

Nonlinear Maneuvering Control Design for a Wheeled Mobile Robot of the type (2,0)

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Abstrac—This paper deals with the kinematic control design of a dynamic compensator to solve the maneuvering problem for a wheeled mobile robot (WMR) of the type (2,0). The maneuvering problem involves two tasks; the first one called the *geometric task* which forces the system output to converge to a desired θ -parametrized path; the second one, called the *dynamic task*, intended to design an algorithm to assign a desired speed profile along the path. Nonlinear control techniques for control design are based on backstepping and Lyapunov functions. The controller performance is evaluated by real time experiments.

Key words: Maneuvering problem, path following, kinematic control, nonlinear control techniques.

I. INTRODUCTION

In many real problems autonomous mobile robots have diverse applications, e.g., military, surveillance, material transportation, etc. In these kind of applications, it is of primary importance to steer the mobile robot (wheeled mobile robots (WMRs), ships, UAV's, etc.) along a desired path. A velocity profile along the path may be of secondary interest. Motion control methodologies which solve these problems are *trajectory tracking* and *path following*.

In Micaelli et al., 1993, is proposed a path-following controller based on Frenet-Serret frames, which only requires to stabilize a single variable; in Jiang et al., 1997, is proposed a trajectory tracking controller for the WMR type (2,0) using the integrator backstepping. Trajectory tracking has some intrinsic problems for physical implementation, the main one, the time variable is the protagonist and not the position, so the tracking controller must force the system to converge to the trajectory and satisfy a velocity profile given by the time derivative of the trajectory, even if system position is not secure. In Aguiar et al., 2008, some performance limitations of the trajectory tracking are shown when is compared with the path following strategy.

In Skjetne et al., 2003, motivated by Hauser et al., 1995, is proposed a particular case of the general path following methodology called *maneuver regulation problem* or *maneuvering problem* which involves two tasks, the first and most important called the *geometric task* which forces the system output to converge to a continuous θ -parametrized path, and the second task, called the *dynamic task*, is aim to satisfy a desired dynamic assignment along the path. In Hespanha et al., 2007, tracking and path following con-

trollers are proposed for a class of underactuated systems, for the path following design is used the maneuvering problem approach. In Akhtar et al., 2011 and Roza et al., 2012, is proposed a maneuver regulation controller using transverse linearization and in Rodríguez-Cortés, H. and Velasco-Villa, M., 2011 is presented a nonlinear controller for maneuver regulation of a car-like mobile robot using input-output feedback linearization and a suitable Lyapunov control function.

Section II presents the maneuvering control approach and its application on a WMR of the type (2,0). In section III is presented the maneuvering control design using nonlinear control techniques. Section IV presents experimental results in a laboratory platform. To measure the position and orientation of the mobile robot we used a visual-based system that provides the absolute localization. Finally, Section V contains the conclusions.

II. MANEUVERING PROBLEM FOR A (2,0) WMR

In this section we present a general overview of the mathematical model used for motion control design in WMRs. The definition of the maneuvering problem in nonholonomic mechanical systems and its solution by the maneuvering control design for WMRs is also presented.

II-A. Mathematical model

Possible mathematical models used for control design in motion control problems are the kinematic and dynamic models. However, most motion control problems in WMRs are solved using the kinematic model (Morin and Samson, 2008) because almost all WMR have an inner control loop, which solves the velocity tracking problem and, from a theoretical framework, the dynamic model may be seen as a dynamic extension of the kinematic model. Some WMRs are nonholonomic mechanical systems, i.e., they have nonintegrable kinematics constrains of the form

$$A(q)^T \dot{q} = 0 \quad (1)$$

where $q : \mathbb{R}_+ \cup \{0\} \rightarrow \mathcal{Q}$ is a vector function of generalized coordinates which takes values in the configuration space (manifold) given by \mathcal{Q} , \dot{q} is the time derivative of q and $A : \mathcal{Q} \rightarrow \mathbb{R}^{n \times m}$ the nonholonomic constrains matrix function. The general kinematic model for WMRs is given as

$$\dot{q} = G(q)u \quad (2)$$

where $u : \mathbb{R}_+ \cup \{0\} \rightarrow \mathcal{U}$ is the input vector function, \mathcal{U} the control space (manifold) and $G : \mathcal{Q} \rightarrow \mathbb{R}^{n \times m}$ is a matrix function where its columns form a base of the null space of the nonholonomic constrains matrix function. In this paper we consider a nonholonomic (2,0)-WMR. Its kinematic model is given by the following set of differential equations

