Optimal Control Design for a Tokamak Plasma Position

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Abstract- Control applied to nuclear fusion, specifically relatively to the control of plasma properties such as current, shape and position within the container, represent since years ago, an interesting application of engineering and control techniques. This paper presents a simplified model of the Tokamak plasma dynamics and an optimal control design, whose objective is to regulate the plasma position to equilibrium.

Keywords—control; PF coils; plasma current; Grad-Shafranov equation; feedback; state-space; lqr

I. INTRODUCTION

By far the most promising fusion reaction is that in which the nuclei of deuterium and tritium fuse to produce an alpha particle with the release of a neutron, that is where the energies given are the kinetic energies of the reaction products.

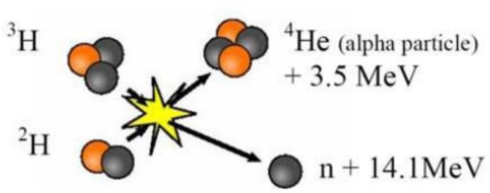

Fig. 1 Nuclear fusion reaction.

For these reactions to happen it is necessary to create first a magnetic confinement of an ionized gas mixture, by different ways, a plasma. Plasma confinement is not always used with compounds to create a nuclear reaction; it can be proved with air, helium or just deuterium. Plasmas can be described as two magnetohydrodynamic (MHD) fluids of ions and electrons with mass, charge and current densities, as well as flow velocities and pressure. Plasmas have certain electric properties like electric resistivity and inductance, depending on the purity of the plasma, the current in it, the pressure etc. (J. Wesson, 2004)

The purpose of this paper is to obtain a state-space model of the plasma position and current dynamics by approximating the plasma and the device that contains it through equations of RL circuits plus some MHD properties that have to be introduced in the model. The objective is that through an optimal control and a modeling based on the real physics it is possible to control the plasma properties and try to get a longer-lived plasma.

This paper is organized as follows: In section II there is a brief description of what a Tokamak is, its main properties, and the approximation of the Tokamak that the control will work with. In section III the main MHD equations that model the equilibrium in a Tokamak are used to carry out the model in state space. In section IV the results in the system through the use of an optimal control over the states are shown.

II. TOKAMAK

The word Tokamak comes from the acronym of "Toroidal chamber with magnetic coils" in russian. A Tokamak consists basically of a toroidal vacuum chamber containing the plasma, a series of toroidal field coils put along and around all over the torus, a transformer iron core, to create the current in the plasma, and the Poloidal Field (PF) coils for the position of the plasma in the torus. Inside the Tokamak there is a toroidal plasma confinement system by a magnetic field. The main magnetic field is the toroidal field. However, this field alone does not allow confinement of the plasma. In order to have an equilibrium in which the plasma pressure is balanced by the magnetic forces it is necessary to also have a poloidal magnetic field. In a Tokamak this field is produced mainly by current in the plasma itself. This current flows in the toroidal direction, but also for this poloidal field, which is the one controlling the plasma shape and position, there is a group of short solenoids called Poloidal Field Coils (PF coils) like those shown in Figure 2 in brown. The combination of the toroidal and poloidal magnetic fields gives rise to the magnetic field lines which have a helical trajectory around the torus.

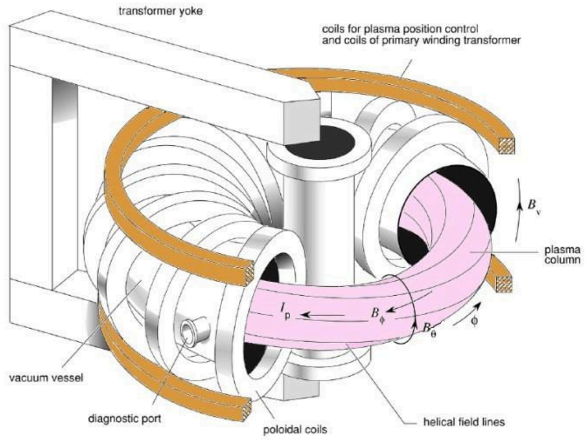

Fig. 2 Diagram of a typical Tokamak (A. Sharma, 2005)

The principal characteristics in the configuration of a Tokamak are the major R and minor radius a. The major radius is the distance from the center of the doughnut Tokamak (vertical axis) to the center of the vacuum chamber and the minor radius of the cross section of the plasma column is determined by a limiter inside the chamber.

