HOMEOSTATIC APPROACH TO OBSERVATION AND CONTROL OF DEGENERATE NONLINEAR SYSTEMS

ALEX SHOSHITAISHVILI

HUGHES RESEARCH LABORATORIES

ALEXSHOSH@AOL.COM

Abstract. We define Qualitative Behavior Control (QBC) problem and reduce to this problem classical problems of synthesis of dynamical regimes of a given nature (like, for example, stable equilibrium or stable periodical motion) and bifurcations of such regimes depending on parameters (like Andronov-Hopf bifurcation). QBC problem consists of finding a feedback such that the phase portrait of the closed system is equivalent to the phase portrait of a given Model (M) system of differential equations. The equivalence means that there exists a mapping $x = F(z)$ of the states z of M-system to the states x of closed loop control system which maps trajectory of the first system to the trajectory of the second system. Maltivaluable mappings in the form of inexplicit relations $\Phi(x, z) = 0$ (compare with explicit relations $x - F(z) = 0$) also are allowed. Note that by definitions $\Phi(x(t), y(t)) = 0$ for any pare of trajectories $x(t), y(t)$ if $\Phi(x(0), y(0)) = 0$. That's why the relations $\Phi(x, z) = 0$ are called homeostatic relations (or homeostasis) for closed loop control and M systems. Control problem which consists of finding a control allowing a homeostasis of the above mentioned systems is called problem of homeostasis (or H-problem).

We argue that QBC problem is one of universal problems of systems theory together with H-problem and optimization problem. Because of the universality each of them can be reformulated in terms of other two. This is an important heuristic principle. It appears that reformulation of QBC problem in terms of homeostasis problem allows to solve problems of synthesis of dynamical regimes of a given nature even in a vicinity of states where the linearization of controlled system is not completely controllable. Namely, join M-system and controlled systems z = w(z) and x = v(x, u) by means of some $u(z)$. Then under proper assumptions systems w, v will have a homeostasis $\Phi(x, z) = 0$. Now one can synthesise a feedback $\tilde{u}(x) = u(z(x))$ which solve OBC problem. Here $z(x)$ is a solution of the equations $\Phi(x, z) = 0$. This solution will have branches (i.e., will be manivalued) in a vicinity of states x at which the linearization of control system v is not completely controllable. Then $u(z(x))$ will also have branches. We describe the process of branch selection to finish the synthesis of the feedback. Similar constructions are described for nonlinear observer in a vicinity of not completely observable state.

Alexander (Alex) Shoshitaishvili received his master's degree (honoris causa and a prize of the Moscow Mathematical Society for students' works) and PhD from Moscow State University (Moscow, USSR) at the department of differential equations. Here he was a participant of Vladimir Arnold's seminar on singularities and obtained a well-known Shoshitaishvili's Reduction Principle which is fundamental in the theory of topological normal forms for singular points of vector fields. In his PhD he investigated local algebras of singularities.

In 1981-1994 he worked in the Laboratory of M. A. Krasnoselskii at Institute of Control Sciences (Moscow, USSR). Here he developed methods of control and observation for complex nonlinear systems with finite codimension degenerations.

Since 1995 Dr. Shoshitaishvili lives in the USA. He has been working on a variety of applications including real time embedded modeling, color imaging, bioinformatic models, data mining, learning on manifolds. Now he continue his research in Hughes Research Laboratories, CA, USA