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Leader-Follower and Leaderless Consensus in Networks of Flexible–Joint Manipulators

Emmanuel Nuño[1], Daniela Valle[2], Ioannis Sarras[3] and Luis Basañez[4]

[1] Department of Computer Science. University of Guadalajara. Guadalajara, Mexico
[2] Electronics Department. University of Guadalajara. Guadalajara, Mexico
[3] IFP, Energies Nouvelles. Department of Control, Signal & Systems. Rueil-Malmaison, France
[4] Institute of Industrial and Control Engineering. Technical University of Catalonia. Barcelona, Spain
emmanuel.nuno@cucei.udg.mx, danny.valler@gmail.com, ioannis.sarras@ifpen.fr, luis.basanez@upc.edu

***Abstract*— This paper reports a solution to the leader-follower consensus problem, provided that at least one follower has a direct access to the leader's position, and to the leaderless consensus problem in networks composed of nonidentical flexible-joint robot manipulators. The network is modeled as an undirected graph and it is assumed that the interconnection does not induce time-delays. The proposed controller has two different terms, one that dynamically compensates the link gravity and another which ensures the desired consensus objective. This last term is a simple Proportional plus damping scheme. Simulations are provided to support the theoretical results.**
Keywords—Network of Robots, Flexible Arms, Decentralized Control.

I. Introduction

For networks of multiple agents, the consensus control objective is to reach an agreement between certain coordinates of interest using a distributed controller. There are mainly two consensus problems: the leader-follower, where a network of follower agents has to be synchronized with a given leader, and the leaderless, where all agents agree at a certain coordinates value. The solutions to these problems have recently attracted the attention of the research community in different fields, such as biology, physics, control theory and robotics (refer to (Olfati-Saber *et al.* 2007, Scardovi and Sepulchre 2009, Ren 2008), for solutions with linear agents, and to (Yu *et al.* 2011, Scardovi *et al.* 2009, Stan and Sepulchre 2007, Zhao *et al.* 2009, Nuño *et al.* 2011*b*), for solutions with classes of nonlinear agents).

The practical applications of the solutions to the consensus problems are diverse and range from formation control of multiple unmaned aerial vehicles to the synchronization of swarms of mobile robots. A particular example is a robot teleoperator, where two mechanical manipulators are coupled by a communication channel (Anderson and Spong 1989). In this last example, the control objective in these systems is that when the human operator moves the *local* manipulator, the *remote* manipulator tracks its position, and the force interaction of this last with the environment is reflected back to the operator (Nuño *et al.* 2011*a*). The results reported in the present paper are an extension to the case of networks of under–actuated Euler-Lagrange (EL) systems of the controller reported in (Nuño *et al.* 2009). A direct application of the consensus controllers presented here is the teleoperation of multiple-remote devices, the collaboration of multiple users via a multiple-local multiple-remote system, among others (Malysz and Sirouspour 2011, Rodriguez-Seda *et al.* 2010).

Consensus of networks of EL-systems without time-delays has been considered in (Ren 2009, Mei *et al.* 2011) using simple proportional controllers together with filtered velocities. The work of (Nuño *et al.* 2011*b*) proposes an adaptive controller for EL-systems that solves the consensus problem with constant time-delays. Further results are those by (Liu and Chopra 2012) and by (Hatanaka *et al.* 2012), which consider the consensus problem in Cartesian space with constant delays. Recently, in (Nuño *et al.* 2012) it has been proved that networks composed by nonidentical EL-systems with variable time–delays can reach a consensus, using P+d controllers, provided enough damping is injected. Nevertheless, it should be underscored that, all these previous results deal with *fully–actuated* EL-systems (fully–actuated robots). However, in diverse applications, including space and surgical robots, the use of thin, lightweight and cable-driven manipulators is increasing. These systems exhibit joint or link flexibility and hence they are *under–actuated* mechanical systems. Furthermore, it must be noted that, as it has been shown in (Tavakoli and Howe 2009), the lumped (linear) dynamics of a flexible link is identical to the (linear) dynamics of a flexible joint.

In this sense, and up to the authors knowledge, the literature on the control of networks of *under–actuated* EL-systems is almost non-existent (in this case, the number of inputs is strictly less than the degrees-of-freedom and designing a controller is far more complicated), few remarkable exceptions being (Nair and Leonard 2008, Avila-Becerril and Espinosa-Pérez 2012) and, more recently, (Avila-Becerril *et al.* 2013). In (Nair and Leonard 2008) the Controlled–Lagrangian technique is employed to solve the leaderless consensus in networks without delays. Via a full-state feedback controller and under the assumption that the initial conditions are known, (Avila-Becerril and Espinosa-Pérez 2012) proposes the first solutions to the leader-follower, provided that all followers have direct access to the leader, and the leaderless problems for directed graphs with constant time–delays. (Avila-Becerril *et al.* 2013) reports the solution to the leaderless consensus eliminating the assumption of the knowledge of the initial conditions of (Avila-Becerril and Espinosa-Pérez 2012).

Under the assumption that the interconnection graph is undirected and inspired by the exact gravity cancelation scheme of (De Luca and Flacco 2010, De Luca and Flacco 2011), this work proposes a simple P+d controller which provides a Globally Asymptotically Stable (GAS) solution to the leader-follower problem, provided that at least one follower has a direct access to the leader position, and to the leaderless consensus problem. Despite that this work focusses on finding the consensus solutions for the case without interconnection time–delays, by using the results of (Nuño *et al.* 2012), the case with variable time–delays can be easily handled.

