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Control reconfiguration for differentially flat systems

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***Abstract*— This paper presents a control reconfiguration approach for nonlinear flat systems. Flatness property affords analytical redundancy and permit to compute the states and control inputs of the system. Fault isolation task is accomplished by comparing real measures and the computed signals obtained using the differentially flat equations. Redundant signals are used to reconfigure the faulty system. Feasibility of this approach is investigated in a three tank system.**

Index Terms: Fault tolerance, Nonlinear systems, Differential flatness.

I. Introduction

In the last decades, demographic explosion and globalization, detonated the necessity to design and operate profitable production systems and reliable transport systems. In order answer this necessity, researchers developed control systems capable of continuing to operate despite faults. Those systems are known as fault tolerant control systems (FTC). Two different approaches exists nowadays, passive and active. The first is known as robust control, here, the controller is insensitive to the occurrence of specific faults. Passive approaches are very restrictive because all the expected faults and their consequence in the controlled system cannot be known a priori. Those techniques are out of the boundaries of this work, interested readers are referred to (Benosman, 2011) and references therein. Active approach in contrast to passive one, adjusts the control loop on-line according to the fault affecting the system.

Process monitoring is necessary because if the active fault tolerant control system (AFTCS) reacts to different faults in different ways, a certain information about the occurring fault has to be available in real time. The mechanism providing this information is known as Fault Detection and Isolation (FDI). In order to accomplish this task three different knowledge-based stategies can be used: quantitative models, (Venkatasubramanian et al., 2003), qualitative models, (Venkatasubramanian et al., 2003a) and historical data, (Venkatasubramanian et al., 2003b).

This work is devoted to quantitative model-based methods, specifically in analytical redundancy. In contrast with hardware redundancy where measurements of parallel physical sensors are compared between them, sensor measurements are compared to analytically generated signals. As a result of this comparison residual signals are obtained, which are indicators of system faults.

Here, analytical redundancy is generated using differential flatness, which is a property of some nonlinear systems and every controllable linear system. If a nonlinear plant is flat, every system variable (states and control inputs) can be computed using a set of variables, called flat outputs. Residues are obtained by comparing those signals and the sensor measures.

If flat outputs are not affected by faults, a fault-free version of faulty states is available (Martínez et al., 2013). By consequence fault reconfiguration is developed by simply changing the faulty reference of the controller by the computed one.

This paper is organized as follows: section II presents the differential flatness property and the flatness motion planning. Control reconfiguration approach is presented in section III. Section IV develops the control reconfiguration approach in a three tank system. Section V contains the conclusion.

II. Differential Flatness

A non-linear or linear system is flat if there exists a set of variables differentially independent, called flat outputs whose number is equal to the quantity of control inputs, such as, the vector state and the control inputs can be expressed as functions of the flat outputs and a finite number of its time derivatives. By consequence, state and control inputs trajectories can be obtained by planning only the flat output trajectories, this property can be particularly exploited on trajectory planning, see (Nieuwstadt and Murray, 1998) and (Milam et al., 2005) and trajectory tracking (Antritter et al., 2004) and (Stumper et al., 2012).

Definition 1: Let us consider the nonlinear system $\dot{x} = f(x,u)$, $x \in \Re^n$ the state vector, $u \in \Re^m$ the control vector and f a C^∞ function of x and u. The system is differentially flat if, and only if, there exists a flat output vector $z \in \Re^m$ such as:

- The flat output vector is expressed as function of the state x and the control input u and a finite number of its time derivatives.

$$z = \phi_z(x, u, \dot{u}, ..., u^{(\gamma)}) \tag{1}$$

- The state x and the control input u are expressed as functions of the vector z and a finite number of its time derivatives.

$$\begin{aligned} x &= \phi_x(z, \dot{z}, ..., z^{(a)}) & (2) \\ u &= \phi_u(z, \dot{z}, ..., z^{(a+1)}) & (3) \end{aligned}$$

Where $z^{(a)}$ denotes the a^{th} time derivative of z.

