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Tracking and force control of a Scara robot under a constraint using sliding mode control

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Abstract— **In this paper are proposed two control algorithms, based on the sliding-mode technique, to track a trajectory and regulate the force ejected by the end effector in a 4-DOF Scara robot system subject to a position constraint. The system may be in non-constrained motion at some time, or in constrained motion at some other time. It is shown that the nonlinear system is globally asymptotically stable and achieves zero steady-state position error. Numerical results show the performance of the proposed controllers.**

Keywords: sliding mode control, force control, constraints, industrial robots.

I. Introduction

Many applications in industry involve mechanical systems interacting with the environment. Examples can be found in manufacturing automation, material handling by robots, and space applications. In these applications, an important issue is to model the complete dynamic behavior so that the system and constraints are presented as a single system (Ben Amor, B. et al., 2009). The formulation given in (Ben Amor, B. and Haded, N.K. and Mnif, F., 2009) describes systematically the complete behavior of a mechanical system interacting with an environment.

A problem shown in experiments performed on single degree of freedom (1-DOF) robots is that force feedback sometimes produces an undesirable bounce behaviour, where the robot repeatedly makes and loses contact with the constraint surface. This behaviour is an example of a limit cycle, and is likely caused by the nonlinearity (i.e., one-sided) constraint (Goldsmith, P.B., 1996).

Friction forces, especially dry friction, which are rarely taken into account in the design of controllers for mechanical systems with constraints, may produce negative effects like tracking errors, limit cycles, or undesired stick-slip motion, reducing considerably the performance of the controlled system, see, e.g., (Canudas de Wit et al., 1995).

To solve the problem of controlling the trajectory and the regulation of the effector force when it makes contact with the constraint, a sliding mode control methodology may be used (V. Utkin, 1992). The main feature of this class of controllers is to allow the sliding mode to occur on a prescribed switching surface, so that the system is governed by the sliding equation only, and remains insensitive to a class of disturbances and parameter variations (V. Utkin, 1978). This control method has been successfully tested for motion control of robotic manipulators, see (Sabanovic A. et al., 2008) and references therein. Besides, a previous work of sliding-mode control in constrained robots can be found in (Lian, Kuang-Yow et al., 1998).

More recently some novel control techniques have been used, for example in (Fateh, Mohammad Mehdi et al., 2012) it is used a decentralized Direct Adaptive Fuzzy Control (DAFC) for tracking performance in a Scara robot, which is electrically driven using the voltage control strategy. Another recent work can be found in (Guangqiang Lu et al., 2012) where an adaptive elastic method and an adaptive viscosity compensation method are proposed in order to save energy for periodic motion in a scara robot.

The problem addressed in the present paper is the trajectory tracking and force regulation of a Scara robot subject to a position constraint, where impacts may occur. This means that the mechanical system may collide with a constraint, represented by a fixed flat surface. Some previous works of mechanical systems with unilateral constraints can be found in (N. Mansard et al., 2008; Menini, L. et al., 2001). Other previous works related to Scara robots can be found in (Visioli, A. et al., 2002) where it is develop a controller for trajectory tracking problem. In (Nakamura, M. et al., 2000) is presented a contour control for a Scara robot considering torque saturation constraints. In (Serhan Yamacli and Huseyin Canbolat, 2008) are used a PD and learning controllers to solve a trajectory tracking problem.

The system presented here is similar to those presented in many mechanical systems, especially in the effector subsystem (see, for example, (Brogliato, B., 1999; Leine, Remco I. and Van de Wouw, Nathan, 2010)). It can display an important dynamical behavior like rebounds, due to collisions with the constraint, which may risk the integrity of the mechanical device. Hence, we design controllers with the aim of having a good trajectory tracking and force regulation of the end effector, hence for the force on the spring.

The main contribution of this paper is to extend some results described before (Brogliato, B., 1999; Goldsmith, P.B., 1996; Lian, Kuang-Yow and Lin, Chia-Ru, 1998) by considering the presence of a not completely characterized Coulomb friction, as well as some kind of matched, external disturbance. In both cases we assume that only some bounds on the friction coefficient and on the disturbance magnitude, are known. Also, different to (Z. Doulgeri et al., 2005), the proposed control law, based on the sliding-mode technique, is designed for the whole system, regardless of the motion phase. That is, the mechanical system and the constraint are modeled as a single system. This model allows us to develop a controller whose purpose is, first to force the system attain the constraint and, once this constraint has been reached, maintain the contact of the system with the constraint. We propose a sliding-mode control algorithm using a sliding mode control which is designed for trajectory tracking and also, a dynamic sliding surface designed for position regulation. The controllers use measurements of positions, velocities, and one controller is also designed to compress the spring against the constraint surface.