Notation. $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$. The spectrum of the square matrix $\mathbf{A}$ is denoted by $\sigma(\mathbf{A})$ while the minimum and the maximum of its spectrum are denoted by $\sigma_{\min}(\mathbf{A})$ and $\sigma_{\max}(\mathbf{A})$, respectively. $||\mathbf{A}||$ denotes the matrix-induced 2-norm. $|\mathbf{x}|$ stands for the standard Euclidean norm of vector $\mathbf{x}$. $\mathbf{I}_k$ is the identity matrix of size $k \times k$. $\mathbf{1}_k$ and $\mathbf{0}_k$ represent column vectors of size k with all entries equal to one and to zero, respectively. For any function $\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\mathbf{f}\|_\infty := \sup_{t\geq 0} |\mathbf{f}(t)|$, $\mathcal{L}_2$-norm as $\|\mathbf{f}\|_2 := (\int_0^\infty |\mathbf{f}(t)|^2 dt)^{1/2}$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_\infty < \infty\}$ and $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_2 < \infty\}$, respectively. The subscript $i \in \bar{N} := \{1, ..., N\}$, where N is the number of nodes of the network.

II. Flexible–Joint Robot Manipulators

Let us consider a network of N non-identical, flexible–joint robot manipulators with n-DOF. Directly actuated, revolute joints robots are assumed and the simplified model for flexibility of (Spong *et al.* 2005) is adopted. For every i, the dynamics of the i-th manipulator is given by

$$\begin{aligned} \mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) + \mathbf{S}_i(\mathbf{q}_i - \boldsymbol{\theta}_i) &= \mathbf{0}_n \\ \mathbf{J}_i\ddot{\boldsymbol{\theta}}_i + \mathbf{S}_i(\boldsymbol{\theta}_i - \mathbf{q}_i) &= \boldsymbol{\tau}_i \end{aligned} \tag{1}$$

where $\mathbf{q}_i \in \mathbb{R}^n$ is the link angular position and $\boldsymbol{\theta}_i \in \mathbb{R}^n$ is the joint (motor) angular position. The matrix $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix, the matrix $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ describes the Coriolis and centrifugal effects (defined via the Christoffel symbols of the first kind), the vector $\mathbf{g}_i(\mathbf{q}_i) := \nabla U_i(\mathbf{q}_i)$ is the gravity force with $U_i : \mathbb{R}^n \to \mathbb{R}$ being the corresponding potential energy, the matrix $\mathbf{J}_i \in \mathbb{R}^{n\times n}$ is the motor inertia at the joints, which is symmetric and positive definite, the matrix $\mathbf{S}_i \in \mathbb{R}^{n\times n}$ is the joint stiffness which is also symmetric and positive definite and the vector $\boldsymbol{\tau}_i \in \mathbb{R}^n$ is the control input.

Dynamics (1) exhibit the following well–known properties (Spong *et al.* 2005, Kelly *et al.* 2005) and thus they are assumed throughout this paper.

- **(P1)** $\mathbf{M}_i(\mathbf{q}_i)$ is symmetric and there exists $\lambda_{mi}, \lambda_{Mi} > 0$ such that $0 < \lambda_{mi}\mathbf{I}_n \leq \mathbf{M}_i(\mathbf{q}_i) \leq \lambda_{Mi}\mathbf{I}_n$.
- **(P2)** The matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew-symmetric.
- **(P3)** There exists $k_{gi} > 0$ such that $\left|\frac{\partial \mathbf{g}_i(\mathbf{q}_i)}{\partial \mathbf{q}_i}\right| \leq k_{gi}$. Hence, for all $\mathbf{q}_{i1}, \mathbf{q}_{i2} \in \mathbb{R}^n$ the following inequality holds $|\mathbf{g}_i(\mathbf{q}_{i1}) - \mathbf{g}_i(\mathbf{q}_{i2})| \leq k_{gi}|\mathbf{q}_{i1} - \mathbf{q}_{i2}|$.

III. Network Interconnection

The interconnection of the N agents is modeled using the Laplacian matrix $\mathbf{L} := [\ell_{ij}] \in \mathbb{R}^{N\times N}$, whose elements are defined as

$$\ell_{ij} = \begin{cases} \sum_{k\in\mathcal{N}_i} a_{ik} & i = j \\ -a_{ik} & i \neq j \end{cases} \tag{2}$$

where $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$ otherwise. $\mathcal{N}_i$ is the set of agents transmitting information to the ith robot.

In order to ensure that the interconnection forces are generated by the gradient of a potential function, the following assumption is used in this paper:

- **(A1)** The robot interconnection graph is *undirected and connected*.

