughout this work, X_{ij} denotes the element of X at its i^{th} row and j^{th} column, X_i represents the i^{th} row of X, and y_i stands for the i^{th} element of y. 0_n represents the origin of $\mathbb{R}^n$, I_n the $n\times n$ identity matrix, and $\mathbb{R}_+$ the set of nonnegative real numbers, *i.e.* $\mathbb{R}_+ = [0,\infty)$. $\|\cdot\|$ denotes the standard Euclidean norm for vectors, *i.e.* $\|y\| = \sqrt{\sum_{i=0}^n y_i^2}$, and induced norm for matrices, *i.e.* $\|X\| = \sqrt{\lambda_{\max}(X^TX)}$, where $\lambda_{\max}(X^TX)$ represents the maximum eigenvalue of X^TX. We denote $\mathcal{B}_r \subset \mathbb{R}^n$ an origin-centered ball of radius $r > 0$, *i.e.* $\mathcal{B}_r = \{x \in \mathbb{R}^n : \|x\| \leq r\}$. Let $\mathcal{D}$ and $\mathcal{E}$ be subsets of some vector spaces $\mathbb{D}$ and $\mathbb{E}$ respectively. We denote $\mathcal{C}^m(\mathcal{D};\mathcal{E})$ the set of m-times continuously differentiable functions from $\mathcal{D}$ to $\mathcal{E}$. For a dynamic or time variable υ, $\dot\upsilon$ and $\ddot\upsilon$ respectively denote its first- and second-order rate of change. For a continuous scalar function $\phi : \mathbb{R} \to \mathbb{R}$, ϕ' denotes its derivative, when differentiable; $D^+\phi$ its upper right-hand (Dini) derivative, *i.e.* $D^+\phi(\varsigma) = \limsup_{h\to 0^+} \frac{\phi(\varsigma+h)-\phi(\varsigma)}{h}$, with $D^+\phi = \phi'$ at points of differentiability (see *e.g.* Appendix C.2 of (Khalil, 2002)); and ϕ^{-1} its inverse, when invertible.

Let us consider the n-degree-of-freedom (n-DOF) manipulator dynamics with viscous friction (Kelly *et al.*, 2005)

$$H(q)\ddot q + C(q,\dot q)\dot q + F\dot q + g(q) = \tau \qquad (1)$$

where q, $\dot q$, $\ddot q \in \mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity, and acceleration vectors, $H(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, and $C(q,\dot q)\dot q$, $F\dot q$, $g(q)$, $\tau \in \mathbb{R}^n$ are, respectively, the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with $F \in \mathbb{R}^{n\times n}$ being a positive definite constant diagonal matrix whose entries $f_i > 0$, $i = 1,\dots,n$, are the viscous friction coefficients. Some well-known properties characterizing the terms of such a dynamical model are recalled here (Kelly *et al.*, 2005). Subsequently, we denote $\dot H$ the rate of change of H, *i.e.* $\dot H : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n} : (q,\dot q) \mapsto \left[\frac{\partial H_{ij}}{\partial q}(q)\dot q\right]$.

Property 1. The inertia matrix $H(q)$ is a positive definite symmetric bounded matrix, *i.e.* $\mu_m I_n \leq H(q) \leq \mu_M I_n$, $\forall q \in \mathbb{R}^n$, for some positive constants $\mu_m \leq \mu_M$.

Property 2. The Coriolis matrix $C(q,\dot q)$ satisfies:

2.1. $\dot q^T\left[\frac{1}{2}\dot H(q,\dot q) - C(q,\dot q)\right]\dot q = 0$, $\forall (q,\dot q) \in \mathbb{R}^n \times \mathbb{R}^n$
2.2. $\dot H(q,\dot q) = C(q,\dot q) + C^T(q,\dot q)$, $\forall (q,\dot q) \in \mathbb{R}^n \times \mathbb{R}^n$
2.3. $C(w,x+y)z = C(w,x)z + C(w,y)z$, $\forall w,x,y,z \in \mathbb{R}^n$
2.4. $C(x,y)z = C(x,z)y$, $\forall x,y,z \in \mathbb{R}^n$
2.5. $\|C(x,y)z\| \leq k_C\|y\|\|z\|$, $\forall (x,y,z) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$, for some constant $k_C \geq 0$.

Property 3. The viscous friction coefficient matrix satisfies $f_m\|x\|^2 \leq x^TFx \leq f_M\|x\|^2$, $\forall x \in \mathbb{R}^n$, where $0 < f_m \triangleq \min_i\{f_i\} \leq \max_i\{f_i\} \triangleq f_M$.

Property 4. The gravity vector $g(q)$ is bounded, or equivalently, every one of its elements satisfies $|g_i(q)| \leq B_{gi}$, $\forall q \in \mathbb{R}^n$, for some positive constants B_{gi}, $i = 1,\dots,n$.[1]

Property 5. The left-hand side of the robot dynamic model in (1) can be rewritten as

[1] Property 4 is satisfied *e.g.* by robots having only revolute joints (Kelly *et al.*, 2005).

$$H(q,\theta)\ddot{q}+C(q,\dot{q},\theta)\dot{q}+F(\theta)\dot{q}+g(q,\theta)=Y(q,\dot{q},\ddot{q})\theta$$

where $\theta\in\mathbb{R}^p$ is a constant vector whose elements depend exclusively on the system parameters, and $Y(q,\dot{q},\ddot{q})\in\mathbb{R}^{n\times p}$ —the regression matrix— is a **continuous matrix function** whose elements depend exclusively on the configuration, velocity, and acceleration variables and do not involve any of the system parameters. As a matter of fact, every term of the left-hand side of (1) can be analogously rewritten as $H(q,\theta)\ddot{q}=Y_H(q,\ddot{q})\theta$, $C(q,\dot{q},\theta)\dot{q}=Y_C(q,\dot{q})\theta$, $F(\theta)\dot{q}=Y_F(\dot{q})\theta$, and $g(q,\theta)=Y_g(q)\theta$, and actually $Y(q,\dot{q},\ddot{q})=Y_H(q,\ddot{q})+Y_C(q,\dot{q})+Y_F(\dot{q})+Y_g(q)$.