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos \psi & 0 \\ \sin \psi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (3)$$

where the vector of generalized coordinates $q = [x \ y \ \psi]^T$ of the vehicle is in the configuration space given by the smooth manifold $\mathcal{Q} = \mathbb{R}^2 \times S^1$ with S^1 the unit sphere and input vector $u = [v \ \omega]^T \in \mathcal{U} \subset \mathbb{R}^2$ is the input vector with v is the linear velocity and ω is the angular velocity. Figure 1 shows the geometric configuration of the robot.

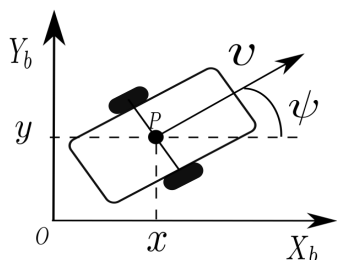


Figure 1. Wheeled mobile robot type (2,0).

II-B. Manuvering Control

In Hauser, J. et al., 1995 is introduced the concept of *maneuver* in the context of motion control, which is stated in the next definition.

Definition 1 (Maneuver): Consider system (2), a maneuver is a curve in the state-control space that is consistent with the system dynamics, i. e.,

$$\eta \triangleq \{[q_r^T(\theta), u_r^T(\theta)]^T \in \mathcal{Q} \times \mathcal{U} : \theta \in \mathbb{R}\} \quad (4)$$

such that

$$\frac{dq_r}{d\theta} = G(q_r(\theta))u_r(\theta). \quad (5)$$

Remark 1 (Consistency with WMRs kinematics): In our problem, we are considering nonholonomic mechanical systems represented by the general kinematic model (2) and the consistency of the curve will satisfy if the nonholonomic constrains are fulfilled.

Now, let $Y = h(q(t))$ be the output of system (2) which takes values in the output space $\mathcal{N} \subseteq \mathcal{Q}$. Let \mathcal{P} be a path in the output space \mathcal{N} represented by the 1-dimensional manifold

$$\mathcal{P}_d \triangleq \{y \in \mathcal{N} : \theta \in \mathbb{D} \text{ such that } Y = Y_r(\theta)\} \quad (6)$$

parametrized by the smooth map $\theta \mapsto Y_r(\theta)$ where $\mathbb{D} = \mathbb{R}$ or $\mathbb{D} = S^1$, according to the path. The output $Y(t)$ is used to specify a task (a trajectory or path).

Remark 2: Consider the task (6) given for specific Y . If $Y(t)$ converges to $Y_r(\theta)$, then the correspondent state-control curve $[q(t) \ u(t)]^T$ converges to $[q_r(\theta) \ u_r(\theta)]^T$.

Under remark 2, it is possible to define the maneuver regulation problem in terms of the system output, as stated in the next definition.

Definition 2 (Maneuvering Problem): Let $Y_d(\theta)$ be a path parametrization map and $v_s(\theta, t)$ a speed profile for (6). The maneuver regulation problem consists of two task:

- **Geometric task:** Force system output Y to converge to a desired path \mathcal{P} ,

$$\lim_{t \rightarrow \infty} \|Y_r(\theta) - Y(t)\| = 0 \quad (7)$$

for any continuous function $\theta(t)$.