Note that, by construction, $\mathbf{L}$ has a zero row sum, *i.e.*, $\mathbf{L}\mathbf{1}_N = \mathbf{0}_N$. Moreover, Assumption (**A1**), ensures that $\mathbf{L}$ is symmetric, has a single zero-eigenvalue and that the rest of the spectrum of $\mathbf{L}$ has positive real parts. Thus, rank$(\mathbf{L}) = N - 1$. Using these facts, it is straightforward to show that, for any $\mathbf{z} \in \mathbb{R}^N$, $\mathbf{z}^\top\mathbf{L}\mathbf{z} = \frac{1}{2}\sum_{i\in\bar{N}}\sum_{j\in\mathcal{N}_i} a_{ij}(z_i - z_j)^2 \geq 0$. Furthermore,

$$\frac{d}{dt}\mathbf{z}^\top\mathbf{L}\mathbf{z} = 2\dot{\mathbf{z}}^\top\mathbf{L}\mathbf{z} = 2\sum_{i\in\bar{N}} \dot{z}_i \sum_{j\in\mathcal{N}_i} a_{ij}(z_i - z_j). \tag{3}$$

IV. Leader-Follower Consensus

In this section it is considered the case when the network of N flexible–joint robot manipulators (followers) has to be regulated at a leader's constant position, denoted $\mathbf{q}_0 \in \mathbb{R}^n$. The leader is regarded as *node 0* of the $N+1$ agent network.

The control objective is to ensure that all link positions of the flexible-joint manipulators asymptotically converge to the constant leader position, *i.e.*, for all $i \in \bar{N}$,

$$\lim_{t\to\infty} |\dot{\mathbf{q}}_i(t)| = 0, \quad \lim_{t\to\infty} \mathbf{q}_i(t) = \mathbf{q}_0,$$

provided that the leader position (node) is only available to a certain set of robot manipulators. With regards to the leader-followers interconnection, this paper has the following assumption:

- **(A2)** At least one of the robot manipulators has access to the leader's position $\mathbf{q}_0$, *i.e.*, in the augmented interconnection graph of $N + 1$ nodes, there exists at least one directed edge from the leader (node 0) to any of the N followers.

Assumptions (**A1**) and (**A2**) ensure that the leader position is *globally reachable* from any other node, *i.e.*, from the leader's node there exist a path to any ith robot-manipulator for $i \in \bar{N}$. Moreover, it also holds that

Fact 1: Suppose that Assumptions (**A1**) and (**A2**) hold. Define matrix $\mathbf{L}_0 := \mathbf{L} + \mathbf{A}_0$, where $\mathbf{A}_0 := \mathrm{diag}(a_{10}, \ldots, a_{N0}) \in \mathbb{R}^{N\times N}$ and $a_{i0} > 0$ if the leader's position $\mathbf{q}_0$ is directly available to the ith flexible-joint manipulator and $a_{i0} = 0$, otherwise. Under these conditions $\mathbf{L}_0$ is symmetric, positive definite and of full rank.

Proof. Assumption (**A1**) ensures that $\sigma_{\min}(\mathbf{L}) = 0$ is a single eigenvalue and the rest of the spectrum is strictly positive. Moreover, $\mathbf{L}$ is symmetric and since $\mathbf{A}_0$ is diagonal, then $\mathbf{L}_0$ is symmetric. Hence, $\sigma(\mathbf{L}_0)$ has only real numbers. Inspired by (Hu and Hong 2007), let us now invoke the Gershgorin Disk Theorem to locate the spectrum of $\mathbf{L}_0$. There exist N disks, denoted $\mathcal{D}_i$, centered in the complex plane at $\ell_{ii} + a_{i0}$ with radius $\sum_{j\in\mathcal{N}_i} |a_{ij}|$, such that

$$\mathcal{D}_i := \left\{ \lambda : |\lambda - \ell_{ii} - a_{i0}| \le \sum_{j\in\mathcal{N}_i} |a_{ij}| \right\},$$

since $\ell_{ii} = \sum_{j\in\mathcal{N}_i} a_{ij}$ and $a_{ij} \ge 0$, then $\lambda \ge 0$ and $\mathcal{D}_i$ may tangentially intersect the origin. Moreover, from Assumption (**A2**), there exists at least one $a_{i0} > 0$ and hence there is at least one disk that does not intersect the origin. Further, since the undirected graph is connected then $\mathbf{L}$ is irreducible and, by Taussky's Theorem, if 0 is an eigenvalue of $\mathbf{L}_0$ then it has to lie in the boundary of *all* the disks (Serre 2010), we conclude that $0 \notin \sigma(\mathbf{L}_0)$, thus $\sigma(\mathbf{L}_0)$ is strictly positive and real. Which, together with symmetry, ensures that $\mathbf{L}_0$ is positive definite. The full rank property follows directly. □

Now, inspired by the exact gravity cancelation scheme of (De Luca and Flacco 2010, De Luca and Flacco 2011), let us define the new variable $\mathbf{x}_i \in \mathbb{R}^n$ as

$$\mathbf{x}_i := \boldsymbol{\theta}_i - \mathbf{S}_i^{-1}\mathbf{g}_i(\mathbf{q}_i). \tag{4}$$

Using (4) and defining the controller

$$\boldsymbol{\tau}_i = \bar{\boldsymbol{\tau}}_i + \mathbf{g}_i(\mathbf{q}_i) + \mathbf{J}_i\mathbf{S}_i^{-1}\ddot{\mathbf{g}}_i(\mathbf{q}_i) - \mathbf{D}_i\dot{\mathbf{x}}_i, \tag{5}$$

where $\mathbf{D}_i \in \mathbb{R}^{n\times n}$ is the damping gain and is symmetric and positive definite and $\bar{\boldsymbol{\tau}}_i \in \mathbb{R}^n$ is the interconnection controller term that will be defined later, each flexible–joint robot manipulator (1) can be written as

$$\begin{aligned} \mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{S}_i(\mathbf{q}_i - \mathbf{x}_i) &= \mathbf{0}_n \\ \mathbf{J}_i\ddot{\mathbf{x}}_i + \mathbf{D}_i\dot{\mathbf{x}}_i + \mathbf{S}_i(\mathbf{x}_i - \mathbf{q}_i) &= \bar{\boldsymbol{\tau}}_i. \end{aligned}$$

In order to achieve the desired control objective, let us define $\bar{\boldsymbol{\tau}}_i$ as

$$\bar{\boldsymbol{\tau}}_i = -k_i a_{i0}(\mathbf{x}_i - \mathbf{q}_0) - k_i \sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{x}_i - \mathbf{x}_j) \tag{6}$$

where $k_i > 0$ and a_{i0} as defined in Fact 1.

