Congreso Nacional de Control Automático 2013
Ensenada, Baja California, Mexico, Octubre 16-18, 2013

# Controller of a DC Servomechanism System with Uncertainties Compensation by Using a Super-Twisting Sliding Mode Observer

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***Abstract*— In this work we propose a proportional derivative controller with a robust uncertain estimator in order to compensate unknown bounded perturbations for the task of tracking a DC servomechanism system, which consists of a pendulum actuated by a DC motor subject to a sustained variable but bounded disturbance, both the velocity of the pendulum and the perturbation are unknown, an extended version of a super-twisting sliding-mode observer is used to estimate them in a finite time. Numerical simulation were included to assesses the effectiveness of the closed-loop system.**

## I. Introduction

**DC**-motor pendulum systems(**DCMP**)are extensively used as test bed to verify the performance of control techniques. Particularly, the second order model associated is an attractive benchmark for testing techniques like sliding mode control, PID-control, adaptive control, robust control, parameters identification, and many others. The reason behind it is that is equivalent to the core of more complex systems, like several actually used industry robots ((J. y W., 1991; Sira-Ramirez y Agrawal, 2004)). This topic has been the seed of many interesting works, as is the case of the paper of (Hernandez y Sira-Ramirez, 2002). There, the Generalized Proportional-Integral controller and position measurements, were applied to solve the tracking control problem for a linear version of the **DCMP**. The authors in (Davila *et al.*, 2005) dealt with the regulation problem, using the sliding-mode super-twisting based observer (**STBO**) method, in conjunction with a PD-controller. In (Davila *et al.*, 2006), an interesting solution to control the **DCMP**, using the **STBO** algorithm, combined with an identification scheme can be found. Finally, we mention two exciting papers, (Davila *et al.*, 2006) and (Garrido, 2011); the first dealt with the on-line estimation of a continuous-time model of the **DCMP**, using a closed-loop input error approach. The other paper, presents a method for the identification of the **DCMP** parameters; the method consists of a discrete-time Least Squares algorithm and a parameterizations using the Operational Calculus. For a detailed review we suggest to the interested reader on control of second-order mechanical manipulator the works (Bartolini *et al.*, 2003; Bartolini *et al.*, 1999; Rafimanzelat y Yadanpanah, 2004) and the reference therein.

From the mentioned works, we can say that the design of an smooth output-feedback stabilization technique for an uncertain and perturbed **DC**-motor pendulum, or a second degree manipulator, as pointed out in the book (Dixon y Dawson, 2004; Krstic *et al.*, 1995), is a very challenge control problem. In broad terms, what makes this problem a difficult one, is that not having information about the time derivative of the uncertainty, as was mentioned in (Ortega, 2002), is impossible to completely compensate it. However, applying a variable structure controller with sliding mode (**SMC**), allows to identify on line the unknown perturbation and compensate it, under certain considerations related to the perturbation bound. We should not forget that the ideas of compensate and cancel perturbation has been previously used on Active Disturbance Rejection (Zhiqiang Gao, 2001; Qing Zheng, 2007).

We developed in this work a smooth controller for output feedback trajectory tracking in a **DC** uncertain servomechanism. To this end, a proportional derivative controller and a robust uncertain estimator that throws out, in finite time, the disturbance by means of compensations. The pendulum non available velocity and the perturbation are, both, recovered using an extended version of the super-twisting sliding-mode observer. The Separation principle was applied to design the control strategy because of the observer finite time convergence and the uncertain term uniform boundedness. The convergence analysis was based on the previous works of (Davila *et al.*, 2006; Levant y Fridman, 2002; Levant, 2003*a*). The remaining of this work continues with Section 2, where the model of the **DC**-motor pendulum system and the problem statement are presented. In Section 3 the control strategy and the corresponding convergence analysis are developed. The numerical simulations that assesses the effectiveness of the obtained results are presented in Section 4, while the conclusions are presented in Section 5.