- **Speed assignment task:** Force the path speed $\dot{\theta}$ to converge to a desired speed $v_s(\theta, t)$,

$$\lim_{t \rightarrow \infty} |\dot{\theta}(t) - v_s(\theta(t), t)| = 0. \quad (8)$$

An important observation about the speed assignment is given in the next remark,

Remark 3 (Speed assignment): For the speed assignment $v_s(\theta, t)$, we let a desired path speed (in m/s) be a commanded input speed $v_d(t)$. Since the identity

$$\|\dot{Y}_r(\theta)\| = |v_s(\theta, t)| \sqrt{x_r'(\theta)^2 + y_r'(\theta)^2} = v_d(t) \quad (9)$$

must hold along the path. Then, we get the speed assignment

$$v_s(\theta, t) = \frac{v_d(t)}{\sqrt{x_r'(\theta)^2 + y_r'(\theta)^2}}. \quad (10)$$

Setting $v_d(t) = 0$ will stop the WMR on the path, while setting $v_d(t) > 0$ will move the WMR in positive direction along the path and $v_d(t) < 0$ will move it in negative direction.

To solve the maneuvering problem, in this paper we define the method *maneuvering control*.

Definition 3 (Maneuvering control): Find a dynamic output feedback controller, such that the *maneuvering controller*

$$u = \alpha(q, \theta, q_r, \omega_s), \quad (11)$$

and the *speed assignment algorithm*,

$$\dot{\omega}_s = S(\omega_s, q, \theta, q_r) \quad (12)$$

with $\dot{\theta} = v_s(\theta, t) - \omega_s$; solve the maneuvering problem.

II-C. Problem Statement

Let $\mathcal{P} \subset \mathbb{R}^2$ be a desired path for (3), parametrized by a function $y_r(\theta) \in C^\infty$, with its first two derivatives bounded and $v_s(\theta, t)$ a speed profile assignment. Design a dynamic output feedback control law as in definition (3) such that all the closed-loop signals are bounded and the position of the WMR given in (3) converges and remains on the desired path. Moreover, in the presence of mobile robot localization errors, the WMR type (2,0) position remains inside a tube centered at the desired path.

III. MANEUVERING CONTROLLER DESIGN

To design the maneuver regulation controller we suppose that the mobile robot follows a virtual reference mobile robot (VMR) with the same kinematics (3) as in (Morin and Samson, 2008). In this case, a necessary condition for the existence of a control solution is that the reference is feasible according to Remark 1. Feasible paths are parameterized by smooth functions $\theta \mapsto [x_r(\theta) \ y_r(\theta) \ \psi_r(\theta)]^T$ on $\theta \in \mathbb{R}$, which are solution of the robot's kinematic model (3) for a specific control $u_r(\theta) = [v_r(\theta) \ \omega_r(\theta)]^T$ called *reference control*. So, the kinematic model for the virtual reference robot is:

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\psi}_r \end{bmatrix} = \begin{bmatrix} \cos \psi_r & 0 \\ \sin \psi_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \quad (13)$$

where (x_r, y_r) are the coordinates of the point P_r , and ψ_r is the orientation angle with respect to X (see Figure 2).

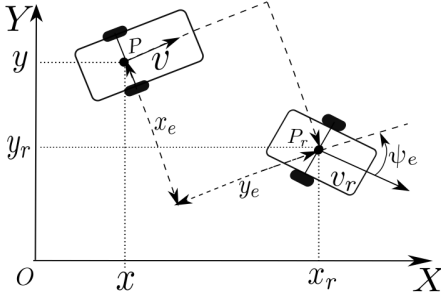


Figure 2. Following a reference vehicle

The reference control $u_r(\theta)$ is a motion planning *open-loop* controller that steers the virtual WMR from initial condition $q_r(\theta_0)$ to describe a path as the one given in (6), satisfying the WMR nonholonomic constraints. To determine the reference or nominal control law, notice that from equation (13) the first two equations can be rewritten as

$$\begin{aligned} x'_r(\theta)\dot{\theta} &= v_r \cos \psi_r \\ y'_r(\theta)\dot{\theta} &= v_r \sin \psi_r \end{aligned} \quad (14)$$

where $z'(\theta) = \frac{dz}{d\theta}$. Equation (4) allows us to define v_r and ψ_r from (14) as

$$\begin{aligned} v_r &= \dot{\theta} [x'_r(\theta) \cos \psi_r + y'_r(\theta) \sin \psi_r] \\ \psi_r &= \arctan \left(\frac{y'_r(\theta)}{x'_r(\theta)} \right). \end{aligned} \quad (15)$$