The closed-loop system (1), (5) and (6) is

$$\left.\begin{aligned} \ddot{\mathbf{q}}_i &= -\mathbf{M}_i^{-1}(\mathbf{q}_i)\left[\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{S}_i(\mathbf{q}_i - \mathbf{x}_i)\right] \\ \ddot{\mathbf{x}}_i &= -\mathbf{J}_i^{-1}\left[\mathbf{D}_i\dot{\mathbf{x}}_i + \mathbf{S}_i(\mathbf{x}_i - \mathbf{q}_i) + k_i a_{i0}(\mathbf{x}_i - \mathbf{q}_0)\right] \\ &\quad - k_i\mathbf{J}_i^{-1} \sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{x}_i - \mathbf{x}_j). \end{aligned}\right\} \tag{7}$$

We are now ready to state one of the main results of this work,

Proposition 1: Consider a network of N flexible–joint manipulators, whose dynamics fulfill (1) and in closed-loop with the controller (5) and (6). Suppose that Assumptions (**A1**) and (**A2**) hold. Under these conditions, the velocities and the link position error, between each robot manipulator and the leader, asymptotically converge to zero, *i.e.*,

$$\lim_{t\to\infty} |\dot{\mathbf{q}}_i(t)| = \lim_{t\to\infty} |\mathbf{q}_i(t) - \mathbf{q}_0| = 0, \qquad \forall i \in \bar{N}.$$

Proof. Every system in (7) exhibits the following energy function

$$\mathcal{E}_i := \mathcal{K}_i(\dot{\mathbf{q}}_i, \dot{\mathbf{x}}_i) + \mathcal{U}_i(\mathbf{q}_i, \mathbf{x}_i) + \mathcal{U}_{i0}(\mathbf{x}_i, \mathbf{q}_0)$$

where $\mathcal{K}_i$ is the kinetic energy, given by

$$\mathcal{K}_i(\dot{\mathbf{q}}_i, \dot{\mathbf{x}}_i) = \frac{1}{2}\left[\dot{\mathbf{q}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i + \dot{\mathbf{x}}_i^\top \mathbf{J}_i\dot{\mathbf{x}}_i\right], \tag{8}$$

$\mathcal{U}_i$ is the potential energy stored in the $\mathbf{x}_i$-coordinate and the link virtual spring, such that

$$\mathcal{U}_i(\mathbf{q}_i, \mathbf{x}_i) = (\mathbf{x}_i - \mathbf{q}_i)^\top \mathbf{S}_i(\mathbf{x}_i - \mathbf{q}_i), \tag{9}$$

$\mathcal{U}_{i0}$ is the potential energy stored in the leader-follower interconnection and it fulfills

$$\mathcal{U}_{i0}(\mathbf{x}_i, \mathbf{q}_0) = k_i a_{i0}|\mathbf{x}_i - \mathbf{q}_0|^2. \tag{10}$$

Evaluating $\dot{\mathcal{E}}_i$ along (7), using Property (**P2**) and since $\dot{\mathbf{q}}_0 = \mathbf{0}_n$, gives $\dot{\mathcal{E}}_i = -\dot{\mathbf{x}}_i^\top \mathbf{D}_i\dot{\mathbf{x}}_i - k_i \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\mathbf{x}}_i^\top(\mathbf{x}_i - \mathbf{x}_j)$.

Let us now define $\mathcal{E}$ as the total energy of all the N system (7) plus the potential energy in the interconnection:

$$\mathcal{E} := \sum_{i\in\bar{N}} \left(\frac{1}{k_i}\mathcal{E}_i + \frac{1}{4}\sum_{j\in\mathcal{N}_i} a_{ij}|\mathbf{x}_i - \mathbf{x}_j|^2 \right). \tag{11}$$

Hence,

$$\dot{\mathcal{E}} = -\sum_{i\in\bar{N}} \left(\frac{1}{k_i}\dot{\mathbf{x}}_i^\top \mathbf{D}_i\dot{\mathbf{x}}_i + \frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}(\dot{\mathbf{x}}_i + \dot{\mathbf{x}}_j)^\top(\mathbf{x}_i - \mathbf{x}_j) \right)$$