## II. System dynamics model

Consider an actuated second order **DC**-motor pendulum, composed by a servomotor attached with a pendulum, a servo-amplifier and a position sensor. The corresponding control model of this system has the following form:

$$\ddot{x} = \frac{1}{J}\left(-f_d\dot{x} - f_c\text{sign}[\dot{x}] + \eta - gmL\sin x + k_u\tau\right) \quad (1)$$

where $x$ and $\dot{x}$ are the pendular angle position and the pendular angle velocity, respectively; while, $\tau$ is the control input voltage. The parameters $m$, $L$, and $g$ are respectively, the pendulum mass, the pendulum arm length and the gravity constant; the coefficient $f_d$ is the pendulum viscous friction, and the coefficient $f_c$ is the Coulomb friction coefficient. The parameter $J$ stands for the system total inertia, that is composed by the arm and the rotor inertias; the parameter $k_u$ is related to the amplifier gain and to the constant motor torque. Finally, $\eta$ is the unknown and bounded, perturbation which may account for model uncertainties and external disturbances.

We remark that in actual applications of this model, the perturbation $\eta$ and the gravitational pair are both unknown. It is the case, for instance, when the pendulum mass and the angle position are changed. Also, the actual velocity is not available or unmeasurable. On the other hand, the torque produced by the DC-motor does not responds to switching actions, like the ones produced when using the control sliding model. These arguments suggest us to use in the real world a continuous and smooth control action, instead of using the **VSS** control strategy. In this context, we desire to solve the regulation pendulum position problem, assuming that we do not know, both, the velocity and the dynamics of the perturbation, defined as:

$$w(x,t) = \frac{1}{J}(-f_c \text{sign}[\dot{x}] + \eta - gmL\sin x), \quad (2)$$

which is a bounded function. In order to simplify the forthcoming developments, the system (1) is re-written in its state space form, as:

$$\begin{array}{l} \dot{x}_1 = x_2; \\ \dot{x}_2 = -f_0 x_2 + w(x,t) + u; \\ y = x_1 \end{array} \quad (3)$$

where, $x_1 = x$, and, $x_2 = \dot{x}$, and

$$f_0 = \tfrac{f_d}{J}; \quad u = \tfrac{k_u}{J}\tau; \quad (4)$$

**Problem statement:** *Consider the uncertain nonlinear system (3) and the corresponding state, $x_1$, regarded as the measured system output, where the perturbation is uniformly bounded, by*

$$|w(x,t)| \leq \frac{1}{J}(\overline{\eta} + \frac{\overline{gmL}}{J} + \overline{f}_c) \leq \delta_0. \quad (5)$$

*Then, the control goal is that the angular pendular position tracks a given smooth reference trajectory $x_r(t)$. In other words, we want to control the pendular angular position $x$ towards a pre-specified desired trajectory $x_r(t)$.*

**Motivation**: It is important to notice that the system (3) can be seen as a general electro-mechanical system, because a wide range of robots admit this configuration. in consequence the propose solution can be applied to more complex configurations, like a manipulator robot controller.

## III. THE CONTROL STRATEGY

In this section we develop and introduce a control scheme that completely compensates de unknown bounded perturbation, by means of an online identification procedure, in conjunction with a traditionally **PD** controller. We pointed out that the proposed control scheme will allows us to accurately estimate, simultaneously, both the unknown **DCMP** angular velocity and the bounded perturbation. To this end, a high-order exact-differentiation filter, based on a variable structure system (proposed in (Davila *et al.*, 2006; Levant y Fridman, 2002; Levant, 2003*a*)) will be deploy. Because we will use an exact differentiator, with finite time convergence, the separation principle will be trivially satisfied, reducing considerably the corresponding stability analysis.

### *A. State Observation - Perturbation Identification*

We underscore two ideas: The first, introduced in (Levant, 2003*a*), says that it is possible to compute the time high-order derivative of a signal with an accuracy dependant on the numerical integration step. In our particular case, we computed the first and the second time derivatives of the pendulum angular position, with high accuracy. The other idea is based on the equivalent output injection, suggested in (Utkin, 1992; Fridman, 1999), which establish that it is possible to assume that the equivalent output injection is equal to the output of the filter.