By taking the time derivative of (15) we get

$$\omega_r = \frac{\dot{\theta} [y''_r(\theta)x'_r(\theta) - y'_r(\theta)x''_r(\theta)]}{x'_r(\theta)^2 + y'_r(\theta)^2} \quad (16)$$

with $\dot{\theta}$ as in Definition 3. Now, the problem is to determinate a feedback control law $(u, \dot{\omega}_s)$ which asymptotically stabilizes the path-following error. In the maneuvering problem definition, the error used to define the geometric task (7) is

given by $e_r(\theta, t) = y(t) - y_r(\theta)$ which is defined with respect to the desired path. However, in this paper we consider and adequate global transformation of the original vector error in the form

$$\begin{bmatrix} x_e \\ y_e \\ \psi_e \end{bmatrix} = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \psi_r - \psi \end{bmatrix} \quad (17)$$

Taking the time derivative of (17), we obtain the error dynamics

$$\dot{x}_e = \omega y_e - v + v_r \cos \psi_e \quad (18a)$$

$$\dot{y}_e = -\omega x_e + v_r \sin \psi_e \quad (18b)$$

$$\dot{\psi}_e = \omega_r - \omega. \quad (18c)$$

Notice that error dynamics y_e is not directly affected by the input v . To overcome this difficulty, we use the idea of integrator *backstepping* (Krstic, Kanellakopoulos, Kokotovic et al., 1995).

Given any fixed $0 < \epsilon < \pi$, let us introduce the set of functions denoted by Φ_ϵ^∞ :

$$\begin{aligned} \Phi_\epsilon^\infty \triangleq \{ \phi : \mathbb{R} \rightarrow (-\pi + \epsilon, \pi - \epsilon) : \phi \in C^\infty, \phi(0) = 0, \\ z\phi(z) > 0 \ \forall z \neq 0 \text{ and } \phi'(z) \text{ is bounded} \}. \end{aligned} \quad (19)$$

Setting $x_e = 0$ and $\psi_e = -\phi(y_e v_r)$, in (18b) implies that $\dot{y}_e = -v_r \sin \phi(y_e v_r)$ is uniformly stable at $y_e = 0$. Then, from the above observation, let ζ be a backstepping error variable defined as

$$\zeta(\psi_e, \phi) \triangleq \psi_e(t) + \phi(v_r y_e). \quad (20)$$

With (18c), the equation (20) is transformed into

$$\begin{aligned} \dot{\zeta} = \omega_r + \phi'(v_r y_e) \left(\frac{\partial v_r}{\partial \theta} (v_s - \omega_s) y_e + \frac{\partial v_r}{\partial v_s} \dot{v}_s y_e \right. \\ \left. + \frac{\partial v_r}{\partial \omega_s} \dot{\omega}_s y_e + v_r^2 \sin(\zeta - \phi) \right) - \omega \left(1 + v_r x_e \phi'(v_r y_e) \right). \end{aligned} \quad (21)$$

To add the desired speed profile $v_s(\theta, t)$ for speed assignment task (8), let ω_s be another backstepping error variable defined as

$$\omega_s(\dot{\theta}, \theta, t) = v_s(\theta, t) - \dot{\theta}. \quad (22)$$

The time derivative $\dot{\omega}_s$ of (22) is going to be used as an additional control input in the control design process.

Consider the candidate Lyapunov function,

$$W(x_e, y_e, \zeta, \omega_s) = \frac{1}{2} x_e^2 + \frac{1}{2} y_e^2 + \frac{1}{2\gamma} \zeta^2 + \frac{1}{2} k_3 \omega_s^2 \quad (23)$$

with $\gamma, k_3 > 0$. It can be directly verified that (23) is a positive-defined, decreasing and radially unbounded function.