II-A. *Motion planning*

Definition 1 implies that every system variable can be expressed in terms of the flat outputs and a finite number of its time derivatives. By consequence, if we want to compute a trajectory whose initial and final conditions are specified, it suffices to construct a flat output trajectory to obtain the open loop control inputs satisfying the desired output of the system

In order to compute all the system variables, the flat output trajectory created needs to be at least r times differentiable, where r is the maximal time derivative of the flat output appearing in the differentially flat equations. Additionally this trajectory is not required to satisfy any differential equation. By consequence the flat outputs trajectories can be created by using the simple polynomial approach. Thus, in this paper, flat outputs trajectories are generated by tunning polynomial functions to cover initial and final conditions on position, velocity and acceleration.

III. FAULT TOLERANCE

Active fault tolerant control systems has to perform at least two main activities in order to reconfigure the faulty system:

- FDI is the element in charge of detect and send the fault information to the reconfiguration mechanism. Afterwards the control action is adapted according to the fault.
- Control reconfiguration, which use the information generated by the FDI block in order to adapt the control action, having as goal at least system stability or at best nominal behavior.

Control reconfiguration can be afforded in two different ways:

- Design a different controller particular to a specific fault.
- Adapt controller parameters.

The complexity of the first one is present in the design stage, because the designer has to take into account all the possible faults affecting the controlled plant. After that create a different controller for each fault case. On the other hand speed of adaptation and fault limitations are the main concerns.

In order to simplify the reconfiguration task, this approach is focused in minimizing the computing time and facilitate the decision making after fault occurrence. To achieve this, the main idea is merge the reconfiguration mechanism with the FDI block. By doing this, a simple controller is synthesized and the reconfiguration is carried out by switching the controller reference. Fig. 1

III-A. *Fault Detection and Isolation*

This task is carried out by using the redundant signals computed using the differentially flat equations.

Let us consider a nonlinear flat model of dimension n, and m control inputs, with z as flat outputs, which corresponds to m components of the state vector, also suppose that the full state is measured, using the states and inputs calculated from (2) and (3), it is always possible to compute n residues:

- n - m state residues, as long as the full state is supposed to be measured.
- m control inputs residues.

The residual signals are computed by using

$$\begin{aligned} r_{jx} &= x_{mk} - \hat{x}_k & (4) \\ r_{ju} &= u_{ml} - \hat{u}_l & (5) \end{aligned}$$

where x_{mk} and u_{ml} are the k_{th} and l_{th} measured state and control input respectively and $\hat{x}_k$ and $\hat{u}_l$ are the k_{th} and l_{th} state and control input calculated using the differentially flat equations.

A nonlinear system composed by four states $[x_1\ x_2\ x_3\ x_4]^T \in \Re^n$ and two control inputs $[u_1\ u_2]^T \in \Re^m$ has two flat outputs, $[z_1\ \ z_2]^T = [x_1\ \ x_2]^T \in \Re^m$, four residuals can be obtained, two state residues and two control inputs residues, see (6).

$$\begin{bmatrix} r_{1x} \\ r_{2x} \\ r_{1u} \\ r_{2u} \end{bmatrix} = \begin{bmatrix} x_{m3} \\ x_{m4} \\ u_{m1} \\ u_{m2} \end{bmatrix} - \begin{bmatrix} \phi_x(z_1, \dot{z}_1, z_2, \dot{z}_2)\ (e_3)^T \\ \phi_x(z_1, \dot{z}_1, z_2, \dot{z}_2)\ (e_4)^T \\ \phi_u(z_1, \dot{z}_1, z_2, \dot{z}_2)\ (c_1)^T \\ \phi_u(z_1, \dot{z}_1, z_2, \dot{z}_2)\ (c_2)^T \end{bmatrix} \tag{6}$$

Where $e_a \in \Re^n$, $\forall a \neq k\ e_a = 0$, $e_a = 1 \Leftrightarrow a = k$, where $a = [1, 2, ..n]$ and $c_b \in \Re^m$, $\forall b \neq l\ c_b = 0$, $c_b = 1 \Leftrightarrow b = l$, where $b = [1, 2, ..m]$.