Property 6. Consider the robot dynamics $Y(q,\dot{q},\ddot{q})\theta=H(q,\theta)\ddot{q}+C(q,\dot{q},\theta)\dot{q}+F(\theta)\dot{q}+g(q,\theta)=\tau$. Let $\theta_{Mj}>0$ represent an upper bound of $|\theta_j|$, such that $|\theta_j|\le\theta_{Mj}$, $j=1,\dots,p$, and let $\theta_M\triangleq\big(\theta_{M1},\dots,\theta_{Mp}\big)^T$ and $\Theta\triangleq[-\theta_{M1},\theta_{M1}]\times\cdots\times[-\theta_{Mp},\theta_{Mp}]$.

a. By Properties 4 and 5, there exist positive constants $B_{gi}^{\theta_M}$, $i=1,\dots,n$, such that $|g_i(w,z)|=|Y_{gi}(w)z|\le B_{gi}^{\theta_M}$, $i=1,\dots,n$, $\forall(w,z)\in\mathbb{R}^n\times\Theta$. Furthermore, there exist positive constants $B_{G_{ij}}$, B_{G_i}, and B_G such that $|Y_{gij}(w)|\le B_{G_{ij}}$, $\|Y_{gi}(w)\|\le B_{G_i}$, and $\|Y_g(w)\|\le B_G$, $\forall w\in\mathbb{R}^n$, $i=1,\dots,n$, $j=1,\dots,p$.
b. Let $\mathcal{X}$ and $\mathcal{Y}$ be any compact subsets of $\mathbb{R}^n$. By Properties 1, 2.5, 5, and 6a, there exist positive constants $B_{Di}^{\theta_M}$, $i=1,\dots,n$, such that $|Y_i(w,x,y)z|\le B_{Di}^{\theta_M}$, $i=1,\dots,n$, $\forall(w,x,y,z)\in\mathbb{R}^n\times\mathcal{X}\times\mathcal{Y}\times\Theta$. Furthermore, there exist positive constants $B_{Y_{ij}}$, B_{Y_i}, and B_Y such that $|Y_{ij}(w,x,y)|\le B_{Y_{ij}}$, $\|Y_i(w,x,y)\|\le B_{Y_i}$, and $\|Y(w,x,y)\|\le B_Y$, $\forall(w,x,y)\in\mathbb{R}^n\times\mathcal{X}\times\mathcal{Y}$, $i=1,\dots,n$, $j=1,\dots,p$.

Remark 1. Let us note that under the considerations of Property 6, by Properties 1, 2.5, 3, 5, and 6a, there exist positive constants $\mu_M^{\theta_M}$, $k_C^{\theta_M}$, and $f_M^{\theta_M}$, such that $|Y_i(w,x,y)z|\le|H_i(w,z)y|+|C_i(w,x,z)x|+|F_i(z)x|+|g_i(w,z)|\le\mu_M^{\theta_M}\|y\|+k_C^{\theta_M}\|x\|^2+f_M^{\theta_M}\|x\|+B_{gi}^{\theta_M}$, $i=1,\dots,n$, $\forall(w,x,y,z)\in\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\times\Theta$. Observe from this expression that for any $T_i>B_{gi}^{\theta_M}$, there always exist sufficiently small positive values a and b (for instance, such that $\mu_M^{\theta_M}b+k_C^{\theta_M}a^2+f_M^{\theta_M}a<T_i-B_{gi}^{\theta_M}$) that guarantee $|Y_i(w,x,y)z|<T_i$, $i=1,\dots,n$, on $\mathbb{R}^n\times\mathcal{B}_a\times\mathcal{B}_b\times\Theta$. ◁

Let us suppose that the absolute value of each input τ_i is constrained to be smaller than a given saturation bound $T_i>0$, *i.e.* $|\tau_i|\le T_i$, $i=1,\dots,n$. In other words, letting u_i represent the control variable (controller output) relative to the i^{th} degree of freedom, we have that

$$\tau_i=T_i\,\mathrm{sat}(u_i/T_i)\qquad(2)$$

$i=1,\dots,n$, where $\mathrm{sat}(\cdot)$ is the standard saturation function, *i.e.* $\mathrm{sat}(\varsigma)=\mathrm{sign}(\varsigma)\min\{|\varsigma|,1\}$.

Functions fitting the following definition will be involved.

Definition 1. Given a positive constant M, a nondecreasing Lipschitz-continuous function $\sigma:\mathbb{R}\to\mathbb{R}$ is said to be a ***generalized saturation*** with bound M if

(a) $\varsigma\sigma(\varsigma)>0$ for all $\varsigma\neq0$;
(b) $|\sigma(\varsigma)|\le M$ for all $\varsigma\in\mathbb{R}$.

Lemma 1. Let $\sigma:\mathbb{R}\to\mathbb{R}$ be a generalized saturation with bound M, and let k be a positive constant. Thus,

1. $\lim_{|\varsigma|\to\infty}D^+\sigma(\varsigma)=0$;
2. $\exists\sigma'_M\in(0,\infty)$ such that $0\le D^+\sigma(\varsigma)\le\sigma'_M$, $\forall\varsigma\in\mathbb{R}$;
3. $|\sigma(k\varsigma+\eta)-\sigma(\eta)|\le\sigma'_Mk|\varsigma|$, $\forall\varsigma,\eta\in\mathbb{R}$;
4. $|\sigma(k\varsigma)|\le\sigma'_Mk|\varsigma|$, $\forall\varsigma\in\mathbb{R}$;
5. $\frac{\sigma^2(k\varsigma)}{2k\sigma'_M}\le\int_0^\varsigma\sigma(kr)dr\le\frac{k\sigma'_M\varsigma^2}{2}$, $\forall\varsigma\in\mathbb{R}$;
6. $\int_0^\varsigma\sigma(kr)dr>0$, $\forall\varsigma\neq0$;
7. $\int_0^\varsigma\sigma(kr)dr\to\infty$ as $|\varsigma|\to\infty$;
8. if σ is strictly increasing, then
   (a) $\varsigma[\sigma(\varsigma+\eta)-\sigma(\eta)]>0$, $\forall\varsigma\neq0$, $\forall\eta\in\mathbb{R}$;
   (b) for any constant $a\in\mathbb{R}$, $\bar{\sigma}(\varsigma)=\sigma(\varsigma+a)-\sigma(a)$ is a strictly increasing generalized saturation function with bound $\bar{M}=M+|\sigma(a)|$.

Proof. Points 3 and 4 are a direct consequence of the Lipschitz-continuity of σ and item 2 of the statement (as analogously stated for instance in Lemma 3.3 of (Khalil, 2002) under continuous differentiability). The rest of the points are proven in (López-Araujo *et al.*, 2013).

Our proposal is based on the following assumptions.