Let us introduce the following state estimator for the states of the system (3):

$$\begin{array}{l} \dot{z} = \begin{bmatrix} 0 & 1 \\ 0 & -f_0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (y - z_y) \\ z_y = \begin{bmatrix} 1 & 0 \end{bmatrix} z \\ \hat{x} = z + K^{-1}\ {}_2v(y - z_y) \end{array} \quad (6)$$

where

$$K = \begin{bmatrix} 1 & 0 \\ -L_1 & 1 \end{bmatrix} \quad (7)$$

and the truncate vector ${}_2v(y - z_y)$ is obtained from the vector $v(y - z_y) = [v_1, v_2, v_3]^T$, whose components are computed using the high-order exact differentiator

(Levant, 2003*b*) as:

$$\begin{array}{l} \dot{v}_1 = -\lambda_1 M^{1/3} |v_1 - (y - z_y)|^{2/3} sign\,(v_1 - (y - z_y)) \\ \dot{v}_1 = \dot{v}_1 + v_2 \\ \dot{v}_2 = -\lambda_2 M^{1/2} |v_2 - \dot{v}_1|^{1/2} sign\,(v_2 - \dot{v}_1) + v_3 \\ \dot{v}_3 = -\lambda_3 M\, sign\,(v_3 - \dot{v}_2) \end{array} \quad (8)$$

with the gains $\lambda_1 = 2$, $\lambda_2 = 1.5$, $\lambda_3 = 1.1$ and $M > \delta_0$. Using the equivalent output injection concept, a disturbance estimated can be obtained from:

$$\hat{w} = v_3 + L_2 v_1 + f_0 v_2 \quad (9)$$

**Theorem 3.1:** Under condition (2) and, with a proper selection of the gains $\lambda_1$, $\lambda_2$, $\lambda_3$ and $M$. The observer (6)-(8), provides an exact estimated of the states ($x_1$, $x_2$) after a finite time transient.

The proof of this theorem can be found in (Fridman *et al.*, 2007).

**Lemma 3.2:** Under conditions of Theorem III-A, the estimation algorithm (9) provides after a finite time transient an exact estimated of the disturbance $w$ in system (3).

## B. Robust controller and convergence analysis

Under the conditions given in Theorem III-A, it is ensured that, after a finite-time transient, the estimated state variables $(\widehat{x}_1, \widehat{x}_2)$, converge to the corresponding actual values. Notice that, the structure of the observer does not vary with respect to the control law. That is to say, the separation principle is trivially satisfied. Therefore, the stability analysis of the control system composed by the plant (3) and the observer (6)-(8) is equivalent to examine the stability of the system (3). Henceforth. without loos of generality, we study the convergence of the system (3) when $\widehat{x}_2 = x_2$.

## C. Output feedback controller

First of all, Let us propose the following control law:

$$u = \frac{1}{k_u}\left[-\widehat{w} - k_p(x_1 - x_r) - k_d(x_2 - \dot{x}_r) + f_0 x_2 + \ddot{x}_r\right], \tag{10}$$

where, $k_p > 0$, and, $k_d > 0$, are positive constants; $x_r$, is the desired angular position and, $x_2$ is the value of the velocity obtained from (6)-(8), $\widehat{w}$ is the perturbation estimated (9).

Substituting (10) into (3), we can obtain the tracking as:

$$\begin{aligned} \dot{x}_{1r} &= x_{2r} \\ \dot{x}_{2r} &= -k_p x_{1r} - k_d x_{2r} \end{aligned} \tag{11}$$

where, $x_{1r} = x_1 - x_r$, and, $x_{2r} = x_2 - \dot{x}$ are the state tracking errors.

The values of the gains $k_p$ and $k_d$ are chosen such that the roots of the characteristic polynomial $s^2 + k_d s + k_p$ belongs to the open left complex semi-plane. Notice that, the damping coefficient of the controlled system is given by the relation $\zeta = \frac{k_d}{2\sqrt{k_p}}$.

# IV. Numerical simulations

In order to asses effectiveness we designed some numerical experiments, whose results we present in this section. The choosen physical parameters are the ones obtained from the actually manufactured device in the Automatic Control Laboratory of the CINVESTAV-IPN, by Garrido and Miranda (Garrido, 2011), whic are:

$$f_0 = 0.1; \quad f_1 = 1.3; \quad \frac{\eta}{J} = 0.84; \quad \frac{gmL}{J} = 14.03; \quad \frac{k_u}{J} = 5.4; \tag{12}$$

Also an external perturbation , $\eta$, were include to prove the robustness of the proposed controller. This perturbation was defined as:

$$\frac{\eta}{J} = 0.84 + 0.2\sin\left(\frac{t}{5}\right). \tag{13}$$

The smooth function, $x_r = \sin(t/2)$, was selected to be trackes as the control goal, where their corresponding parameters were chosen, as:

$$L_1 = 49/10; \quad L_2 = 551/100; \quad M = 1000; \quad k_p = 4000; \quad k_d = 130; \tag{14}$$

The eigenvalues of, $(A - LC)$, provided by these parameters, are, $-3$, $-2$. The damping coefficient is 1. In order to compensate the discontinuous effect of the dry friction, both the differentiator and the controller gains were selected large enough. The initial conditions were chosen as, $x_1 = -1.57$[rad], and, $x_2 = 0$[rad/sec]; and the nonlinear observer were set at the origin. To perform the simulation, the Euler method, with a sampling integration interval of, $1 \times 10^{-4}$[sec], was used.