Proposition 1: Faults affecting flat outputs can be detected but cannot be isolated, since the n residues will be affected, however faults affecting actuators or state sensors are or are not isolated depending on the specific system.

III-B. *Control reconfiguration*

The hypothesis presented in the section above implies that:

- n residues are always available.
- State sensor faults not affecting flat outputs can be isolated and reconfigured.
- Actuators faults can also be isolated and reconfigured.

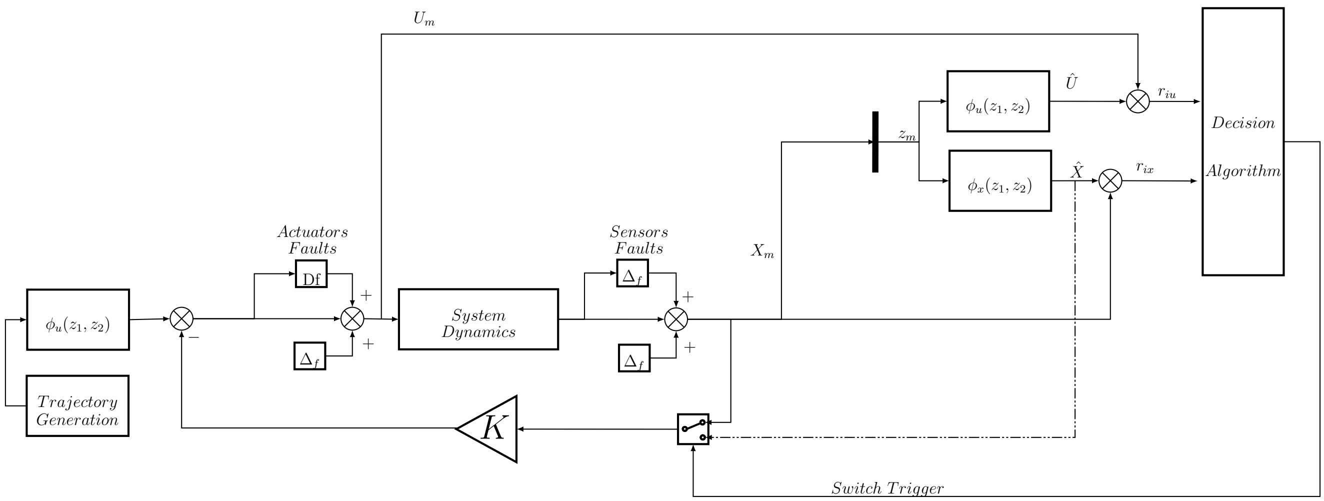

Figure 1. Control reconfiguration schema

- Flat output sensor faults can always be detected, fault isolation depends on the system and control reconfiguration is not possible by using the proposed methodology.

Control reconfiguration stage is assured if every fault not-affecting flat outputs are isolable. Reconfiguration is possible by using the analytically redundant states system obtained through flat systems properties.

In this way if a fault affects one state sensor and the flat outputs are fault-free, it is always possible to compute an unfaulty reference for the controller.

Let us revisit the example of section III-A.

Equation (6) shows that if a fault affects measure x_{m3}, only r_{1x} will be triggered, by consequence the fault is detected and isolated. Besides since the fault does not affect any flat output a fault-free version of x_3 is available to reconfigure the controller by simply switching between the faulty measure and the unfaulty reference computed with the flat outputs. see Fig. 1. Faults affecting x_{m4} can be treated in the same manner. On the other hand, the right side of (6) will be always affected when a flat output measure is faulty. By consequence the n residues will exceed the threshold. The fault can be detected but isolation is not possible. Since the measure affected is a flat output, no redundant signal is computed. As a result control reconfiguration is not possible in this case. Hence actuators faults are rejected by the controller.