The rest of the paper is outlined as follows: In Section II we describe the dynamic model of the Scara robot with position constraint. The state feedback design and its stability analysis is presented in Section III. Section IV presents experimental results, through numerical simulations performed with MATLAB®. Section V includes some final comments.

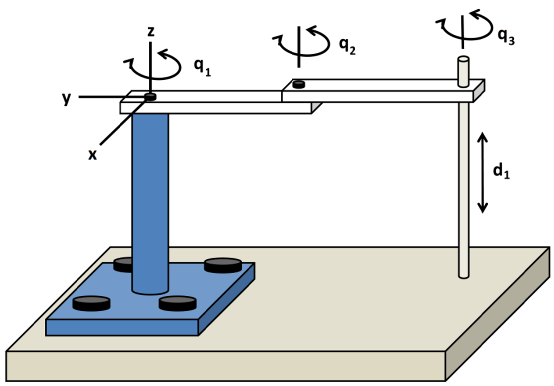

Figure 1. Four dof scara robot with a position constraint

II. Dynamic Model

The dynamic model of the 4 DOF scara robot with a position constraint (see Figure 1), can be expressed in Euler-Lagrange equations as follows

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) \quad = \tau + w(t) \tag{1}$$

$$m_3\ddot{d}_1 - m_3 g + f(\dot{d}_1) + f_c \quad = \tau_4 + w_4(t) \tag{2}$$

where $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \mathbb{R}^3$ are the displacements, velocities, and joint accelerations of the mechanical system rotational links; $M(q) \in \mathbb{R}^{3\times 3}$ is the inertia matrix, which is symmetric and positive-definite for each $q \in \mathbb{R}^3$; $C(q,\dot{q})\dot{q}$ is the vector of Coriolis and centrifugal forces; $G(q) \in \mathbb{R}^3$ is the gravitational forces vector; $F(\dot{q})$ is the vector of friction forces given by Coulomb friction type vector $[\alpha_1 \text{sign}(q_1); \alpha_2 \text{sign}(q_2); \alpha_3 \text{sign}(q_3)]$, where $\alpha_1 \ldots \alpha_3 \in \mathbb{R}^+$, friction forces are considered bounded; $w(t) \in \mathbb{R}^3$ is the external perturbations vector, perturbations are considered unknown but bounded by an upper limit.

Figure 2. Possible equilibrium points of the end efector.

The prismatic joint acts as a decoupled system of (1), where d_1 is the position of the body with mass $m_3 \in \mathbb{R}^+$, $g \in \mathbb{R}$ is the gravitational constant, and the Coulomb friction level is proportional to $f(\dot{d}_1) = \alpha_4 \text{sign}(\dot{d}_1)$, where $\alpha_4 \in \mathbb{R}^+$. The $\text{sign}(\dot{d}_1)$ denotes the signum function defined as

$$\text{sign}(\dot{d}_1) = \begin{cases} 1 & \dot{d}_1 > 0 \\ [-1,1] & \dot{d}_1 = 0 \\ -1 & \dot{d}_1 < 0 \end{cases} \tag{3}$$

Moreover f_c is the force acting on the spring, calculated from

$$f_c = \begin{cases} 0, & \text{if } d_1 < d_0, \\ k(d_1 - d_0), & \text{if } d_1 \geq d_0. \end{cases} \tag{4}$$

Parameter k is a positive stiffness coefficient, which we assume to be known. The position d_1 of the mechanical system is set to d_0 when the force sensor is in contact with the surface and the spring is not compressed; that is, when it delivers zero force. The mass m_3 is driven by $\tau_4 \in \mathbb{R}$. We consider that the constraint is a rigid surface. To account for discrepancies in the model, an unknown external disturbance $w_4(t) \in \mathbb{R}$ has been introduced, with an upper bound M_4 that is assumed known a priori, so it satisfies

$$\sup_t |w_i(t)| \quad \leq A_i, \tag{5}$$

$$\sup_t |w_4(t)| \quad \leq A_4, \tag{6}$$

for all t, with $i = 1 \ldots 3$ and some constants $M_i > 0$.