Taking the time derivative of (23) along the solutions of (18a), (18b) and (21) yields

$$\begin{aligned} W(x_e, y_e, \zeta, \omega_s) &= x_e[\omega y_e - v + v_r \cos(\zeta - \phi)] \\ &+ y_e[-\omega x_e + v_r \sin(\zeta - \phi)] + \frac{1}{\gamma} \zeta \left[\omega_r + \right. \\ &+ \phi'(v_r y_e) \left(\frac{\partial v_r}{\partial \theta} (v_s - \omega_s) y_e + \frac{\partial v_r}{\partial v_s} \dot{v}_s y_e + \right. \\ &+ \left. \frac{\partial v_r}{\partial \omega_s} \dot{\omega}_s y_e + v_r^2 \sin(\zeta - \phi) \right) + \\ &\left. - \omega \left(1 + v_r x_e \phi'(v_r y_e) \right) \right] + k_3 \omega_s \dot{\omega}_s. \end{aligned} \quad (24)$$

After some computations

$$\begin{aligned} W(x_e, y_e, \zeta, \omega_s) &= x_e[-v + v_r \cos(\zeta - \phi) + \\ &- \frac{1}{\gamma} \zeta v_r \phi'(v_r y_e) + y_e v_r \sin(\zeta - \phi) + \frac{1}{\gamma} \zeta \left[\omega_r + \right. \\ &+ \phi'(v_r y_e) \left(\frac{\partial v_r}{\partial \theta} (v_s - \omega_s) y_e + \frac{\partial v_r}{\partial v_s} \dot{v}_s y_e + \right. \\ &+ \left. \frac{\partial v_r}{\partial \omega_s} \dot{\omega}_s y_e + v_r^2 \sin(\zeta - \phi) \right) - \omega] + k_3 \omega_s \dot{\omega}_s. \end{aligned} \quad (25)$$

By choosing the maneuvering controllers v and ω as

$$v = v_r \cos(\zeta - \phi) - \frac{1}{\gamma} \zeta \omega v_r \phi'(v_r y_e) + k_1 x_e \quad (26)$$

$$\begin{aligned} \omega &= \omega_r + \phi'(v_r y_e) \left(\frac{\partial v_r}{\partial \theta} (v_s - \omega_s) y_e + \frac{\partial v_r}{\partial v_s} \dot{v}_s y_e + \right. \\ &+ \left. v_r^2 \sin(\zeta - \phi) \right) + \gamma y_e v_r \frac{\sin(\zeta)}{\zeta} \cos(\phi) + k_2 \zeta \quad (27) \end{aligned}$$

with $k_1, k_2 \in \mathbb{R}_+$. It is obtained

$$\begin{aligned} \dot{W}(x_e, y_e, \zeta, \omega_s) &= -k_1 x_e^2 - y_e v_r \cos(\zeta) \sin(\phi) - \frac{k_2}{\gamma} \zeta^2 + \\ &+ \left(k_3 \omega_s + \frac{1}{\gamma} \zeta \phi'(v_r y_e) \frac{\partial v_r}{\partial \omega_s} y_e \right) \dot{\omega}_s. \end{aligned} \quad (28)$$

Defining $\dot{\omega}_s$ as

$$\dot{\omega}_s = - \left(k_3 \omega_s + \frac{1}{\gamma} \zeta \phi'(v_r y_e) \frac{\partial v_r}{\partial \omega_s} y_e \right) \quad (29)$$

we have that

$$\begin{aligned} \dot{W}(x_e, y_e, \zeta, \omega_s) &= -k_1 x_e^2 - y_e v_r \cos(\zeta) \sin(\phi) + \\ &- k_2 \zeta^2 - \left(k_3 \omega_s + \frac{1}{\gamma} \zeta \phi'(v_r y_e) \frac{\partial v_r}{\partial \theta} y_e \right)^2 \end{aligned} \quad (30)$$

which finally implies

$$\dot{W}(x_e, y_e, \zeta, \omega_s) \leq -y_e v_r \cos(\zeta) \sin(\phi). \quad (31)$$

For initial conditions $[x(t_0) \ y(t_0) \ \psi(t_0)]^T$ near to the path (6), i.e., in a ϵ -neighborhood of any $y_r(\theta) \in \mathcal{P}$, it is easy to verify that $Im\phi, Im\zeta \subset \{\beta \in \mathbb{R} : |\beta| < \epsilon\}$. Then, due to equation (19), we have that

$$\dot{W}(x_e, y_e, \zeta, \omega_s) = -y_e v_r \phi(y_e v_r) < 0. \quad (32)$$

Thus, the maneuvering problem has been solved by the maneuvering control approach. The maneuvering controller is $u = [v \ w]^T$, where u, ω are defined in (26) and (27) respectively and the speed assignment algorithm is given by (29).