IV. EXAMPLE: THREE TANK SYSTEM

IV-A. *Nonlinear state space model*

The feasibility of the proposed approach is studied in a three tank system, see Fig. 2 , the system equations are expressed as follows:

$$S\dot{x}_1 = -Q_{10}(x_1) - Q_{13}(x_1, x_3) + u_1 \quad (7)$$

$$S\dot{x}_2 = -Q_{20}(x_2) + Q_{32}(x_2, x_3) + u_2 \quad (8)$$

$$S\dot{x}_3 = Q_{13}(x_1, x_3) - Q_{32}(x_2, x_3) - Q_{30}(x_3) \quad (9)$$

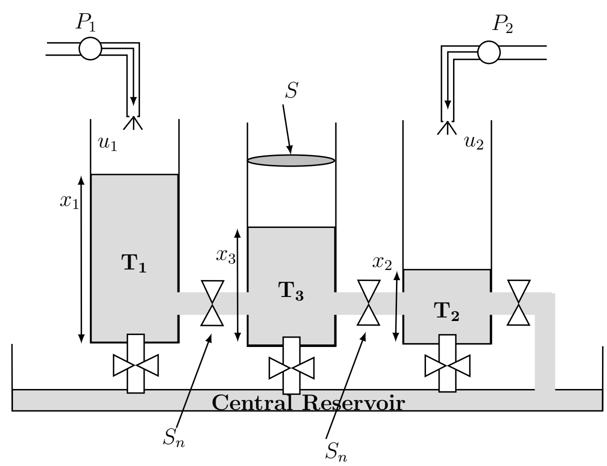

Figure 2. Three Tank schema

Where S is the transverse section of the tanks, x_i, $i = 1, 2, 3$, Q_{i0}, $i = 1, 2, 3$ the outflow between each tank and the central reservoir, Q_{13} and Q_{32} are the outflow between tank 1 and tank 3 and the outflow between tanks 3 and 2 respectively, u_1 and u_2 are the incoming flows of each pump.

The valves connecting tanks one and three with the central reservoir are considered closed, so Q_{10} and Q_{30} are always equal to zero. The flows Q_{13}, Q_{32} and Q_{20} can be expressed as follows:

$$Q_{13}(x_1, x_3) = a_{z1} S_n \sqrt{2g(x_1 - x_3)} \quad (10)$$

$$Q_{20}(x_2) = a_{z2} S_n \sqrt{2g(x_2)} \quad (11)$$

$$Q_{32}(x_2, x_3) = a_{z3} S_n \sqrt{2g(x_3 - x_2)} \quad (12)$$

where S_n represents the transverse section of the pipes connecting the tanks and a_{zj}, $j = 1, 2, 3$ represents the flow coefficients.

IV-B. *Flat model*

The flat model is computed by defining x_1 and x_3 as flat outputs, $z = [x_1 \; x_3]^T$, so the differentially flat equations can be written as follows:

$$x_1 = z_1 \quad (13)$$

$$x_2 = z_2 - \frac{1}{2g}\left(\frac{a_{z1}S_n\sqrt{2g(z_1-z_2)} - S\dot{z}_2}{a_{z3}S_n}\right)^2 \quad (14)$$

$$x_3 = z_2 \quad (15)$$

$$u_1 = S\dot{z}_1 + a_{z1}S_n\sqrt{2g(z_1-z_2)} \quad (16)$$

$$u_2 = S\dot{x}_2 - a_{z3}S_n\sqrt{2g(z_2-x_2)} + a_{z2}S_n\sqrt{2gx_2} \quad (17)$$