For the model in this paper, basically the state space model made for the TCV Tokamak located in Switzerland (A. Coutlis, 1999) will be used, and the Tokamak parameters will be the ones of the "Novillo" Tokamak located in Mexico, whose construction parameters are in (J. Ramos, 1982). Tokamak Novillo has a major radius R=23 (cm) and a minor radius a= 6 (cm).

III. MODEL

When the plasma is in steady state and the flow velocity is zero, the Tokamak equilibrium is given by:

$$\nabla p = j \times B \ , \qquad (1)$$

$$\nabla \times B = \mu_0 j \ , \qquad (2)$$

where B is the magnetic field, p is the plasma pressure, j is the current density and μ_0 is the magnetic permeability in vacuum ($4\pi \times 10^{-7}\ \left[\frac{H}{m}\right]$). The magnetic field in cylindrical coordinates *(R,θ,z)* is given b $B = \nabla\psi(R,\theta) + F(\psi)\nabla\theta$,

where axisymmetry is assumed ($\frac{\partial}{\partial\theta} = 0$), and $\psi(R,Z)$ is the poloidal magnetic flux function. Given this form of the magnetic field, equations (1) and (2) are rewritten as the so called *Grad-Shafranov equation:*

$$\left(R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial}{\partial R} + \frac{\partial^2}{\partial z^2}\right)\psi + \mu_0 R^2 \frac{\partial p(\psi)}{\partial\psi} + F\frac{\partial F(\psi)}{\partial\psi} = 0 \ (3)$$

Using the Solovev solution for equation (3) and according to the boundary conditions and adjustment done for the pressure $p(\psi)$ and for the function $F(\psi)$(A. Rahimirad, 2010), the Grad-Shafranov solution resulting is

$$\psi(R,Z) = 0.00055 - 0.08313R^2 + 0.7246R^4 - 0.2267(R^4 - 4R^2Z^2) - 0.02803(-Z^2 + R^2\ln[R]), \ (4)$$

whose contour lines are shown in Fig. 3. This figure shows the ψ surface against the position (*R,z*). The darker surfaces are the ones with the lesser value of Ψ.

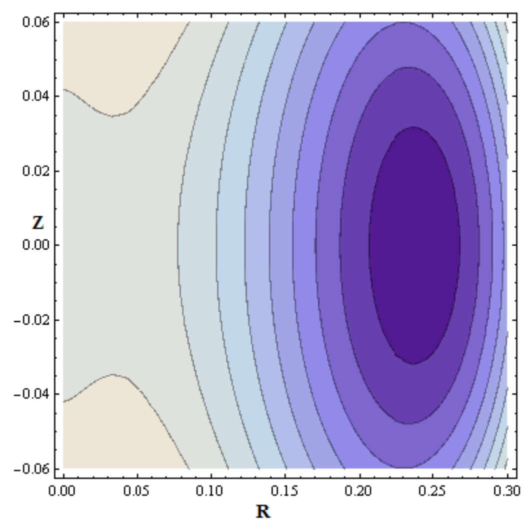

Fig. 3 Plot of different magnetic field surfaces ψ in the Tokamak region.

The numeric coefficients of the GS equation are related to the geometric configuration of the Tokamak (major and minor radius) and the plasma current in equilibrium. With the solution of the GF equation it is possible to obtain the poloidal field in the Tokomak as

$$B_p = B_z + B_R \ , \qquad (5)$$

$$B_z = \frac{1}{R}\frac{\partial\psi}{\partial R} \ , \qquad (6)$$

$$B_R = -\frac{1}{R}\frac{\partial\psi}{\partial z} \ . (7)$$

These components of the magnetic field will be used in the state space model.

Through the solution of the GS equation it is possible to find the value of the position for the maximum magnetic field surface. This value will be the axis of the plasma and it is taken to be the equilibrium radius for the model. For the case of the geometric configuration we are working with, the equilibrium radius has the value R_0=0.2393 (m).

Fig. 4 shows an approximation of the PF coils arrangement for the model. It has two couples of coils, with the same geometric and electric characteristics, fixed around the vacuum chamber containing the plasma. Their electrical properties and the plasma ones are shown in Tables 1, 2 and 3.