IV. EXPERIMENTAL RESULTS

To show the maneuver regulation controller performance, let us consider the path

$$\begin{aligned} \mathcal{P}_r &= \{[x \ y]^T \in \mathbb{R}^2 : \\ Y_r(\theta) &= [a \cos \theta, b \cos(\theta) \sin(\theta)]^T, \theta, r \in \mathbb{R}\}. \end{aligned} \quad (33)$$

The correspondent maneuver is

$$\begin{aligned} q_r(\theta) &= \left[a \cos \theta \quad b \cos \theta \sin \theta \quad \arctan \left(\frac{-\sin \theta}{b \cos(2\theta)} \right) \right]^T \\ u_r(\theta) &= [r(v_s - \omega_s)^2 \quad v_s - \omega_s]^T. \end{aligned} \quad (34)$$

Together with the following specifications :

$$\frac{\partial v_r}{\partial v_s} = \left[x'_r(\theta) \cos \psi_r + y'_r(\theta) \sin \psi_r \right] \quad (35a)$$

$$\frac{\partial v_r}{\partial \omega_s} = - \left[x'_r(\theta) \cos \psi_r + y'_r(\theta) \sin \psi_r \right] \quad (35b)$$

$$\phi(v_r y_e) = \arctan(v_r y_e) \quad (35c)$$

$$\begin{aligned} \frac{\partial v_r}{\partial \theta} &= \left[\frac{\partial v_s}{\partial \theta} - \frac{\partial \omega_s}{\partial \theta} \right] \left[x'_r(\theta) \cos \psi_r + \right. \\ &+ \left. y'_r(\theta) \sin \psi_r \right] + \end{aligned}$$

$$\begin{aligned} &+ (v_s - \omega_s) \left[x''_r(\theta) \cos \psi_r + \right. \\ &+ \left. y''_r(\theta) \sin \psi_r + \left(y_r(\theta) \cos \psi_r + \right. \right. \end{aligned} \quad (35d)$$

$$\left. - x'_r(\theta) \sin \psi_r \right) \frac{\partial \psi_r}{\partial \theta} \left. \right]$$

$$\begin{aligned} \frac{\partial v_s}{\partial \theta} &= \frac{v_s}{\sqrt{x'_r(\theta)^2 + y'_r(\theta)^2}} \left[x'_r(\theta) x''_r(\theta) + \right. \\ &+ \left. y'_r(\theta) y''_r(\theta) \right] \end{aligned} \quad (35e)$$

$$\frac{\partial \omega_s}{\partial \theta} = \frac{\omega_s(k) - \omega_s(k-1)}{\theta(k) - \theta(k-1)}. \quad (35f)$$

where $\omega_s(k)$ and $\theta(k)$ are the numerical values of the variables in the k -th step of the program execution.

In the original system coordinates, the control input

vector components are

$$\omega = \omega_r + \phi'(v_r y_e) \left(\frac{\partial v_r}{\partial \theta} (v_s - \omega_s) y_e + \frac{\partial v_r}{\partial v_s} \dot{v}_s y_e + v_r^2 \sin(\psi_e) \right) + \gamma y_e v_r \frac{\sin(\psi_e + \phi)}{\psi_e + \phi} \cos(\phi) + k_2(\psi_e + \phi) \quad (36)$$

$$v = v_r \cos(\psi_e) - \frac{1}{\gamma} (\psi_e + \phi) \omega v_r \phi'(v_r y_e) + k_1 x_e \quad (37)$$

And the speed assignment algorithm is

$$\dot{\omega}_s = - \left[k_3 \omega_s - \frac{2r}{\gamma} (\psi_e + \phi) \phi'(v_r y_e) (v_s - \omega_s) \right] \quad (38)$$

To implement the maneuvering controller, we need to transform the linear velocity v and angular velocity ω of the robot into angular velocities ω_{right} and ω_{left} for the right and left wheels of the WMR respectively. This is done by the transformation,

$$\begin{aligned} \omega_{right} &= \frac{1}{r} (v + m\omega) \\ \omega_{left} &= \frac{1}{r} (v - m\omega) \end{aligned} \quad (39)$$

where r is the wheel radius and m the length between the wheels of the robot.