$$\phi_x(z_1, z_2) = \left[\; x_1 \; x_2 \; x_3 \; \right]^T \quad (18)$$

$$\phi_u(z_1, z_2) = \left[\; u_1 \; u_2 \; \right]^T \quad (19)$$

IV-C. *Simulation results*

Additive and Multiplicative faults affecting level sensors and flow actuators are considered. For multiplicative faults $20\,\%$ loss is considered for water level measures and $30\,\%$ for input flows. For additive faults a $+8cm$ fault is considered in sensors and an extra flow of $1*10^{-5}m^3/s$ is added to input flows. For simplicity sake only a single fault may be present at a time, once the fault appears (at $250s$) it is recurrent until the end of the simulation.

Two classic PID controllers are connected to high measures of tanks 1 and 2, see (20). Faults are detected by simply comparing the residual signal amplitude versus the threshold amplitude. The detection threshold was defined by changing the flow parameters in the range of $\pm$ $10\,\%$, afterwards the maximal value for each residue (positive and negative) plus an error margin is used as the final amplitude of the detection threshold. This margin adds robustness and avoids false alarms, if the threshold is exceeded, the fault is consider detected.

$$\begin{aligned} PID_{U1} &= -1*10^{-3}\frac{11{,}5s+1}{11{,}5s} \\ PID_{U2} &= -1*10^{-3}\frac{12{,}5s+1}{12{,}5s} \end{aligned} \quad (20)$$

Flow coefficients a_{z1} and a_{z3} are equal to $0{,}75$, a_{z2} value is $0{,}76$, the transverse section of the tanks and the transverse section of the connecting pipes are $15{,}4*10^{-3}$ and $5*10^{-5}$ respectively.

Flat outputs trajectories were generated by using a fifth order polynomial. White noise is added to the measured outputs with a relevant level to the real process measure level. Derivatives are estimated by using a high-gain observer (Vasiljevic and Khalil, 2008) coupled to a low-pass filter to reduce the amplitude of the noise and improve the derivative estimation.

Once the fault is detected the signal measure is changed by switching between this and the computed one. All the residual signals are normalized.

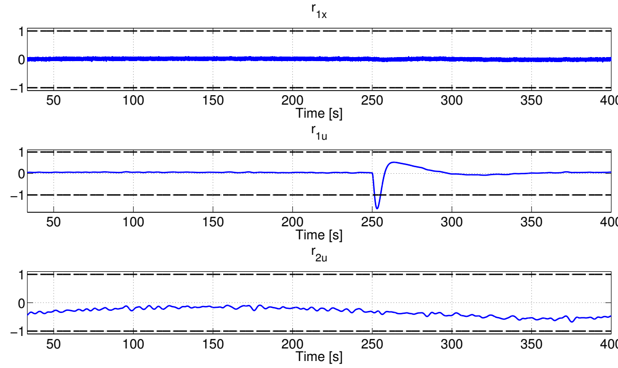

Figure 3. u_1 Actuator multiplicative fault

IV-D. *Fault detection and isolation*

One state residue and two inputs residues are obtained as follows:

$$\begin{bmatrix} r_{1x} \\ r_{1u} \\ r_{2u} \end{bmatrix} = \begin{bmatrix} x_{m2} \\ u_{m1} \\ u_{m2} \end{bmatrix} - \begin{bmatrix} \phi_x(z_1,\dot{z}_1,z_2,\dot{z}_2)^T[0\;1\;0]^T \\ \phi_u(z_1,\dot{z}_1,z_2,\dot{z}_2)^T[1\;0]^T \\ \phi_u(z_1,\dot{z}_1,z_2,\dot{z}_2)^T[0\;1]^T \end{bmatrix} \quad (21)$$

Actuators faults and high measure of sensor in tank 2 are detected and isolated by simply comparing residues amplitude versus threshold detection. See Fig. 3 to 5 for multiplicative faults and 8 to 10 for additive faults. Faults affecting flat outputs x_1 and x_3 can be detected but cannot be isolated by simply comparing the residual signal versus the threshold. See Fig. 6, 7, 11 and 12. However none of the PID are connected to the measure of tank 3, so this fault will no affect the final position, this results in a non-optimal isolation between faults in tank 1 and tank 3. By this way every sensor and actuator fault can be detected and isolated. Table I presents a summary of residues triggered by every fault. The fault signature is the same regardless the fault type.