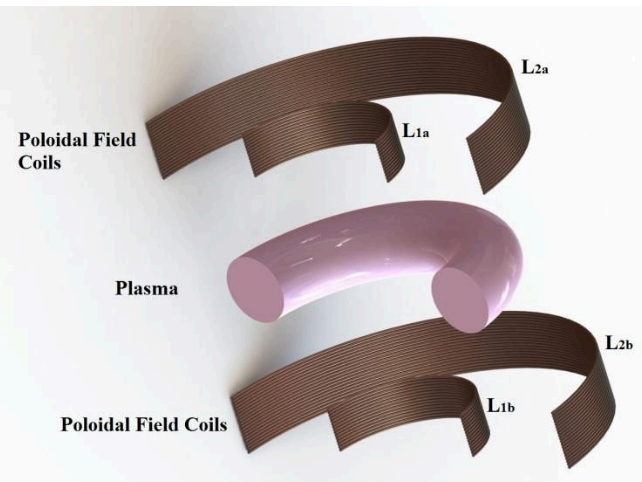

Fig. 4 Approximate diagram for the coils arrangement of the Tokamak model used.

In this system the only variables that can be physically controlled are the voltages applied to the PF coils, through a device such as a microcontroller or a data acquisition board.

There is another parameter in the system associated with the magnetic forces for the equilibrium (K. Miyamoto, 2007). The change in the parameter

$$\Gamma = \ln\left(\frac{8R}{a}\right) + \beta_p + \frac{li}{2} \quad (8)$$

will be introduced in the system as a perturbation (A. Coutlis,1999). The force balance ($\sum F = 0$) is applied in:

$$\frac{\partial M_p}{\partial R} I_{p0}\dot{I}_c + \left(\frac{\mu_0 I_{p0}^2}{2}\frac{1}{R_0} + 2\pi I_{p0}B_{z0} + 2\pi R_0 I_{p0}\frac{\partial B_z}{\partial R}\right)\dot{R}$$

$$\dot{+}(\mu_0 I_{p0}\Gamma_0 + 2\pi R_0 B_{z0})\dot{I}_p = -\frac{\mu_0 I_{p0}^2}{2}\dot{\Gamma} \quad (9)$$

Where B_{Z0} refers to the magnetic field component in z valuate in the equilibrium and $\dot{\Gamma}$ is the change in the parameter associated with the perturbation in the force balance.

Since the PF Coils are approximated as RL circuits, the currents in them need to be state variables. In addition, the plasma current is also a state variable (the plasma is an RL circuit too) and the radial position R is the last state variable to control, and the most important. Equation (10) shows the relation of the state variables due to the RL circuits in the Tokamak.

$$M_c\dot{I}_c + \left(\frac{\partial M_p}{\partial R}\right)^T I_{p0}\dot{R} + M_p^T\dot{I}_p + \Omega_c I_c = V_c. \quad (10)$$

Since the plasma is also considered a circuit, it has to be modeled in a similar way, considering linearization

$$M_p\dot{I}_c + [\mu_0(1+f_0)I_{p0} + 2\pi R_0 B_{z0}]\dot{R} + L_{p0}\dot{I}_p$$
$$+ I_{p0}\frac{\partial \Omega_p}{\partial R}R + \Omega_p I_p = 0\,. \quad (11)$$

Equations (9), (10) and (11) describe completely the system as a state space system.

The parameter l_i is an internal inductance due to the magnetic field energy and is related to the poloidal field. In this case it will be taken as unity, as proposed in (J. Ramos, 1982).

$$f_0 = \ln\left(\frac{8R_0}{a}\right) + \frac{li}{2} - 1\ , \quad (12)$$

$$\beta_p = \frac{2\pi_0 < p(\psi) >}{B_p(a)^2}\ . \quad (13)$$

The parameter β_p[1] is called *poloidal beta,* the ratio between the plasma average pressure and the poloidal magnetic field pressure, and is the main parameter which originates the variation of the parameter of perturbation Γ.

Substituting the values for the equilibrium through the electrical parameters, and the values coming from the GS equation, and taking it into the matrix form

$$M\dot{x} + \Omega x = B_c u + E\xi\ , \quad (14)$$

allows us to write the matrixes M , Ω and E.