Using the fact that

$$(\dot{\mathbf{x}}_i + \dot{\mathbf{x}}_j)^\top(\mathbf{x}_i - \mathbf{x}_j) = \rho_i - \rho_j + \dot{\mathbf{x}}_i^\top(\mathbf{x}_i - \mathbf{x}_j) - \mathbf{x}_i^\top(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_j),$$

where $\rho_i := \dot{\mathbf{x}}_i^\top \mathbf{x}_i$, $\rho_j := \dot{\mathbf{x}}_j^\top \mathbf{x}_j$ and, by the Laplacian properties,

$$\begin{aligned} \sum_{i\in\bar{N}}\sum_{j\in\mathcal{N}_i} a_{ij}(\dot{\mathbf{x}}_i + \dot{\mathbf{x}}_j)^\top(\mathbf{x}_i - \mathbf{x}_j) &= \dot{\mathbf{x}}^\top(\mathbf{L}\otimes\mathbf{I}_n)\mathbf{x} - \\ &\quad - \mathbf{x}^\top(\mathbf{L}\otimes\mathbf{I}_n)\dot{\mathbf{x}} - \mathbf{1}_N^\top\mathbf{L}\boldsymbol{\rho} = 0, \end{aligned}$$

where $\mathbf{x} := \text{col}(\mathbf{x}_1^\top, \ldots, \mathbf{x}_N^\top)$, $\dot{\mathbf{x}} := \text{col}(\dot{\mathbf{x}}_1^\top, \ldots, \dot{\mathbf{x}}_N^\top)$, $\boldsymbol{\rho} := \text{col}(\rho_1, \ldots, \rho_N)$ and $\otimes$ is the standard Kronecker product. Thus, $\dot{\mathcal{E}} = -\sum_{i\in\bar{N}} \frac{1}{k_i}\dot{\mathbf{x}}_i^\top \mathbf{D}_i \dot{\mathbf{x}}_i \leq 0$.

It should be noted that, for all $i \in \bar{N}$, $\mathcal{E}$ is positive definite and radially unbounded with regards to the signals: $\dot{\mathbf{q}}_i, \dot{\mathbf{x}}_i, |\mathbf{x}_i - \mathbf{q}_i|$ and $|\mathbf{x}_i - \mathbf{q}_0|$. This last, and the fact that $\dot{\mathcal{E}} \leq 0$ ensure that such signals are bounded. Moreover, $\dot{\mathbf{x}}_i \in \mathcal{L}_2$. From the closed-loop dynamics (7) it also holds that $\ddot{\mathbf{q}}_i, \ddot{\mathbf{x}}_i \in \mathcal{L}_\infty$. Now, $\dot{\mathbf{x}}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and $\ddot{\mathbf{x}}_i \in \mathcal{L}_\infty$ imply that $\lim_{t\to\infty} |\dot{\mathbf{x}}_i(t)| = 0$.

The time-derivative of $\ddot{\mathbf{x}}_i$ satisfies

$$\begin{aligned}\frac{d}{dt}\ddot{\mathbf{x}}_i = &-\mathbf{J}_i^{-1}\left[\mathbf{D}_i\ddot{\mathbf{x}}_i + \mathbf{S}_i(\dot{\mathbf{x}}_i - \dot{\mathbf{q}}_i) + k_i a_{i0}\dot{\mathbf{x}}_i\right] - \\ &- k_i \sum_{j\in\mathcal{N}_i} a_{ij}\mathbf{J}_i^{-1}(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_j),\end{aligned} \tag{12}$$

and, since $\ddot{\mathbf{x}}_i, \dot{\mathbf{x}}_i, \dot{\mathbf{q}}_i \in \mathcal{L}_\infty$, $\frac{d}{dt}\ddot{\mathbf{x}}_i \in \mathcal{L}_\infty$. This last and the fact that $\lim_{t\to\infty}\int_0^t \ddot{\mathbf{x}}_i(\sigma)d\sigma = \lim_{t\to\infty}(\dot{\mathbf{x}}_i(t) - \dot{\mathbf{x}}_i(0)) = -\dot{\mathbf{x}}_i(0)$, supports the claim that $\lim_{t\to\infty}|\ddot{\mathbf{x}}_i(t)| = 0$, according to Barbalat's Lemma.

Since all signals, except $\dot{\mathbf{q}}_i$, in the right-hand-side of (12) asymptotically converge to zero, if we can prove that $\lim_{t\to\infty}|\frac{d}{dt}\ddot{\mathbf{x}}_i(t)| = 0$, then $\lim_{t\to\infty}|\dot{\mathbf{q}}_i(t)| = 0$. For, because convergence to zero of $\ddot{\mathbf{x}}_i$ implies that $\lim_{t\to\infty}\int_0^t \frac{d}{d\sigma}\ddot{\mathbf{x}}_i(\sigma)d\sigma = \lim_{t\to\infty}(\ddot{\mathbf{x}}_i(t) - \ddot{\mathbf{x}}_i(0)) = -\ddot{\mathbf{x}}_i(0)$, it only rests to show that $\frac{d^2}{dt^2}\ddot{\mathbf{x}}_i \in \mathcal{L}_\infty$. Indeed, (12) and $\frac{d}{dt}\ddot{\mathbf{x}}_i, \ddot{\mathbf{x}}_i, \ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$ ensures that $\frac{d^2}{dt^2}\ddot{\mathbf{x}}_i \in \mathcal{L}_\infty$, as needed. Finally, $\lim_{t\to\infty}|\dot{\mathbf{q}}_i(t)| = 0$ and boundedness of $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \dot{\mathbf{x}}_i$ ensures, from (7), that $\lim_{t\to\infty}|\ddot{\mathbf{q}}_i(t)| = 0$.