TABLE I
RESIDUES MATRIX

Fault	r_{1x}	r_{1u}	r_{2u}
F_{x1}	1	1	1
F_{x2}	1	0	1
F_{x3}	1	1	1
F_{u1}	0	1	0
F_{u2}	0	0	1

IV-E. *Control reconfiguration*

Redundant high measure of tank 2 is available thanks to flat systems properties. As explained in section III-B, the control reconfiguration can be obtained by changing the faulty measure by the non-faulty one. Fig 13 shows a comparison between configuration and no reconfiguration simulations for a multiplicative fault. Figure 14 presents the fault additive case. It is straightforward to see that in both figures in the first case the final position follows the trajectory reference, this is not the result in the second

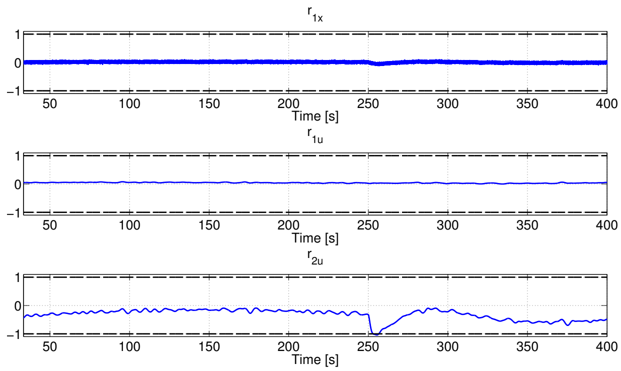

Figure 4. u_2 Actuator multiplicative fault

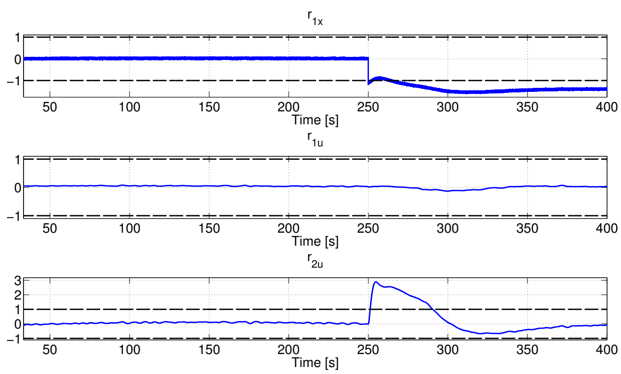

Figure 5. x_2 Level sensor multiplicative fault

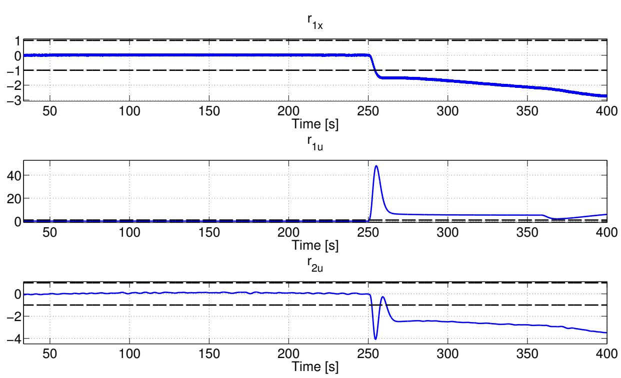

Figure 6. x_1 Level sensor multiplicative fault