Assumption 1. $T_i>B_{gi}$, $\forall i\in\{1,\dots,n\}$.

Assumption 2. The desired trajectory $q_d(t)$ (to be tracked) belongs to $\mathcal{Q}_d\triangleq\big\{q_d\in\mathcal{C}^2(\mathbb{R}_+;\mathbb{R}^n):\|\dot{q}_d(t)\|\le B_{dv},\|\ddot{q}_d(t)\|\le B_{da},\forall t\ge0\big\}$ for some positive constants B_{da} and $B_{dv}<\frac{f_m}{k_C}$ (see Properties 2.5 and 3).

Remark 2. Observe that Assumption 2 does not restrict the location of the target trajectory q_d but rather its first-order and second-order rates of change. Hence, under Assumption 2, desired trajectories defined anywhere in the configuration space may be tracked as long as they give rise to sufficiently slow motions. Let us further point out that the need to restrict the target trajectories is a direct consequence of the bounded nature of the inputs. Indeed observe, from (1) and (2) under the consideration of accurate tracking, *i.e.* $q(t)\equiv q_d(t)$, that only desired trajectories giving rise to left-hand sides of (1) with elements having absolute values lower than the input bounds T_i can be tracked through a control vector u subject to (2). This leads to the need for additional adjustments on the desired trajectory first-order and second-order rate bounds, B_{dv} and B_{da}. Specifications are given in Section 3 in the context of the approach presented here. The additional restriction on B_{dv} stated through Assumption 2, namely $B_{dv}<f_m/k_C$, emerges from the closed-loop stability analysis as a complement to the previously described sufficiently slow desired motion requirement naturally arisen in a bounded-input context. ◁

3. THE PROPOSED ADAPTIVE SCHEME

Let $M_a\triangleq\big(M_{a1},\dots,M_{ap}\big)^T$, and $\Theta_a\triangleq[-M_{a1},M_{a1}]\times\cdots\times[-M_{ap},M_{ap}]$, with M_{aj}, $j=1,\dots,p$, being positive constants such that

$$|\theta_j|<M_{aj}\quad\forall j\in\{1,\dots,p\}\qquad(3)$$

$$B_{gi}^{M_a}<T_i\quad\forall i\in\{1,\dots,n\}\qquad(4)$$

where, in accordance to Property 6a, $B_{gi}^{M_a}$, $i=1,\dots,n$, are positive constants such that $|g_i(w,z)|=|Y_{gi}(w)z|\le B_{gi}^{M_a}$, $\forall(w,z)\in\mathbb{R}^n\times\Theta_a$, and consider (small enough) desired-trajectory-related bound values B_{dv} and B_{da} (in accordance to Assumption 2) such that

$$\big|Y_i\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\vartheta\big|\le B_{Di}^{M_a}<T_i\qquad(5)$$

$i = 1, \ldots, n$, $\forall q \in \mathbb{R}^n$, $\forall \vartheta \in \Theta_a$, $\forall t \geq 0$, where, in accordance to Property 6b, B_{Di}^{Ma}, $i = 1, \ldots, n$, are positive constants such that $|Y_i(w, x, y)z| \leq B_{Di}^{M_a}$, $\forall (w, x, y, z) \in \mathbb{R}^n \times \mathcal{B}_{B_{dv}} \times \mathcal{B}_{B_{da}} \times \Theta_a$. Let us note that Assumption 1 ensures the existence of such positive values M_{aj}, $j = 1, \ldots, p$, satisfying (3) and (4) while, under Assumption 2, through the fulfillment of (4), inequalities (5) can always be satisfied through sufficiently small values of B_{dv} and B_{da} (see Remark 1). Notice further that inequalities (4) are satisfied if $\sum_{j=1}^{p} B_{G_{ij}} M_{aj} < T_i$, $B_{G_i}\|M_a\| < T_i$, or $B_G\|M_a\| < T_i$, $i = 1, \ldots, n$; actually, $\sum_{j=1}^{p} B_{G_{ij}} M_{aj}$, $B_{G_i}\|M_a\|$, or $B_G\|M_a\|$, may be taken as the value of $B_{gi}^{M_a}$ as long as (4) is fulfilled. Similarly, inequalities (5) are satisfied if $\sum_{j=1}^{p} B_{Y_{ij}} M_{aj} < T_i$, $B_{Y_i}\|M_a\| < T_i$, or $B_Y\|M_a\| < T_i$, $i = 1, \ldots, n$, where, in accordance to Property 6b, $B_{Y_{ij}}$, B_{Y_i}, and B_Y are positive constants such that $|Y_{ij}(w, x, y)| \leq B_{Y_{ij}}$, $\|Y_i(w, x, y)\| \leq B_{Y_i}$, and $\|Y(w, x, y)\| \leq B_Y$, respectively, $\forall (w, x, y) \in \mathbb{R}^n \times \mathcal{B}_{B_{dv}} \times \mathcal{B}_{B_{da}}$; in fact, $\sum_{j=1}^{p} B_{Y_{ij}} M_{aj}$, $B_{Y_i}\|M_a\|$, or $B_Y\|M_a\|$, may be taken as the value of $B_{Di}^{M_a}$ as long as (5) is fulfilled.

The proposed adaptive control scheme is defined as

$$u(t, q, \dot{q}, \hat{\theta}) = -s_D(K_D\dot{\bar{q}}) - s_P(K_P\bar{q}) + Y\big(q, \dot{q}_d(t), \ddot{q}_d(t)\big)\hat{\theta} \quad (6)$$

where $\bar{q} = q - q_d(t)$, the position error vector variable; $Y(\cdot,\cdot,\cdot)$ is the regression matrix characterizing the system open-loop structure in accordance to Property 5; $K_P \in \mathbb{R}^{n\times n}$ and $K_D \in \mathbb{R}^{n\times n}$ are positive definite diagonal matrices, *i.e.* $K_P = \mathrm{diag}[k_{P1}, \ldots, k_{Pn}]$ and $K_D = \mathrm{diag}[k_{D1}, \ldots, k_{Dn}]$ with $k_{Pi} > 0$ and $k_{Di} > 0$ for all $i = 1, \ldots, n$; $s_P(x) = \Big[\sigma_{P1}(x_1), \ldots, \sigma_{Pn}(x_n)\Big]^T$ and $s_D(x) = \Big[\sigma_{D1}(x_1), \ldots, \sigma_{Dn}(x_n)\Big]^T$, with $\sigma_{Pi}(\cdot)$, $i = 1, \ldots, n$, being **continuously differentiable generalized saturation functions** with bounds M_{Pi}, and $\sigma_{Di}(\cdot)$, $i = 1, \ldots, n$, being **generalized saturation functions** with bounds M_{Di}, such that