The previous convergence claims ensure that the equilibrium point is Globally Asymptotically Stable (GAS). Furthermore, one part of this GAS equilibrium point fulfills $(\ddot{\mathbf{x}}_i, \dot{\mathbf{x}}_i, \ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i) = (\mathbf{0}_n, \mathbf{0}_n, \mathbf{0}_n, \mathbf{0}_n)$ and the rest satisfies

$$\begin{aligned}\mathbf{q}_i &= \mathbf{x}_i \\ a_{i0}(\mathbf{x}_i - \mathbf{q}_0) + \sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{x}_i - \mathbf{x}_j) &= \mathbf{0}_n,\end{aligned}$$

which, by piling up the N vectors $\mathbf{q}_i$ and $\mathbf{x}_i$ as $\mathbf{q} := \text{col}(\mathbf{q}_1^\top, \ldots, \mathbf{q}_N^\top)$ and $\mathbf{x} := \text{col}(\mathbf{x}_1^\top, \ldots, \mathbf{x}_N^\top)$, respectively, it can be written as $\mathbf{q} = \mathbf{x}$ and

$$(\mathbf{A}_0 \otimes \mathbf{I}_n)(\mathbf{x} - (\mathbf{1}_N \otimes \mathbf{q}_0)) + (\mathbf{L} \otimes \mathbf{I}_n)\mathbf{x} = \mathbf{0}_{Nn}.$$

Moreover, from the fact that $\mathbf{L}\mathbf{1}_N = \mathbf{0}_N$ it also holds that

$$\begin{aligned}\mathbf{q} &= \mathbf{x} \\ (\mathbf{L}_0 \otimes \mathbf{I}_n)(\mathbf{x} - (\mathbf{1}_N \otimes \mathbf{q}_0)) &= \mathbf{0}_{Nn}.\end{aligned}$$

This, together with Fact 1, ensures that the only possible solution to these equations is $\mathbf{q} = \mathbf{x} = (\mathbf{1}_N \otimes \mathbf{q}_0)$. Thus, for all $i \in \bar{N}$, $\mathbf{q}_i = \mathbf{q}_0$. This concludes the proof. □

V. Leaderless Consensus

The control objective in the leaderless consensus problem is to show that, in the absence of a leader, all the N robot manipulators link position asymptotically converge to a common consensus point, denoted $\mathbf{q}_c \in \mathbb{R}^n$, and that velocities asymptotically converge to zero, *i.e.*, for all $i \in \bar{N}$ there exist $\mathbf{q}_c \in \mathbb{R}^n$ such that

$$\lim_{t\to\infty}\mathbf{q}_i(t) = \mathbf{q}_c, \quad \lim_{t\to\infty}|\dot{\mathbf{q}}_i(t)| = 0. \tag{13}$$

The controller is the same as in (5) and (6) with $a_{i0} = 0$ for all $i \in \bar{N}$. In this case, the closed-loop system becomes

$$\begin{aligned}\ddot{\mathbf{q}}_i &= \mathbf{M}_i(\mathbf{q}_i)^{-1}\left[\mathbf{S}_i(\mathbf{x}_i - \mathbf{q}_i) - \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i\right] \\ \ddot{\mathbf{x}}_i &= \mathbf{J}_i^{-1}[\mathbf{S}_i(\mathbf{q}_i - \mathbf{x}_i) - \mathbf{D}_i\dot{\mathbf{x}}_i - k_i\sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{x}_i - \mathbf{x}_j)]\end{aligned} \tag{14}$$

Proposition 2: Consider a network of N flexible–joint manipulators, whose dynamics fulfill (1) and in closed-loop with the controller (5) and (6) with $a_{i0} = 0$ for all $i \in \bar{N}$. Suppose that Assumption (**A1**) hold. Under these conditions, all link positions asymptotically converge to a common consensus point and velocities asymptotically converge to zero, *i.e.*, (13) holds for all $i \in \bar{N}$.

Proof. The proof follows *verbatim* the proof of Proposition 1, hence only the main steps are given. In this case,

$$\mathcal{E}_i := \mathcal{K}_i(\dot{\mathbf{q}}_i, \dot{\mathbf{x}}_i) + \mathcal{U}_i(\mathbf{q}_i, \mathbf{x}_i)$$

where $\mathcal{K}_i$ and $\mathcal{U}_i$ have been defined in (8) and (9), respectively. Moreover, using $\mathcal{E}$ as in (11) yields

$$\dot{\mathcal{E}} = -\sum_{i\in\bar{N}}\frac{1}{k_i}\dot{\mathbf{x}}_i^\top\mathbf{D}_i\dot{\mathbf{x}}_i \leq 0.$$

Using, systematically, Barbalat's Lemma with

$$\mathbf{J}_i\frac{d}{dt}\ddot{\mathbf{x}}_i = -\mathbf{D}_i\ddot{\mathbf{x}}_i - \mathbf{S}_i(\dot{\mathbf{x}}_i - \dot{\mathbf{q}}_i) - k_i\sum_{j\in\mathcal{N}_i} a_{ij}(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_j),$$

it can be shown that the part of the equilibrium point given by $(\ddot{\mathbf{x}}_i, \dot{\mathbf{x}}_i, \ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i) = (\mathbf{0}_n, \mathbf{0}_n, \mathbf{0}_n, \mathbf{0}_n)$ is GAS. The rest of the equilibrium satisfies $\mathbf{q}_i = \mathbf{x}_i$ and $\sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{x}_i - \mathbf{x}_j) = \mathbf{0}_n$, which implies that $\mathbf{q} = \mathbf{x}$ and $(\mathbf{L} \otimes \mathbf{I}_n)\mathbf{x} = \mathbf{0}_{Nn}$. This last, together with the Laplacian properties, ensures that the only possible solution to these equations is $\mathbf{q} = \mathbf{x} = (\mathbf{1}_N \otimes \mathbf{q}_c)$, for any $\mathbf{q}_c \in \mathbb{R}^n$. Hence, for all $i \in \bar{N}$, $\mathbf{q}_i = \mathbf{q}_c$. This concludes the proof. □