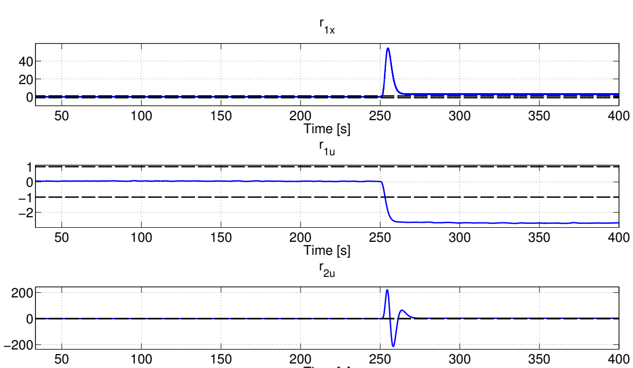

Figure 7. x_3 Level sensor multiplicative fault

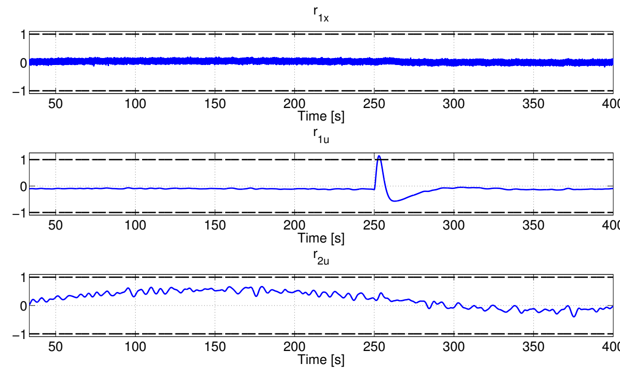

Figure 8. u_1 Actuator additive fault

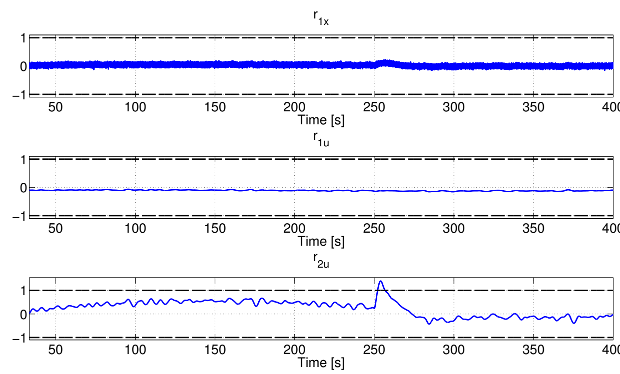

Figure 9. u_2 Actuator additive fault

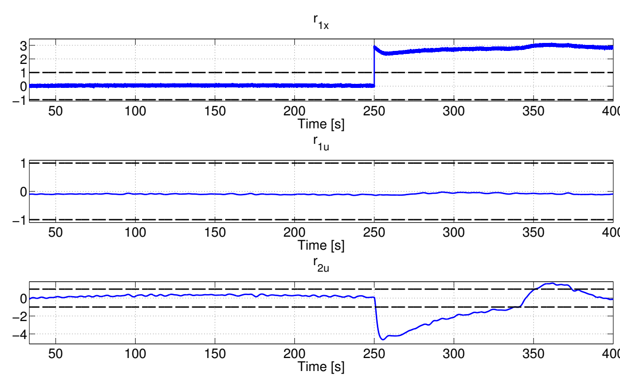

Figure 10. x_2 Level sensor additive fault

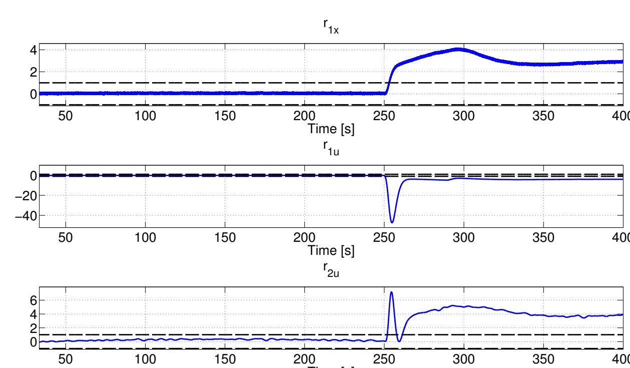

Figure 11. x_1 Level sensor additive fault

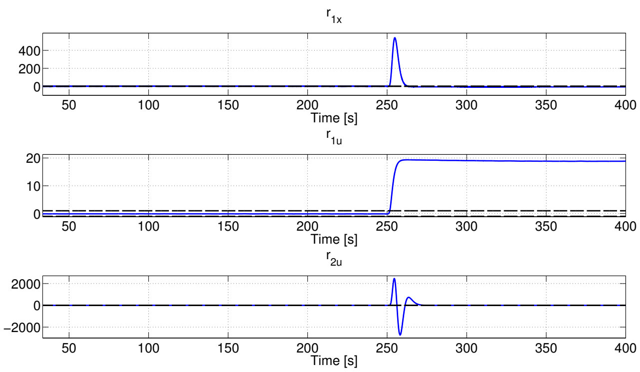

Figure 12. x_3 Level sensor additive fault

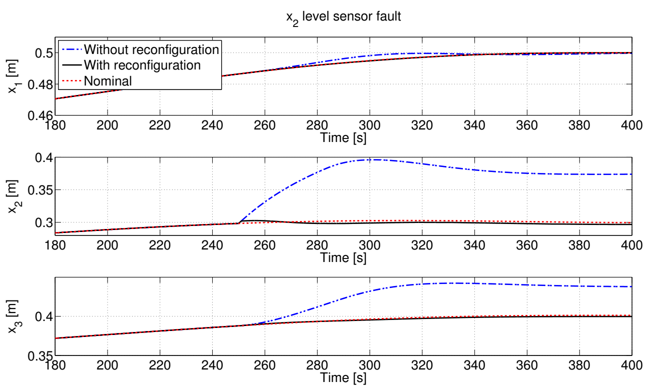

Figure 13. x_2 Fault reconfiguration. Multiplicative fault

one, where the difference between the measured position and the trajectory generated is remarkable. Actuators faults are compensated by the controller, tank 3 sensor fault does not affect the final position and faults affecting water level measure of tank 1 can be isolated, but a non-faulty measure is not available. By consequence if such fault affects the system, the system cannot be reconfigured.