[1] $\beta p = 0.9942$ for the equilibrium obtained through the solution to the GS equation

Finally the linearized model is given by equation (14), where $\xi = -\frac{\mu_0 I_{p0}^2}{2}\dot{\Gamma}$, and the inputs are $u = \begin{bmatrix} Vc_{1a} \\ V_{c2a} \\ V_{c1b} \\ V_{c2b} \end{bmatrix}$,

$$M = \begin{pmatrix} L_1 & M_{aa} & M_1 & M_{ab} & \frac{\partial M_{p1}}{\partial R} & M_{p1} \\ M_{aa} & L_2 & M_{ab} & M_2 & \frac{\partial M_{p2}}{\partial R} & M_{p2} \\ M_1 & M_{ab} & L_1 & M_{aa} & \frac{\partial M_{p1}}{\partial R} & M_{p1} \\ M_{ab} & M_2 & M_{aa} & L_2 & \frac{\partial M_{p2}}{\partial R} & M_{p2} \\ \frac{\partial M_{p1}}{\partial R} & \frac{\partial M_{p2}}{\partial R} & \frac{\partial M_{p1}}{\partial R} & \frac{\partial M_{p2}}{\partial R} & 374.6 & 0.067 \\ M_{p1} & M_{p2} & M_{p1} & M_{p2} & 0.059 & L_p \end{pmatrix}, (15)$$

$$\Omega = \begin{pmatrix} R_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{p0}\frac{\partial \Omega_p}{\partial R} & \boldsymbol{\Omega_p} \end{pmatrix}, \quad (16)$$

$$E = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} . \quad (17)$$

The state space vector is compound by the PF coils currents, the current plasma and the radial position. The state space vector , translated to the equilibrium point, is:

$$x = \begin{pmatrix} I_{c1a} - I_{c1a0} \\ I_{c2a} - I_{c2a0} \\ I_{c1b} - I_{c1b0} \\ I_{c2b} - I_{c2b0} \\ R - R_0 \\ I_p - I_{p0} \end{pmatrix} . (18)$$

For the conventional form of the state space matrices, it is necessary to apply matrix algebra to take the matrices M and Ω to A and B.

$$A = -M^{-1}R \ , \ (19)$$

$$B = M^{-1}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (20)$$

In order to obtain the electrical parameters of the coils, which includes the self and mutual inductances, its resistivity and the mutual inductance between the coils and the plasma considering the plasma, as a conductor of a single turn, the formulas for short solenoids (G. Jaramillo, 2007) and the ones for mutual inductances using elliptic integrals (J. Jackson, 1962) were used. The results are shown in Tables 1, 2 and 3.

Table 1 Geometrics Properties of the Coils

N1	N2	Material	Radius 1	Radius 2	R1	R2
15 turns	20 turns	Cooper AWG /4	15 (cm)	36 (cm)	0.113 (Ω)	0.362 (Ω)

Table 2 Coils' Inductive Properties

L1	L2	M1	M2	Mab	Maa
0.1001 (mH)	0.5127 (mH)	1.4188 (μH)	41.858 (μH)	7.0127 (μH)	39.720 (μH)

Table 3 Plasma Resistive and Inductive Properties

Lp	Rp	li	Mp1	Mp2
0.8909 (μH)	2.5831 (μΩ)	1	0.8133 (μH)	3.1404 (μH)

IV. Control and Results

It is assumed that the six states can be measured. In first place, the coil currents can be obtained via suitable sensors, the plasma current can be measured using Rogowski coils, as well as the radial position of the plasma, where Mirnov coils are typically used to obtain it. In this sense, a state feedback control with fully measurable states is proposed as $u = -kx$.

In order to design a feedback control for this MIMO system a LQR strategy with infinite time horizon is used, where the main objective of the design is that the plasma position bias from its equilibrium point is corrected in less than 2 seconds, while the input voltage does not surpass more or less 20 (V). Based on this design criteria for the

Tokamak "Novillo"[2], a suitable matrix election was the one in equation (20).

$$Q = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 \end{pmatrix} \quad (21)$$

The matrix R chosen for the best "energy" weight at the input is:

$$R = \begin{pmatrix} 0.3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \quad (22)$$

The simulation of the control was made using MATLAB 2011 software, with an i3-Core Intel processor PC; and Mathematica 8 software was used for the analytic calculus of the coefficients.

Figure 5 shows the behavior of the states with an LQR feedback control applied for 2 seconds. It is observed that the change in the position δR is equal to zero and the plasma current δI_p is almost equal to zero. While the changes in the coils currents (δI_{c1a}, δI_{c2a}, δI_{c1b} and δI_{c2b}) are about 20-100 (A), which implies a low error, considering that coils currents in the equilibrium are about tens of (KA.)