For a constant force input $\tau_4 = \bar{\tau}_4$ and zero disturbance ($w_4 = 0$), the system (2) has the following equilibrium points if we want the tip to be on the constraint, in steady state, if it is satisfied $m_3 g > \alpha_4$ then it is enough to choose $\bar{\tau}_4 = f_d = 0$, if it is satisfied $m_3 g \leq \alpha_4$ then we must choose $\bar{\tau}_4 = f_d > \alpha_4 - m_3 g$ where f_d is the desired force in the spring. This force must be a positive constant. Therefore, the equilibrium region of interest can be considered as $(\bar{d}_1 \in [(-\alpha_4 + m_3 g + f_d)/k + d_0, (\alpha_4 + m_3 g + f_d)/k + d_0], \dot{\bar{d}}_1 = 0)$, as shown in Figure 2. Given that in system (2) the position feedback is used, instead of force feedback, then the position of the surface and the stiffness of the sensor must be known to achieve the desired contact force. A rebound occurs when a transition from constrained motion ($d_1 \geq d_0$) to free motion ($d_1 < d_0$) exists.

III. Control Design

First of all, it is going to be designed a trajectory tracking controller for the links q_1, q_2, and q_3, the control methodology chosen for this purpose is sliding mode control due its robustness properties again external perturbations, parametric uncertainties and friction. Later, considering as a decoupled system the translational link d_1, it is designed a force regulation controller through integral sliding mode control methodology due to the aforementioned properties.

A. Trajectory Tracking Control

Let us suppose that the disturbances $w_1(t)\dots w_3(t)$ affecting system (1) satisfy (5), and the Coulomb friction coefficients are such that $0<\alpha_i\le B_i$, for some known bound B_i with $i=1\dots 4$. The control objective is to find a control τ, depending on the angular positions $q=[q_1,q_2,q_3]^T$ and velocities $\dot q=[\dot q_1,\dot q_2,\dot q_3]^T$, such that the closed-loop response of system (1) satisfies

$$\lim_{t\to\infty}|q(t)-q_d(t)| \quad = \quad 0. \tag{7}$$

where $q_d(t)\in \mathbb{R}^3$ are periodic desired trajectories to be followed by each link q_i.

A sliding mode control law is proposed for the system (1), first let us consider a sliding surface given by

$$s=\mu e+\dot e \tag{8}$$

where $e=[q_1-q_{d1},q_2-q_{d2},q_3-q_{d3}]^T$, and $\dot e=[\dot q_1-\dot q_{d1},\dot q_2-\dot q_{d2},\dot q_3-\dot q_{d3}]^T$, $\mu\in\mathbb{R}^{3\times3}$ is a diagonal gain matrix with positive coefficients.

One control law that can assure us the fulfillment of (7) is given by

$$\tau=C(q,\dot q)\dot q-\beta\frac{s}{\|s\|}-M(q)(\mu\dot e+\lambda s-\ddot q_d); \tag{9}$$

where β, $\lambda\in\mathbb{R}^{3\times3}$ are diagonal gain matrixes with positive coefficients, $s=[s_1,s_2,s_3]^T$.

The objective for proposing a controller with such structure is to satisfied that $s^T\dot s<-s^T\lambda s-\beta\|s\|$. The closed loop system (1) using the controller takes the form

$$\ddot q=M^{-1}(q)\left[-F(\dot q)-\beta\frac{s}{\|s\|}+w\right]-\mu\dot e-\lambda s+\ddot q_d \tag{10}$$

B. Stability Analysis

Lets analyze in this section the stability of the closed-loop system (10), controlled by (III-A), and conclude about the stability. Now, can be ensured the existence of sliding modes by verifying $s^T\dot s<0$. To this end, note that, from (5) and the fact that $\alpha_i\le B_i$, then

$$\begin{aligned} s^T\dot s \;&=\; s\left(-M^{-1}(q)\left[F(\dot q)+\beta\tfrac{s}{\|s\|}-w\right]-\lambda s\right)\\ &\le\; -\lambda_{min}\{M^{-1}(q)\}\lambda_{min}\{\beta\}\|s\|\\ &\quad +\lambda_{max}\{M^{-1}(q)\}\sum_{i=1,2,3}(A_i+B_i)\|s\|\\ &\quad -s^T\lambda s\\ &\le\; -\Big(\lambda_{min}\{M^{-1}(q)\}\lambda_{min}\{\beta\}\\ &\quad -\lambda_{max}\{M^{-1}(q)\}\sum_{i=1,2,3}(A_i+B_i)\Big)\|s\|\\ &\quad -\lambda\|s\|^2 \end{aligned}$$