Remark 1: If link accelerations are not available for measurement then, to implement the proposed controllers, the term $\ddot{\mathbf{g}}_i(\mathbf{q}_i)$ can be algebraically computed as

$$\begin{aligned}\ddot{\mathbf{g}}_i(\mathbf{q}_i) &= -\frac{\partial\mathbf{g}_i(\mathbf{q}_i)}{\partial\mathbf{q}_i}\ddot{\mathbf{q}}_i + \sum_{k=1}^n\frac{\partial^2\mathbf{g}_i(\mathbf{q}_i)}{\partial\mathbf{q}_i\partial q_{i_k}}\dot{\mathbf{q}}_i\dot{q}_{i_k} \\ &= -\frac{\partial\mathbf{g}_i(\mathbf{q}_i)}{\partial\mathbf{q}_i}\mathbf{M}_i^{-1}\left[\mathbf{C}_i\dot{\mathbf{q}}_i + \mathbf{S}_i(\mathbf{q}_i - \mathbf{x}_i)\right] \\ &\quad + \sum_{k=1}^n\frac{\partial^2\mathbf{g}_i(\mathbf{q}_i)}{\partial\mathbf{q}_i\partial q_{i_k}}\dot{\mathbf{q}}_i\dot{q}_{i_k}.\end{aligned}$$

This algebraic manipulation does not induce an algebraic loop because the relative degree is four.

VI. Simulations

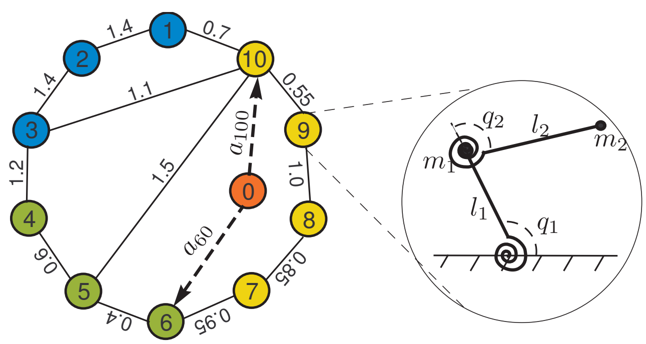

Fig. 1. Weighted network composed of ten 2-DOF nonlinear flexible-joint manipulators with revolute joints and a leader at node 0. There are three different groups of manipulators and the members of each group are equal.

By means of some numerical simulations, this section shows the consensus performance of the proposed controllers. For, we consider a weighted network composed of ten 2-DOF nonlinear flexible-joint manipulators with revolute joints. The network has three different groups of manipulators and the members of each group are equal. The inertia matrix, the Coriolis and centrifugal effects matrix and the gravity vector are, respectively,

$$\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix} \alpha_i + 2\beta_i c_{i_2} & \delta_i + \beta_i c_{i_2} \\ \delta_i + \beta_i c_{i_2} & \delta_i \end{bmatrix},$$

$$\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \begin{bmatrix} -2\beta_i s_{i_2}\dot{\mathbf{q}}_{i_2} & -\beta_i s_{i_2}\dot{\mathbf{q}}_{i_2} \\ \beta_i s_{i_1}\dot{\mathbf{q}}_{i_2} & 0 \end{bmatrix}$$

and

$$\mathbf{g}_i(\mathbf{q}_i) = \text{col}(l_{i_1}(m_{i_1}+m_{i_2})c_{i_1} + g l_{i_2} m_{i_2} c_{i_{12}}, g l_{i_2} m_{i_2} c_{i_{12}}),$$

where c_{i_k}, s_{i_k} are the short notation for $\cos(\mathbf{q}_{i_k})$ and $\sin(\mathbf{q}_{i_k})$; $c_{i_{12}}$ stands for $\cos(q_{i_1} + q_{i_2})$; $\mathbf{q}_{i_k}$ represents the angular position of link k of manipulator i, with $k \in 1, 2$; $\alpha_i = l_{i_2}^2 m_{i_2} + l_{i_1}^2(m_{i_1} + m_{i_2})$, $\beta_i = l_{i_1} l_{i_2} m_{i_2}$ and $\delta_i = l_{i_2}^2 m_{i_2}$, where l_{i_k} and m_{i_k} are the respective lengths and masses of each link and $g = 9.81$ is the acceleration of gravity constant.

The physical constant parameters are: $m_1 = 2.5$kg, $m_2 = 1.5$kg and $l_1 = l_2 = 0.4$m for agents 1, 2 and 3; $m_1 = 2$kg, $m_2 = 1$kg and $l_1 = l_2 = 0.4$m for agents 4, 5 and 6; $m_1 = 3$ kg, $m_2 = 2$ kg and $l_1 = l_2 = 0.4$m for agents 7, 8, 9 and 10. The motor inertia and the joint stiffness have been set to $\mathbf{J}_i = \text{diag}(0.7, 0.7)$ and $\mathbf{S}_i = \text{diag}(100, 100)$, respectively, for all the agents.