$$M_{Pi} + M_{Di} < T_i - B_{Di}^{M_a} \quad (7)$$

(recall (5)) $i = 1, \ldots, n$; and $\hat{\theta}$ (the parameter estimator) being the output variable of an auxiliary (adaptation) dynamic subsystem defined as

$$\begin{aligned} \dot{\phi} &= -\Gamma Y^T\big(q, \dot{q}_d(t), \ddot{q}_d(t)\big)\big[\dot{\bar{q}} + \varepsilon s_P(K_P\bar{q})\big] \\ \hat{\theta} &= s_a(\phi) \end{aligned} \quad (8)$$

where ϕ is the (internal) state of the auxiliary subsystem in (8); $s_a = \Big(\sigma_{a1}(x_1)\ ,\ \ldots\ ,\ \sigma_{ap}(x_p)\Big)^T$, with $\sigma_{aj}(\cdot)$, $j = 1, \ldots, p$, being **strictly increasing generalized saturation functions** with bounds M_{aj} as defined above, *i.e.* such that inequalities (3)–(5) are satisfied; $\Gamma \in \mathbb{R}^{p\times p}$ is a positive definite diagonal constant matrix, *i.e.* $\Gamma = \mathrm{diag}[\gamma_1 \ldots, \gamma_p]$ with $\gamma_j > 0$ for all $j = 1, \ldots, p$; and ε is a positive constant satisfying

$$\varepsilon < \varepsilon_M \triangleq \min\{\varepsilon_1, \varepsilon_2\} \quad (9)$$

$$\varepsilon_1 \triangleq \sqrt{\frac{\mu_m}{\mu_M^2\beta_P}} \quad , \quad \varepsilon_2 \triangleq \frac{f_m - k_C B_{dv}}{\beta_M + \left(\frac{f_M + \beta_D}{2} + k_C B_{dv}\right)^2}$$

(note that Assumption 2 ensures positivity of ε_2), where $\beta_P \triangleq \max_i\{\sigma'_{PiM} k_{Pi}\}$, $\beta_M \triangleq k_C B_P + \mu_M \beta_P$, $\beta_D \triangleq \max_i\{\sigma'_{DiM} k_{Di}\}$, $B_P \triangleq \big[\sum_{i=0}^{n} M_{Pi}^2\big]^{1/2}$, with σ'_{PiM} and σ'_{DiM} being the bounds of $\sigma'_{Pi}(\cdot)$ and $D^+\sigma_{Di}(\cdot)$ respectively, in accordance to item 2 of Lemma 1, and μ_m, μ_M, k_C, f_m, and f_M as defined through Properties 1–3.

Remark 3. Observe that discontinuous changes are avoided throughout the control scheme in (6),(8). Note further that the proposed controller does not involve the exact values of the elements of θ. It only requires the fulfillment of inequalities (3)–(5). In other words, only strict bounds M_{aj} of $|\theta_j|$, $j = 1, \ldots, p$, (satisfying (4)-(5)) are involved. Note further that an appropriate choice of ε does not require the exact knowledge of the system parameters either. Indeed observe, on the one hand, that an estimation of the right-hand side of (9) may be obtained by means of upper and lower bounds of the system parameters and viscous friction coefficients. On the other hand, the fulfillment of (9) is not necessary but only sufficient for the closed-loop analysis to hold, as shown in the following section. Actually, proving global tracking through (9) is tantamount to show the existence of some $\varepsilon^* \geq \varepsilon_M$ such that, for any $\varepsilon \in (0, \varepsilon^*)$, global stabilization is guaranteed. Hence, the proposed scheme permits successful implementations with values of ε higher than ε_M (up to certain limit, ε^*) without destabilizing the closed loop. Furthermore, by previous arguments, the satisfaction of the restriction on B_{dv} stated through Assumption 2 does not require the exact knowledge of the system parameters either. ◁

Remark 4. The passive character of the generalized saturations (through item (a) of Definition 1), involved in the s_P and s_D vector functions, ensures the required correction role of the SP and SD terms, namely the opposition to displacement and motion errors respectively, furnishing the required stiffness and damping (with respect to the desired trajectory). This is so irrespective to the bounded nature of the involved generalized saturations. Further, under ideal conditions, through the auxiliary dynamics in (8), post-transient errors due to parameter estimation imprecisions are eliminated. Though, (reduced) post-transient variations could still take place in practice due to unmodelled phenomena (such as Coulomb friction), as will be observed through the experimental results shown in Section 5. ◁

4. CLOSED-LOOP ANALYSIS

Consider system (1)-(2) taking $u = u(t, q, \dot{q}, \hat{\theta})$ as defined through (6),(8). Observe that (under Assumptions 1 and 2, the satisfaction of inequalities (3)–(5), and the consideration of (2)) the fulfilment of (7) shows that

$$\begin{aligned} T_i > \big|u_i\big(t, q, \dot{q}, s_a(\phi)\big)\big| = |u_i| = |\tau_i| \quad i = 1, \ldots, n \\ \forall (t, q, \dot{q}, \phi) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p \end{aligned} \quad (10)$$

Thus, under the consideration of Property 5, the closed-loop system takes the form

$$\begin{aligned} H(q)\ddot{\bar{q}} + \big[C(q, \dot{q}) + C\big(q, \dot{q}_d(t)\big)\big]\dot{\bar{q}} + F\dot{\bar{q}} = -s_P(K_P\bar{q}) \\ - s_D(K_D\dot{\bar{q}}) + Y\big(q, \dot{q}_d(t), \ddot{q}_d(t)\big)\bar{s}_a(\bar{\phi}) \end{aligned} \quad (11)$$

$$\dot{\bar{\phi}} = -\Gamma Y^T\big(q, \dot{q}_d(t), \ddot{q}_d(t)\big)\big[\dot{\bar{q}} + \varepsilon s_P(K_P\bar{q})\big] \quad (12)$$

where [2] $\bar{\phi} = \phi - \phi^*$ and $\bar{s}_a(\bar{\phi}) = s_a(\bar{\phi} + \phi^*) - s_a(\phi^*)$, with $\phi^* = \big(\phi_1^*, \ldots, \phi_p^*\big)^T$ such that $s_a(\phi^*) = \theta$, or equivalently,

[2] Observe that, in the error variable space, $q = \bar{q} + q_d(t)$ and $\dot{q} = \dot{\bar{q}} + \dot{q}_d(t)$. However, for the sake of simplicity, $H(q)$, $C(q, \cdot)$, $C(\cdot, \dot{q})$, $\dot{H}(q, \dot{q})$ and $Y(q, \cdot, \cdot)$ will be used.