Can be concluded the existence of sliding modes on the surface $s=\mu e+\dot e$ while the condition $0<\lambda_{max}\{M^{-1}(q)\}\,(A_i+B_i)<\lambda_{min}\{M^{-1}(q)\}\lambda_{min}\{\beta\}$ be satisfied using $i=1,2,3$. This gives a guide to tune the coefficients of β matrix in the controller (III-A). In fact, it can be demonstrated that the trajectories reach the surface $s=0$, in finite time, using the quadratic function

$$V(s)=s^Ts, \tag{11}$$

and compute its time derivative along the solutions of (10),

$$\begin{aligned} \dot V(s(t))&\le -2\lambda\|s\|^2-2\Big(\lambda_{min}\{M^{-1}(q)\}\lambda_{min}\{\beta\}\\ &\quad-\lambda_{max}\{M^{-1}(q)\}\sum_{i=1,2}(A_i+B_i)\Big)\|s\|\\ &\le -2\Big(\lambda_{min}\{M^{-1}(q)\}\lambda_{min}\{\beta\}\\ &\quad-\lambda_{max}\{M^{-1}(q)\}\sum_{i=1,2}(A_i+B_i)\Big)\|s\|\\ &= -2\Big(\lambda_{min}\{M^{-1}(q)\}\lambda_{min}\{\beta\}\\ &\quad-\lambda_{max}\{M^{-1}(q)\}\sum_{i=1,2}(A_i+B_i)\Big)\sqrt{V(s(t))}. \end{aligned} \tag{12}$$

From (12) it follows that $V(t)=0$ for

$$\begin{aligned} t\ge& t_0+\frac{\sqrt{V(t_0)}}{\lambda_{min}\{M^{-1}(q)\}\lambda_{min}\{\beta\}-\lambda_{max}\{M^{-1}(q)\}\sum_{i=1,2}(A_i+B_i)}\\ t_f=&t_0+\frac{\sqrt{V(t_0)}}{\lambda_{min}\{M^{-1}(q)\}\lambda_{min}\{\beta\}-\lambda_{max}\{M^{-1}(q)\}\sum_{i=1,2}(A_i+B_i)}. \end{aligned} \tag{13}$$

Hence, $V(t)$ converges to zero in finite time and, in consequence, a motion along the manifold $s=[0,0,0]^T$ occurs in the discontinuous system (10). Notice that the reaching time can be reduced by increasing the value of the main diagonal coefficients in β matrix. Thus, in the following development, we assume that system (10) is in sliding mode; therefore, $s=\dot s=0$ for $t\ge t_f$.

Now let us show that, while the system remains in $s=0$, the trajectories $(q,\dot q)$ converge to zero as $t\to\infty$. From (8), and the dynamics of system (10), once in sliding mode, are reduced to

$$\dot e=-\mu e. \tag{14}$$

Note that system (14) has only one equilibrium point placed at the origin, which is locally asymptotically stable.

C. Force Regulation Control

Now the disturbance $w_4(t)$ affecting system (2) satisfies (6), and the Coulomb friction coefficient is such that $0<\alpha_4\le B_4$, for some known bound B_4. The control objective is to find a control τ_4, depending on the desired force $f_d\in\mathbb{R}^+$ (via a desired position d_{d1}), the generalized displacement d_1 and velocity $\dot d_1$, such that the closed-loop response of system (2) satisfies

$$\lim_{t\to\infty}|f(t)-f_d| \quad = \quad 0. \tag{15}$$

For control purposes the expression for f_c in (4) can be rewritten as

$$f_c(d_1, d_0) = \frac{k}{2}(d_1 - d_0 + |d_1 - d_0|) \ \ \forall \ d_1 \in \mathbb{R}. \quad (16)$$

A integral sliding mode control law is now proposed for system (2), first let us consider a sliding surface given by

$$s_4 = \mu_4 d_1 + \dot{d}_1 + \gamma \int_0^T [f(d_1, d_0) - f_d]\, dt, \quad (17)$$

where μ_4, $\gamma \in \mathbb{R}^+$ are gain constants.