For the experiment, we used a Garcia Robot (GR) from *Acroname* which is a WMR of the type (2,0). To measure the absolute position of GR, we used the vision based system *OptiTrack-Flex13* (see Figure 3), with a resolution of 1.3 million pixels, 120 FPS sample rate, and 56° field of view.



Figure 3. On the left OptiTrack and on the right GR.

In Table (I) the parameters for the experiment are defined.

Table I

EXPERIMENT PARAMETERS		
$x(0) = 0,4946m$	$\theta(0) = 0rad$	$k = 1,22$
$y(0) = 1,2820m$	$\omega_s(0) = 0,1$	$k_2 = 1,55$
$\psi(0) = -2,8705rad$	$\gamma = 50$	$k_3 = 1,1$
$v(0) = 0m/s$	$\omega(0) = 0rad/s$	$v_d = 0,25m/s$

Figure 4 shows the error behavior with respect to time, subject to the input vector u . Error components converge to zero, which implies system state converges to the desired maneuver (33) with a settling time of 10 seconds.

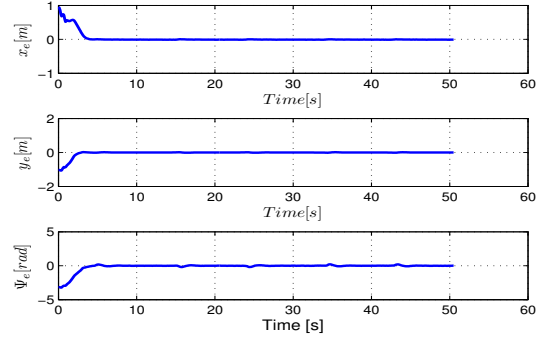


Figure 4. Evolution of the error components.

In Figure 5 the behavior of the speed assignment variable is shown. It is clear that this variable is appropriately bounded. Also, the evolution of the path variable θ depicted.

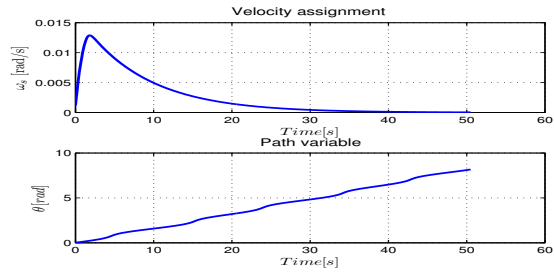


Figure 5. Speed assignment v_s and path variable θ .

Figure 6 shows the vector control components behavior along the time.

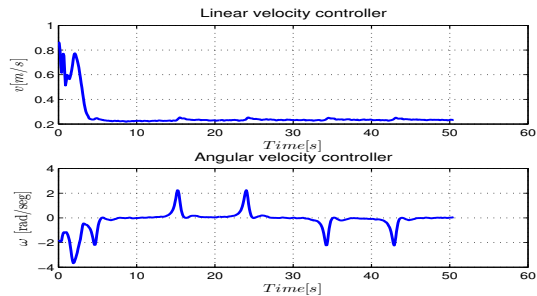


Figure 6. Linear velocity controller v and angular velocity controller ω .

In Figure 7 it is possible to see that the speed along the curve for the point P of the WMR converges to v_d , signal was obtained by numerical differentiation.

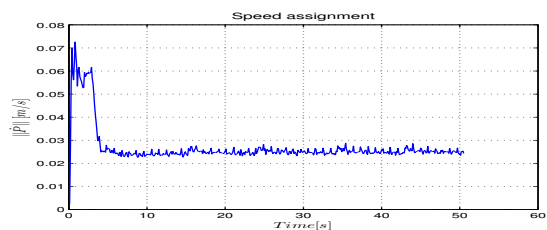


Figure 7. Speed assignment for the point $P = (x, y)$ of the WMR

In (8) is the path that the system configuration followed.