$\phi_j^* = \sigma_{aj}^{-1}(\theta_j)$, $j = 1, \ldots, p$ (the invertibility is ensured by the strictly increasing character of σ_{aj}). Observe that, by item 8b of Lemma 1, the elements of $\bar{s}_a(\bar{\phi})$, *i.e.* $\bar{\sigma}_{aj}(\bar{\phi}_j) = \sigma_{aj}(\bar{\phi}_j + \phi_j^*) - \sigma_{aj}(\phi_j^*)$, $j = 1, \ldots, p$, turn out to be strictly increasing generalized saturation functions.

Proposition 1. Consider the closed-loop system in (11)-(12) under the satisfaction of Assumptions 1 and 2. Thus, for any positive definite diagonal matrices K_P, K_D and Γ, and any ε satisfying (9), the trivial solution $(\bar{q}, \bar{\phi})(t) \equiv (0_n, 0_p)$ is uniformly stable and, for any initial condition $\big(t_0, \bar{q}(t_0), \dot{\bar{q}}(t_0), \bar{\phi}(t_0)\big) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, the closed loop solution $(\bar{q}, \bar{\phi})(t)$ is bounded and such that $\bar{q}(t) \to 0_n$ as $t \to \infty$ with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq t_0$.

Proof. By (10), we see that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $\forall t \geq 0$. This proves that, under the proposed adaptive scheme, input saturation is avoided. Now, in order to develop the stability/convergence analysis, let us define the scalar function

$$V(t, \bar{q}, \dot{\bar{q}}, \bar{\phi}) = \frac{1}{2}\dot{\bar{q}}^T H(q)\dot{\bar{q}} + \varepsilon s_P^T(K_P\bar{q})H(q)\dot{\bar{q}} + I_P(\bar{q}) + I_a(\bar{\phi}) \quad (13)$$

$$I_P(\bar{q}) \triangleq \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr = \sum_{i=1}^{n} \int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i$$

$$I_a(\bar{\phi}) \triangleq \int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr = \sum_{j=1}^{p} \int_0^{\bar{\phi}_j} \bar{\sigma}_{aj}(r_j)\gamma_j^{-1}dr_j$$

From Property 1 and items 4 and 5 of Lemma 1, we have $W_1(\bar{q}, \dot{\bar{q}}, \bar{\phi}) \leq V(t, \bar{q}, \dot{\bar{q}}, \bar{\phi}) \leq W_2(\bar{q}, \dot{\bar{q}}, \bar{\phi})$, where

$$W_1(\bar{q}, \dot{\bar{q}}, \bar{\phi}) = W_{01}(\bar{q}, \dot{\bar{q}}) + (1 - \alpha)I_P(\bar{q}) + I_a(\bar{\phi}) \quad (14)$$

$$W_{01}(\bar{q}, \dot{\bar{q}}) = \frac{\mu_m}{2}\|\dot{\bar{q}}\|^2 - \varepsilon\mu_M\|s_P(K_P\bar{q})\|\|\dot{\bar{q}}\| + \frac{\alpha}{2\beta_P}\|s_P(K_P\bar{q})\|^2$$

with α being a positive constant satisfying

$$\frac{\varepsilon^2}{\varepsilon_1^2} < \alpha < 1 \quad (15)$$

(see (9)), and

$$W_2(\bar{q}, \dot{\bar{q}}, \bar{\phi}) = W_{02}(\bar{q}, \dot{\bar{q}}) + I_a(\bar{\phi})$$

$$W_{02}(\bar{q}, \dot{\bar{q}}) = \frac{\mu_M}{2}\|\dot{\bar{q}}\|^2 + \varepsilon\mu_M\beta_P\|\bar{q}\|\|\dot{\bar{q}}\| + \frac{\beta_P}{2}\|\bar{q}\|^2$$

Moreover, $W_{01}(\bar{q}, \dot{\bar{q}})$ and $W_{02}(\bar{q}, \dot{\bar{q}})$ may be rewritten as

$$W_{01}(\bar{q}, \dot{\bar{q}}) = \frac{1}{2}\begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T Q_1 \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}$$

$$W_{02}(\bar{q}, \dot{\bar{q}}) = \frac{1}{2}\begin{pmatrix}\|\bar{q}\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T Q_2 \begin{pmatrix}\|\bar{q}\| \\ \|\dot{\bar{q}}\|\end{pmatrix}$$

with $Q_1 = \begin{pmatrix}\frac{\alpha}{\beta_P} & -\varepsilon\mu_M \\ -\varepsilon\mu_M & \mu_m\end{pmatrix}$ and $Q_2 = \begin{pmatrix}\beta_P & \varepsilon\mu_M\beta_P \\ \varepsilon\mu_M\beta_P & \mu_M\end{pmatrix}$.

Note that, by (9) and point 6 of Lemma 1, $W_{01}(\bar{q}, \dot{\bar{q}})$ and $W_{02}(\bar{q}, \dot{\bar{q}})$ are positive definite (since, with $\varepsilon < \varepsilon_M \leq \varepsilon_1$, any α satisfying (15) renders Q_1 positive definite, while the referred condition on ε renders Q_2 positive definite as well), and observe that $W_{01}(0_n, \dot{\bar{q}}) \to \infty$ as $\|\dot{\bar{q}}\| \to \infty$. From this, inequality (15), and points 6, 7, and 8b of Lemma 1 (through which one sees that the integral terms in the right-hand side of (14) are radially unbounded positive definite functions in their respective arguments), $V(t, \bar{q}, \dot{\bar{q}}, \bar{\phi})$ is concluded to be positive definite, radially unbounded, and decrescent. Its derivative along the system trajectories is given by