One control law that can assure us the fulfillment of (15) is given by

$$\tau_4 = f_c - \beta_4 \text{sign}(s_4) - m_3 \left(g + \lambda_4 s_4 + \gamma(f_c - f_d) + \mu_4 \dot{d}_1 \right); \quad (18)$$

where λ_4, β_4, γ and $\mu \in \mathbb{R}$ are tunable gain coefficients.

The objective for proposing a controller with such structure is to satisfied that $s\dot{s} < -\lambda_4 s_4^2 - \beta_4 |s_4|$. Since the sliding surface (17) is a dynamical variable, we can add s as another state in (2). This leads to the extended system

$$\begin{aligned} \ddot{d}_1 &= -\lambda_4 s_4 - \beta_4 \text{sign}(s_4) - \gamma \tilde{f} - \mu_4 \dot{d}_1 + \frac{w_4}{m_3} - \frac{\alpha_4}{m_3}\text{sign}(\dot{d}_1) \\ \dot{s} &= \mu_4 \dot{d}_1 + \ddot{d}_1 + \gamma \tilde{f} \end{aligned} \quad (19)$$

where $\tilde{f} = f_c(d_1, d_0) - f_d$. Now, we ensure the existence of sliding modes by verifying $s_4 \dot{s}_4 < 0$. To this end, note that, from (6) and the fact that $\alpha_4 \leq B_4$, then

$$\begin{aligned} s_4 \dot{s}_4 &= s_4 \left(-\lambda_4 s_4 - \tfrac{\beta_4}{m_3}\text{sign}(s_4) + \tfrac{w_4}{m_3} - \tfrac{\alpha_4}{m_3}\text{sign}(\dot{d}_1) \right) \\ &\leq -\lambda_4 s_4^2 - \tfrac{\beta_4}{m_3}|s_4| + \tfrac{A_4 + B_4}{m_3}|s_4| \\ &\leq -\lambda_4 s_4^2 - \left(\tfrac{\beta_4 - (A_4 + B_4)}{m_3} \right) |s_4|. \end{aligned}$$

Can be concluded the existence of sliding modes on the surface $s_4 = \mu_4 d_1 + \dot{d}_1 + \gamma \int_0^T [f(d_1, d_0) - f_d]\, dt$ while the condition $\beta_4 > A_4 + B_4$ be satisfied. This gives a guide to tune the parameter β_4 of the controller (III-C). It can be demonstrated that the trajectories reach the surface $s_4 = 0$, in finite time, using the quadratic function $V(s_4) = s_4^2$. The development is very similar to the previous one of the trajectory tracking controller, therefore it is omitted.

Now let us show that, while the system remains in $s_4 = 0$, the trajectories $(d_1, \dot{d}_1)$ converge to zero as $t \to \infty$. From (16), (17) and, given that $f_d = k(d_{d1} - d_0 + |d_{d1} - d_0|)/2 > 0$, we have that the dynamics of system (19), once in sliding mode, and considering $y_1 = d_1 - d_{d1}$, $y_2 = \dot{d}_1$ are reduced to

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= -\frac{k\gamma}{2}(y_1 + |y_1 + d_{d1} - d_0| - |d_{d1} - d_0|) - \mu_4 y_2. \end{aligned} \quad (20)$$

Note that system (20) has only one equilibrium point placed at the origin, which is locally asymptotically stable, due to $d_{d1} > 0$.

Notation	Description	Value	Units
m_1	Mass of link 1	0.38	kg
m_2	Mass of link 2	0.34	kg
m_3	Mass of link 3	0.25	kg
l_1	Length of link 1	0.297	m
l_2	Length of link 2	0.297	m
I_1	Inertia of link 1	0.243×10^{-3}	kg m^2
I_2	Inertia of link 2	0.068×10^{-3}	kg m^2
I_3	Inertia of link 3	0.015×10^{-3}	kg m^2
g	Gravity	9.80665	m/s^2
k	Spring stiffness constant	500	N/m
d_0	Distance from link 3 to the constraint surface	0.5	m

TABLE I
4-DOF ROBOT SCARA PARAMETERS.