The closed-loop response of the whole state is shown in figure 1; it can be seen that the proposed controller effectively makes the pendulum to follow the reference signal, $x_r = \sin(t/2)$, after one second elapsed. We also see that after one second elapsed the velocity state tracks the defined reference. The corresponding state estimation error is presented in figure 2; we can see that accuracy state reconstruction is in the order $1 \times 10^{-4}$.

In figure 3 the tracking position and the tracking error are displayed; there we can see that, after controller convergence, the reference signal is tracking with an error of order $1 \times 10^{-4}$. Finally, in figure 4 we show the control signal and the disturbance.

# V. Conclusions

We solved the trajectory tracking and stabilization problems for an uncertain and continuously perturbed **DC** servomechanism. This perturbation was unknown but bounded. The solution was accomplished by an output feedback control scheme. We used a derivative controller with a robust uncertain estimator, which compensates the perturbation by using a version of the super-twisting sliding-mode observer.

The application of the Separation principle was justified because the observer convergence is finite time and the uncertain term is uniformly bonded.

The performance of our controller was assessed with some numerical simulations; we claim with the results that the goal was accomplished.

# VI. Thanks

We would like to thank our respective donors: CONACYT and COFAA, with their grants this work have been accomplished.

This research was supported by the Centro de Investigación en Computación of the Instituto Politecnico Nacional (CIC-IPN); by the Secretaría de Investigación y Posgrado of the Instituto Politecnico Nacional (SIP-IPN)

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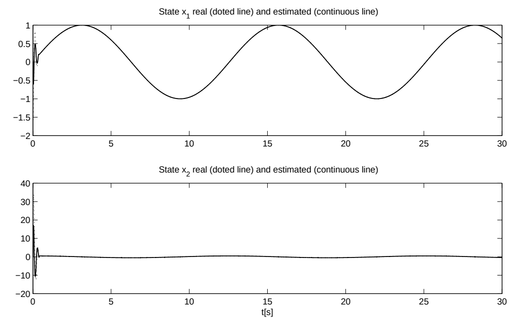


Figura 1. State estimation of DC servomechanism

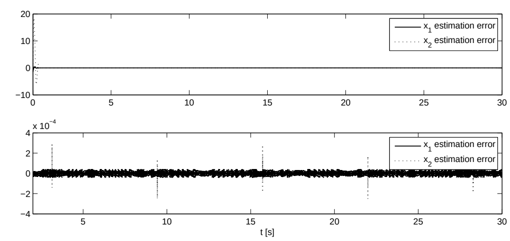


Figura 2. State estimation error

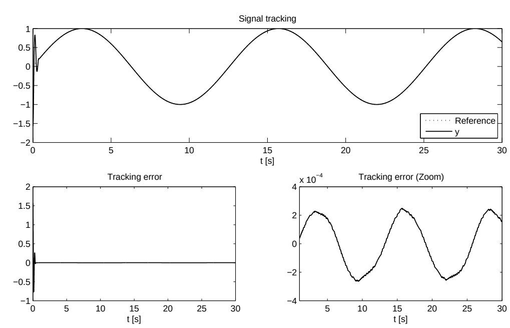


Figura 3. Tracking of DC servomechanism

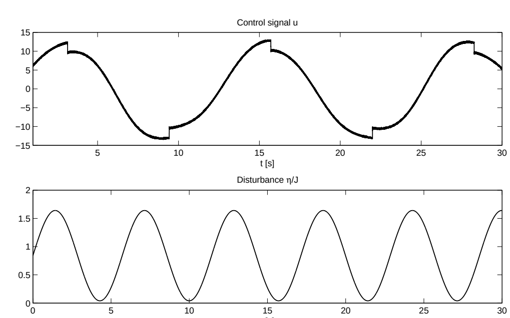


Figura 4. Control signal (above) and the disturbance $\eta/J$ (below).