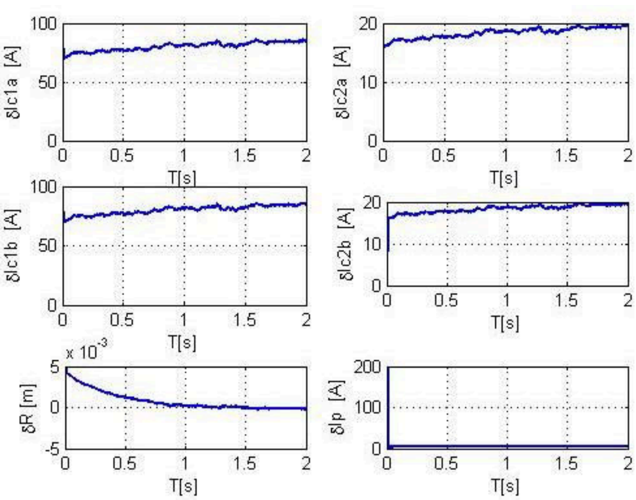

Fig. 5 Change in the linearized states about the equilibrium point.

Figure 6 shows the behavior of the input controls, which are the voltages applied to the PF coils δ_{Vc1a}, δ_{Vc2a}, δV_{c1b} and δV_{c2}. These voltages also have a low error about 10(V). The error in the linearized controls is very acceptable too, as in the coil currents.

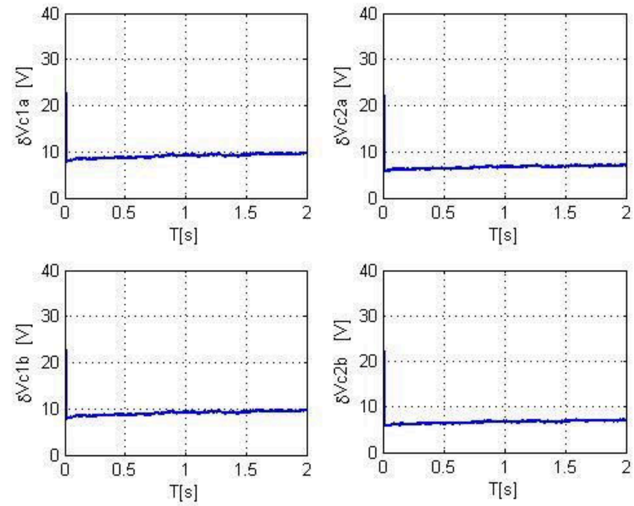

Fig. 6 Change in the linearized control inputs about the equilibrium point .

The perturbation ξ is related to the beta poloidal and it represents the change in the force balance in (19). It was simulated by a random number input, and through the numerical calculus for the beta poloidal and an approximation of the different experimental values, the beta takes during a shot in a Tokamak, it was decided to apply a variance of 200 (N). It is also possible to see in the Figures 5 and 6 that the perturbation does not cause a problem to the control while taking the system to the equilibrium.

It can be seen from these results that the control of the position, which is in part the most important state, converges quite well to zero even though the perturbation is big. The plasma current converges very well too and much quicker to zero.

In the case of the currents in the PF coils and the control voltages the values have a small numerical error, probably due to the numerical approximation made by the software.

The initial conditions for the system were none current at the PF coils, and initial position in R of 5 (mm) and an initial plasma current of 200 (A).

[2] The "Novillo" Tokamak is located at the Instituto Nacional de Investigaciones Nucleares near Mexico City. Even though it has never been tested for feedback control, it was decided to take its geometrical parameters.

V. CONCLUSIONS

In this paper we have presented an optimal linear control design with infinite time horizon for a linearized Tokamak model. The results show that it is possible to control this kind of variables with a very acceptable convergence, even though it is a system which may cause numeric problems to the software simulation and has been simulated with a considerable perturbation due to the Grad-Shafranov parameters.

By modeling the approximate dynamics based on experimental parameters from a Tokamak, and having its linear model, an optimal control was designed. However, in order to obtain a more flexible control it would be necessary to consider a faster response, as in real experiments in small Tokamaks as the one considered in this paper, where the time span of the discharge is in the range of 25-100 (ms).

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