V. Conclusion

This paper presents a flatness-based control reconfiguration technique. The technique was applied in a classical three tank system. Fault detection and isolation is assured for actuators and non-flat outputs measures. Faults affecting flat outputs (x_1 and x_3) are detected but cannot be isolated by simply analyzing the residual signals, nevertheless using the information of the final position of the water level inside the tanks, the operator can certainly make the difference between flat outputs.

Full recovery is not possible since an unfaulty measure of x_1 is not available, this problem can be afforded by using an algebraically independent set of flat outputs. Future work will be focus in study the case where two or more set of flat outputs are available.

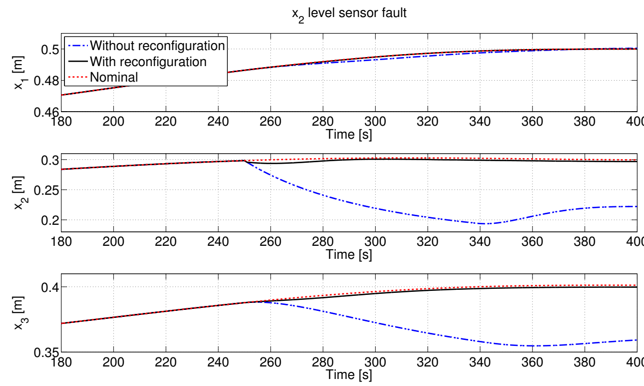

Figure 14. x_2 Fault reconfiguration. Additive fault

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