The following simulations are performed using three different sets of initial conditions and, in all cases, with initial zero velocities and $\boldsymbol{\theta}(0) = \mathbf{q}(0)$. For the first set

$$\begin{aligned} \mathbf{q}^\top(0) &= [1, 2, 2.5, 1.5, 3, 0.5, -1, -1.5, 0.5, 3, 3, -2 \\ &- 2.5, , -0.5, 0.5, 1, -2, 3.5, 3.5, 2.5]; \end{aligned} \quad (15)$$

for the second set, $\mathbf{q}(0)$ in (15) has been multiplied by 2 and, for the third set, $\mathbf{q}(0)$ in (15) has been multiplied by -2. The interconnection gains have been set as $k_i = 15$ and the damping gains as $\mathbf{D}_i = \text{diag}(8, 8)$, for all $i \in [1, 10]$.

A. Leader-Follower Consensus

The interconnection graph of the followers and the leader is shown in Fig. 1. The leader constant position is $\mathbf{q}_0 = [3, 0]^\top$. Only followers number 10 and 6 receive the desired leader position and the leader-follower interconnection gains have been set to $a_{100} = a_{60} = 2$.

The dynamic behavior of the followers link and joint positions is depicted in Fig. 2. Such behavior with the three different sets of initial conditions is shown in columns A, B and C, respectively, and in all cases the followers asymptotically reach the desired leader joint position.

B. Leaderless Consensus

For these simulations, the interconnection graph is the same of Fig. 1 but with $a_{100} = a_{60} = 0$. The leaderless consensus results are depicted in Fig. 3, from which it can be observed that the agents asymptotically reach a consensus point and such consensus point changes if the initial conditions change.

VII. Conclusions

Under the assumption that the interconnection graph is undirected and does not induce time-delays, a globally asymptotically stable solution to the leader-follower and the leaderless consensus problem, for networks composed of nonidentical flexible-joint robot manipulators, is reported in this paper. The proposed controller is composed of two different terms, one that dynamically compensates the link gravity and another which ensures the desired consensus objectives. The last term is a simple Proportional plus damping scheme.

Using a ten flexible-joint robot network, numerical simulations show the performance of the proposed controller for both consensus problems.

A clear extension of this work is the inclusion of variable time-delays in the interconnection. A possible solution to this issue is to incorporate the recent results in (Nuño *et al.* 2012). This extension is underway and will be reported elsewhere.

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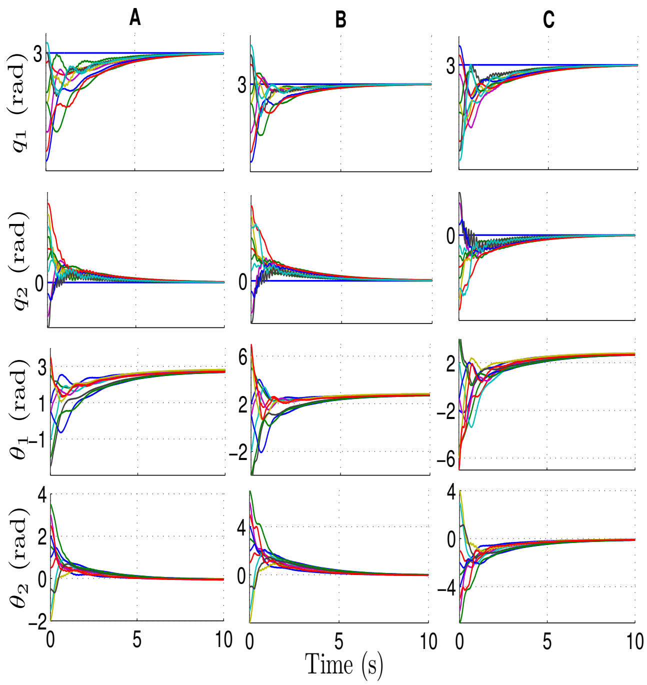

Fig. 2. Leader-follower simulation results. The leader constant position is $\mathbf{q}_0 = [3, 0]^\top$ and is only available at followers 10 and 6.

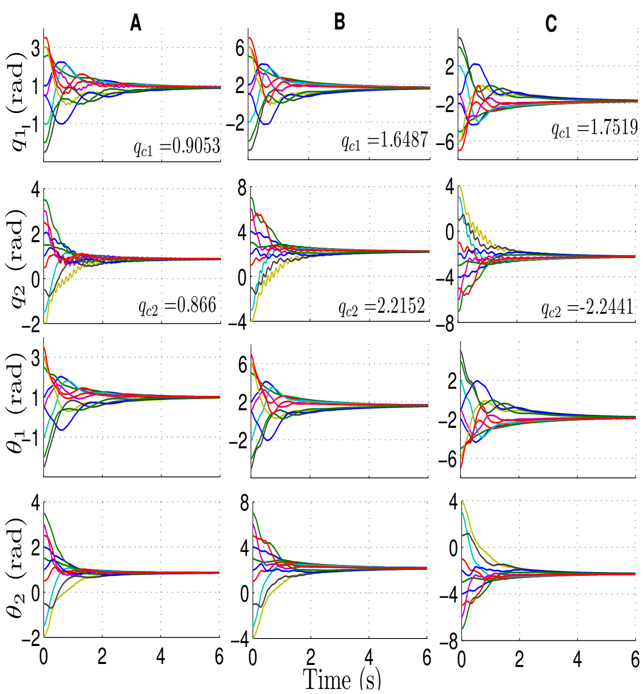

Fig. 3. Leaderless simulation results for three different sets of initial conditions.

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