IV. SIMULATION RESULTS

Performance issues and robustness properties of the proposed sliding mode controllers have been tested with some numerical experiments under the following parameters of the Scara robot as can be seen in Table I

The dynamical model, based on the positions, velocities and the dynamics of the sliding surface s as in (10) and (19), is studied with the following parameters: desired trajectories $q_{d1} = 0.1\sin(t)$ rad, $q_{d2} = 0.2\cos(t)$ rad, $q_{d3} = 0.1\sin(t)$ rad, desired force $f_d = 5$ N, friction coefficients $\alpha = [0.2N.m, 0.1N.m, 0.1N.m, 0.1N]$. The feedback controllers coefficients are set to $\lambda = [40N.m, 40N.m, 40N.m, 10N]$, the sliding surface coefficients $\mu = [5kg/s, 20kg/s, 30kg/s, 2kg/(m.s)]$, the gain coefficients of the signum function $\beta = [0.6N.m, 0.6N.m, 0.7N.m, 1N]$, and the gain of force $\gamma = 1$ m^{-1}. The desired position is set to $d_{d1}0.51$ m. The initial values of the position error, velocity and sliding surface s were set to $q_1(0) = \pi/4$rad $q_2(0) = \pi/4$rad, $q_3(0) = \pi/4$rad, $d_1(0) = 0$m , $\dot{q}(0) = [0, 0, 0]$rad/s, $\dot{d}_1 = 0$m/s and $s_4(0) = 0.1$, respectively. Perturbations were set to $w_1 = 0.3\sin(t)$rad, $w_2 = 0.4\cos(t)$, $w_3 = 0.4\sin(t)$ and $w4 = 0.2\sin(t)$.

Figure 3 displays the numerical responses of the system, such trajectory tracking, position error, sliding mode and control input, all of them are the link q_1 parameters. Figures 4, 5 and 6 show the same parameters as the aforementioned Figure 3 but for links q_1, q_3 and d_1, respectively. Note also that, even that Coulomb frictions and external disturbances are present, the controlled system exhibits robustness against perturbations and Coulomb frictions in each link. Finally, Figure 7 shows how the end effector reaches the desired contact force in approximately 5 s.

V. COMMENTS

In an experimental platform, one of the disadvantages of using these types of controllers are the theoretically endless commutations of the control signal, that may lead to a damage on the actuators. Although in control signal the number of commutations are theoretically infinite, in real life it presents a finite number of commutations due to physical limitations of the actuators.

In real experiments if the control signal does not reach the switching frequency specified by the theoretical analysis,

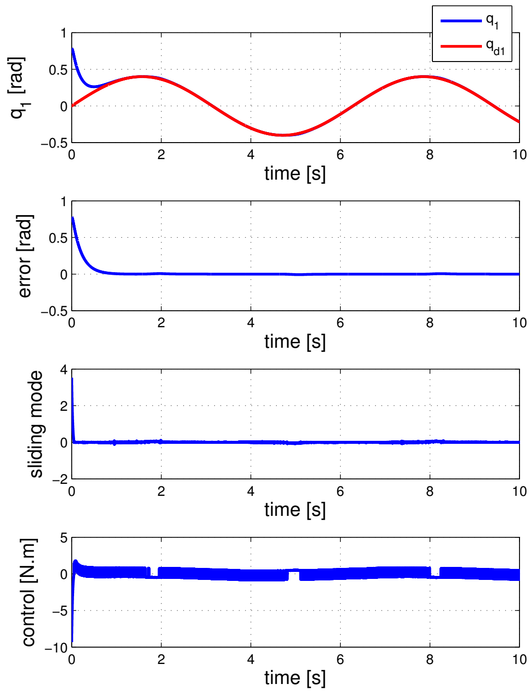

Figure 3. Rotational link q_1.

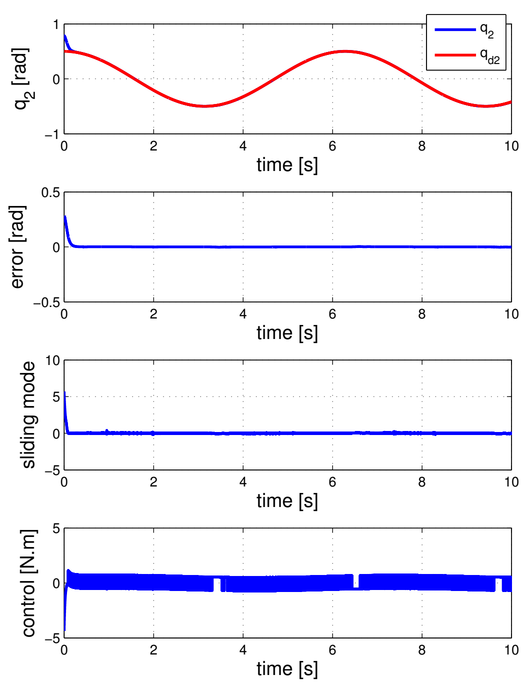

Figure 4. Rotational link q_2.