$$\begin{aligned}
&\dot{V}(t, \bar{q}, \dot{\bar{q}}, \bar{\phi}) \\
&\quad = -\dot{\bar{q}}^T C\big(q, \dot{q}_d(t)\big)\dot{\bar{q}} - \dot{\bar{q}}^T F\dot{\bar{q}} - \dot{\bar{q}}^T s_D\big(K_D\dot{\bar{q}}\big) \\
&\qquad - \varepsilon s_P^T(K_P\bar{q})C\big(q, \dot{q}_d(t)\big)\dot{\bar{q}} - \varepsilon s_P^T(K_P\bar{q})F\dot{\bar{q}} \\
&\qquad - \varepsilon s_P^T(K_P\bar{q})s_D\big(K_D\dot{\bar{q}}\big) - \varepsilon s_P^T(K_P\bar{q})s_P(K_P\bar{q}) \\
&\qquad + \varepsilon\dot{\bar{q}}^T C(q, \dot{\bar{q}})s_P(K_P\bar{q}) + \varepsilon\dot{\bar{q}}^T C\big(q, \dot{q}(t)\big)s_P(K_P\bar{q}) \\
&\qquad + \varepsilon\dot{\bar{q}}^T H(q)s_P'(K_P\bar{q})K_P\dot{\bar{q}}
\end{aligned}$$

where $H(q)\ddot{\bar{q}}$ and $\dot{\bar{\phi}}$ have been replaced by their equivalent expression from the closed-loop dynamics in (11)-(12), Properties 2.1–2.3 have been used, and $s_P'(K_P\bar{q}) \triangleq \mathrm{diag}[\sigma_{P1}'(k_{P1}\bar{q}_1), \ldots, \sigma_{Pn}'(k_{Pn}\bar{q}_n)]$. Observe that from Assumption 2, Properties 1–3, and the satisfaction of items 2 and 4 of Lemma 1 and (b) of Definition 1, we have that $\dot{V}(t, \bar{q}, \dot{\bar{q}}, \bar{\phi}) \leq -\dot{\bar{q}}^T s_D\big(K_D\dot{\bar{q}}\big) - W_3(\bar{q}, \dot{\bar{q}})$, where

$$\begin{aligned}
&W_3(\bar{q}, \dot{\bar{q}}) \\
&\quad = -k_C B_{dv}\|\dot{\bar{q}}\|^2 + f_m\|\dot{\bar{q}}\|^2 - \varepsilon k_C B_{dv}\|s_P(K_P\bar{q})\|\|\dot{\bar{q}}\| \\
&\qquad - \varepsilon f_M\|s_P(K_P\bar{q})\|\|\dot{\bar{q}}\| - \varepsilon\beta_D\|s_P(K_P\bar{q})\|\|\dot{\bar{q}}\| \\
&\qquad + \varepsilon\|s_P(K_P\bar{q})\|^2 - \varepsilon k_C B_P\|\dot{\bar{q}}\|^2 \\
&\qquad - \varepsilon k_C B_{dv}\|s_P(K_P\bar{q})\|\|\dot{\bar{q}}\| - \varepsilon\mu_M\beta_P\|\dot{\bar{q}}\|^2 \\
&\quad = \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T Q_3 \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}
\end{aligned}$$

$$Q_3 = \begin{pmatrix}\varepsilon & -\varepsilon\left(\frac{f_M + \beta_D}{2} + k_C B_{dv}\right) \\ -\varepsilon\left(\frac{f_M + \beta_D}{2} + k_C B_{dv}\right) & f_m - k_C B_{dv} - \varepsilon\beta_M\end{pmatrix}$$

Note further that, from the satisfaction of (9), $W_3(\bar{q}, \dot{q})$ is positive definite (since any $\varepsilon < \varepsilon_M \leq \varepsilon_2$ renders Q_3 positive definite). From this and item (a) of Definition 1, we have that $\dot{V}(t, \bar{q}, \dot{\bar{q}}, \bar{\phi}) \leq -\dot{\bar{q}}^T s_D\big(K_D\dot{\bar{q}}\big) - W_3(\bar{q}, \dot{\bar{q}}) \leq 0$, $\forall (t, \bar{q}, \dot{\bar{q}}, \bar{\phi}) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, with $\dot{V}(t, \bar{q}, \dot{\bar{q}}, \bar{\phi}) = -\dot{\bar{q}}^T s_D\big(K_D\dot{\bar{q}}\big) - W_3(\bar{q}, \dot{\bar{q}}) = 0 \iff (\bar{q}, \dot{\bar{q}}) = (0_n, 0_n)$. Therefore, by Lyapunov stability theory (applied to nonautonomous systems, see for instance Theorem 4.8 of (Khalil, 2002)), the trivial solution $(\bar{q}, \bar{\phi})(t) \equiv (0_n, 0_p)$ is concluded to be uniformly stable. Finally, by Theorem 8.4 of (Khalil, 2002), we conclude that for any initial condition $\big(t_0, \bar{q}(t_0), \dot{\bar{q}}(t_0), \bar{\phi}(t_0)\big) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, the closed-loop system solution $(\bar{q}, \bar{\phi})(t)$ is bounded and such that $\bar{q}(t) \to 0_n$ as $t \to \infty$.

Readers interested in a more thorough presentation of the proof and a generalized version of the proposed scheme are invited to consult (López-Araujo *et al.*, 2014).

5. EXPERIMENTAL RESULTS

The efficiency of the proposed approach was corroborated through real time control implementations using a 2-DOF direct drive manipulator. The experimental setup is a prototype of the 2-revolute-joint robot arm used in (Reyes and Kelly, 1997), located at the *Instituto Tecnológico de la Laguna*. The actuators are direct-drive brushless motors operated in torque mode, so they act as torque source and accept an analog voltage as a reference of torque signal.

The control algorithm is executed at a 2.5 ms sampling period in a control board (based on a DSP 32-bit floating point microprocessor from Texas Instrument) mounted on a PC-host computer. Further technical information on (the hardware and software of) the experimental setup is given in (Reyes and Kelly, 1997).

For this manipulator, Properties 1–5 are satisfied with [3]
$Y^T(q, \dot{q}, \ddot{q}) =$

$$\begin{pmatrix} \ddot{q}_1 & 0 \\ (2\ddot{q}_1 + \ddot{q}_2)\cos(q_2) - \dot{q}_2(2\dot{q}_1 + \dot{q}_2)\sin(q_2) & \ddot{q}_1\cos(q_2) + \dot{q}_1^2\sin(q_2) \\ \ddot{q}_2 & \ddot{q}_1 + \ddot{q}_2 \\ \dot{q}_1 & 0 \\ 0 & \dot{q}_2 \\ \sin(q_1) & 0 \\ \sin(q_1 + q_2) & \sin(q_1 + q_2) \end{pmatrix}$$

$$\theta^T = (2.351\ 0.084\ 0.102\ 2.288\ 0.175\ 38.465\ 1.825)$$

$\mu_m = 0.088$ kg m^2, $\mu_M = 2.533$ kg m^2, $k_C = 0.1455$ kg m^2, $f_m = 0.175$ kg m^2/s, $f_M = 2.288$ kg m^2/s, $B_{g1} = 40.29$ Nm and $B_{g2} = 1.825$ Nm. The maximum allowed torques (input saturation bounds) are $T_1 = 150$ Nm and $T_2 = 15$ Nm for the first and second links respectively. From these data, one easily corroborates that Assumption 1 is fulfilled.