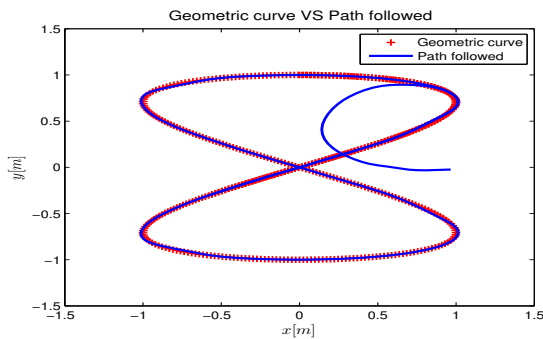


Figure 8. Real cartesian evolution and desired path \mathcal{P}_r

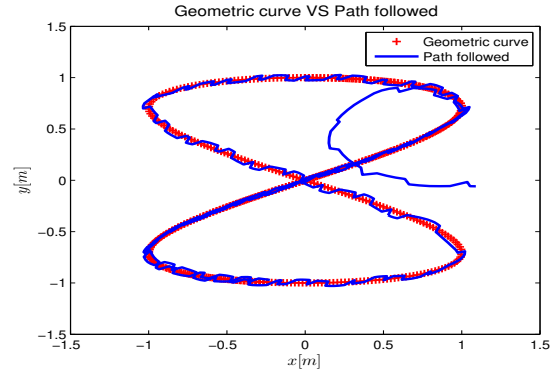


Figure 12. Real cartesian evolution with CPME and desired path \mathcal{P}_r

Now, let's suppose the absolute cartesian position is obtained by the Xsen sensor MTi-G with a global positioning system (GPS), then after a characterization of the sensor, it has a cartesian position measurement error (CPME) with variance $0,8m$. In the scale of the experiment, that variance is equivalent to $0,0175m$. However, for the experiment we consider $0,04cm$ of variance. After the experiment with the similar initial conditions, the following results were obtained

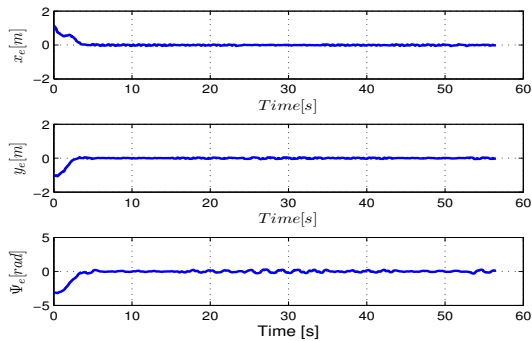


Figure 9. Evolution of the error components with CPME.

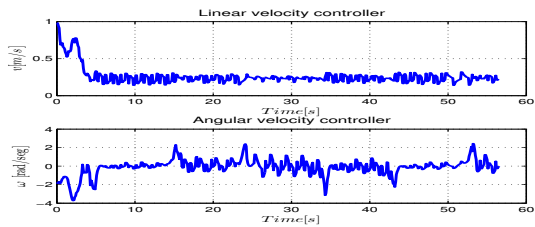


Figure 10. Control components subject to CPME.

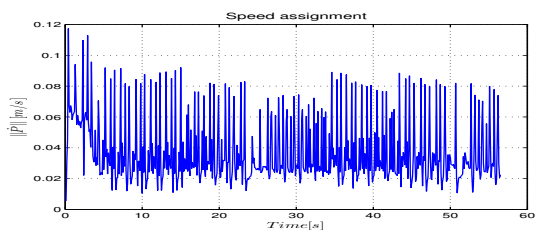


Figure 11. Speed of point $P = (x, y)$ with CPME.

V. CONCLUSIONS

The maneuvering problem was solved by a dynamic feedback controller which consists of a state feedback controller called maneuvering controller plus a speed assignment algorithm. The WMR configuration converged to the correspondent maneuver to follow and remain at the desired path. The experiments were carried out in an indoor platform with the absolute position and orientation of the system measured by a vision-based system. Experimental results confirm the theoretical development of the work. When the controller was subject to CPME, the system position P converged and remained inside a tube centered at the path.

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