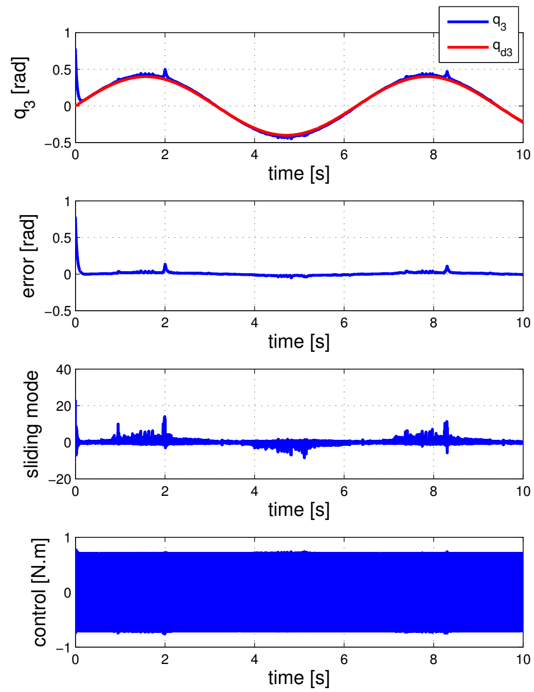

Figure 5. Rotational link q_3.

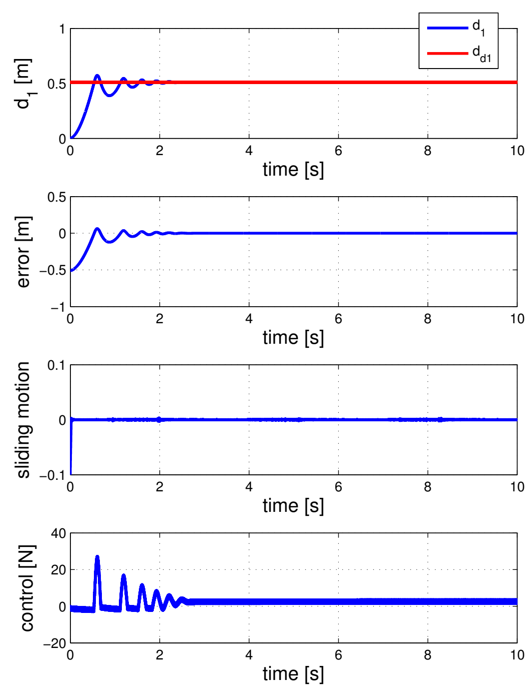

Figure 6. Translational link d_1.

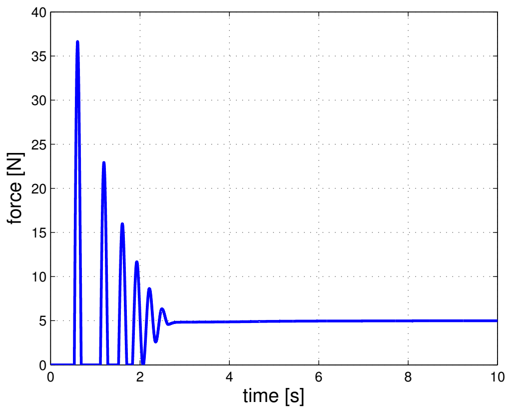

Figure 7. Contact force of the end effector.

it may cause an error in steady state. Nowadays there are brushless actuators that could be implemented using sliding mode control, this type of control render a robust close-loop system, even in the presence of certain type of parametric variations and not completely know parameters.

By the other hand, in our numerical approach the two controllers synthesized, one of them using sliding-mode control and the other one dynamic sliding-mode control technique, rendered a robust closed-loop system. The convergence in finite time of the controlled system trajectories to the sliding surface, and asymptotically to the unique equilibrium point, was proved. Some numerical results were performed, showing good agreement and robustness performance.

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