Letting $\sigma_h(\varsigma; M) = M\,\mathrm{sat}(\varsigma/M)$ and

$$\sigma_s(\varsigma; L, M) = \begin{cases} \varsigma & \forall|\varsigma| \le L \\ \rho(\varsigma; L, M) & \forall|\varsigma| > L \end{cases}$$

where $\rho(\varsigma; L, M) = \mathrm{sign}(\varsigma)L + (M - L)\tanh\left(\frac{\varsigma - \mathrm{sign}(\varsigma)L}{M-L}\right)$, for $0 < L < M$, the involved saturation functions were defined as $\sigma_{Pi}(\varsigma) = \sigma_s(\varsigma; L_{Pi}, M_{Pi})$, $\sigma_{Di}(\varsigma) = \sigma_h(\varsigma; M_{Di})$, $i = 1, 2$, and $\sigma_{aj}(\varsigma) = \sigma_s(\varsigma; L_{aj}, M_{aj})$, $j = 1, \ldots, 7$. Let us note that with these saturation functions we have $\sigma'_{PiM} = \sigma'_{DiM} = 1$, $\forall i \in \{1, 2\}$.

For comparison purposes, additional tests were implemented considering the adaptive controller proposed by (Dixon *et al.*, 1999) —referred to as D$_e$99— (choice made in terms of the analog nature of the compared algorithms: bounded adaptive), *i.e.*

$$u = Y_d(t)\hat{\theta} - K_P T_h(\Lambda_P \bar{q}) - K_D T_h(\Lambda_D r) \qquad (16)$$
$$\dot{\hat{\theta}} = P\big(Q(t, r), \hat{\theta}\big)$$

where $Y_d(t) = Y\big(q_d(t), \dot{q}_d(t), \ddot{q}_d(t)\big)$; $T_h(x) = \big(\tanh(x_1), \ldots, \tanh(x_n)\big)^T$; $\Lambda_P = \mathrm{diag}(\lambda_{P1}, \ldots, \lambda_{Pn})$ and $\Lambda_D = \mathrm{diag}(\lambda_{D1}, \ldots, \lambda_{Dn})$ with $\boldsymbol{\lambda_{Pi} = 1}$ [rad]$^{-1}$ and $\boldsymbol{\lambda_{Di} = 1}$ s/rad, $\forall i \in \{1, \ldots, n\}$; $r = \dot{\bar{q}} + \varepsilon T_h(\bar{q})$, with ε being a positive constant; $Q(t, r) = -\Gamma Y_d^T(t) r$; K_P, $K_D \in \mathbb{R}^{n\times n}$ and $\Gamma \in \mathbb{R}^{p\times p}$ are positive definite diagonal matrices; the elements of P are defined as

$$P_j(Q, \hat{\theta}) = \begin{cases} Q_j & \text{if } \theta_{jm} < \hat{\theta}_j < \theta_{jM} \text{ or } C_1 \\ 0 & \text{if } C_2 \end{cases}$$

$$C_1 : \big(\hat{\theta}_j \le \theta_{jm} \text{ and } Q_j \ge 0\big) \text{ or } \big(\hat{\theta}_j \ge \theta_{jM} \text{ and } Q_j \le 0\big)$$
$$C_2 : \big(\hat{\theta}_j \le \theta_{jm} \text{ and } Q_j < 0\big) \text{ or } \big(\hat{\theta}_j \ge \theta_{jM} \text{ and } Q_j > 0\big)$$

[3] For the sake of simplicity, the units of the elements of θ, their estimation variables and related bounds and saturation function parameters, the auxiliary states, and the control and adaptation gains are omitted. The angles are expressed and measured in radians.

Table 1. Control parameter and RMS values

parameter	SP-SD+	D$_e$99
ε	1.0167×10^{-7}	3
K_D	diag[20, 5]	diag[120, 20]
K_P	diag[1500, 300]	diag[70, 7.9]
Γ	diag[20, 0.5, 0.1, 1.5, 0.1, 10, 0.25]	
Λ_P		diag[20, 10]
Λ_D		diag[3, 3]
RMS	0.0138	0.0314

$j = 1, \ldots, p$, with θ_{jm} and θ_{jM} being known lower and upper bounds of θ_j respectively; and the initial auxiliary state values are taken such that $\hat{\theta}_j(0) \in [\theta_{jm}, \theta_{jM}]$, $j = 1, \ldots, p$. The parameter bounds were fixed at $(\theta_{1m}\ \theta_{2m}\ \theta_{3m}\ \theta_{4m}\ \theta_{5m}\ \theta_{6m}\ \theta_{7m}) = (0.588\ \ 0.021\ \ 0.025\ \ 0.572\ \ 0.044\ \ 9.616\ \ 0.456)$ and $\theta_{jM} = M_{aj}$, $j = 1, \ldots, 7$, (these values are specified below).

At every implementation, the initial link positions and velocities were taken as $q_i(0) = \dot{q}_i(0) = 0$, $i = 1, 2$. The auxiliary states were initiated at $\phi^T(0) = (2.88\ 0.103\ 0.125\ 2.803\ 0.214\ 47.119\ 2.235)$ in the SP-SD+ case and $\hat{\theta}^T(0) = (2.88\ \ 0.103\ \ 0.125\ \ 2.803\ \ 0.214\ \ 47.119\ \ 2.235)$ for the D$_e$99 algorithm. The desired trajectory for both implemented controllers was defined as

$$q_d(t) = \begin{pmatrix} q_{d1}(t) \\ q_{d2}(t) \end{pmatrix} = \begin{pmatrix} \frac{\pi}{2} + \sin(\omega t) \\ \cos(\omega t) \end{pmatrix} \quad \text{[rad]}$$

where $\omega = 1.2$. Let us note that with this desired trajectory, Assumption 2 is satisfied with $B_{dv} = \omega < 1.2027$ rad/s ($\approx f_m/k_C$) and $B_{da} = \omega^2$.

For the proposed algorithm a sufficiently small value of ε (satisfying (9)) was taken and the saturation-function parameters were fixed such that inequalities (7) and (3)–(5) were satisfied. The control gains in K_P and K_D were fixed after several trial-and-error tests so as to have the best possible closed-loop performance; guidelines on the tuning are given in (López-Araujo *et al.*, 2014). As for the D$_e$99 controller, a similar procedure was followed taking small enough control gains to avoid input saturation. [4] The resulting tuning values for all the implemented schemes are shown in Table 1. As for the saturation function parameters involved in the proposed algorithm, the selected values were: $M_{P1} = 40$, $M_{D1} = 40$, $M_{P2} = 4$, and $M_{D2} = 4$; $M_a^T = (2.939\ 0.105\ 0.127\ 2.86\ 0.219\ 48.081\ 2.281)$, and $L_{aj} = 0.9M_{aj}$, $j = 1, \ldots, 7$. With these values, inequalities (7) and (3)–(5) were corroborated with $\omega = 1.2$ rad/s, taking $B_{gi}^{M_a} = \sum_{j=1}^{7} B_{G_{ij}} M_{aj}$, $i = 1, 2$, *i.e.* $B_{g1}^{M_a} = M_{a6} + M_{a7} = 50.362$ Nm, $B_{g2}^{M_a} = M_{a7} = 2.281$ Nm, and $B_{Di}^{M_a} = \sum_{j=1}^{7} B_{Y_{ij}} M_{aj}$, $i = 1, 2$, *i.e.* $B_{D1}^{M_a} = \big(M_{a1} +$

[4] For the D$_e$99, controller, the *saturation avoidance* inequalities were indeed taken into account (recall that in this approach, the control gains in K_P and K_D respectively bound the P and D terms; see (16)). Unfortunately, preliminary tests gave rise to extremely slow motions, even though the stability tuning conditions were not strictly applied to avoid their conservativeness. Thus, with the aim to speed up the closed-loop responses, gains λ_{Pi} and λ_{Di}, $i = 1, 2$, greater than unity were fixed (recall that (16) was originally proposed with $\Lambda_P = I_n$ [rad]$^{-1}$ and $\Lambda_D = I_n$ s/rad); this increases the effective gain applied to the error variables in each correction (SP/SD) action. Such modifications to the D$_e$99 controller were carried out in an effort to improve as much as possible the resulting responses.

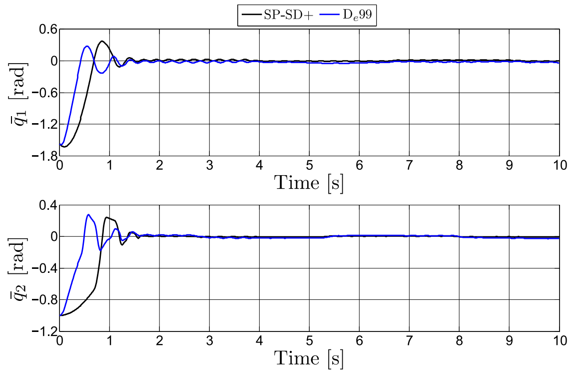

Fig. 1. Position errors

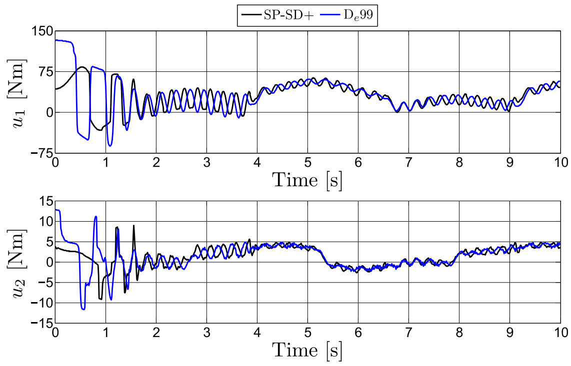

Fig. 2. Control signals

$\sqrt{10}M_{a2} + M_{a3})\omega^3 + M_{a4}\omega + M_{a6} + M_{a7} = 58.6872$ Nm, $B_{D2}^{M_a} = \left(M_{a2} + \sqrt{2}M_{a3}\right)\omega^2 + M_{a5}\omega + M_{a7} = 2.9536$ Nm.

Figures 1 and 2 show the position error evolution and control signals obtained at the performed experimental test. Observe that both schemes achieve the trajectory tracking objective in less than 2 seconds avoiding input saturation. Considerably slower transients were observed through the D_e99 controller with unitary internal gains (results not shown here) as originally proposed (recall footnote 4). Notice further that post-transient effects due to unmodelled phenomena, such as Coulomb friction, were present at the closed-loop responses. They are observed as small oscillations in the position errors graphs. In order to evaluate and compare the performance of the implemented controllers in relation to such a post-transient effect, the root mean square (RMS) of the position errors, *i.e.* $\sqrt{\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\|\bar{q}(t)\|^2 dt}$ was calculated from $t_1 = 2$ s to $t_2 = 10$ s. The resulting values are shown in table 1. Note that under such a criterion, the best performance was obtained through the proposed algorithm. As for the parameter estimators, a considerably slow evolution was observed due to the small value of ε. Such a slow adaptation rate, together with the high number of parameters involved in θ and the unmodelled dynamics gave rise to biased parameter estimations. This can be improved by selecting desired trajectories leading to the satisfaction of *persistency of excitation* conditions, which is out of the scope of this work. It is important to keep in mind that parameter estimation/identification is not part of the motion goal considered in this work. Furthermore, neither the slow evolution, nor the biased convergence of the parameter estimators prevented the trajectory tracking objective to be accomplished —avoiding input saturation— or to be achieved in a considerably short time.

6. CONCLUSION

In this work, an adaptive SP-SD-type scheme for the global motion control of robot manipulators with bounded inputs was proposed. Compared to previous bounded adaptive tracking approaches, the proposed adaptive scheme guarantees the motion control objective for any initial condition (globally), avoiding discontinuities in the control expression as well as in the adaptation auxiliary dynamics, preventing the inputs to reach their natural saturation bounds, imposing no *saturation-avoidance* restriction on the control gains, and permitting the use of any *saturation function* within a well-specified set to achieve the required boundedness. The efficiency of the proposed adaptive controller was corroborated through real-time experimental tests on a 2-DOF manipulator.

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