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# Use of the Cell Transmission Model in Urban Networks with Flow Discontinuities

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***Abstract* – Even though the Cell Transmission Model has been performed in freeway-type traffic scenarios, it also has been in urban traffic simulations. However, most of these macroscopic simulations are performed for continuous states of the well-known variables such as density or flow of highway stretches or road networks that are mainly continuous in their nature.**

**This work explores approximations to the application of this model to urban traffic networks with discontinuities caused mainly by traffic lights which imply zero-values in calculations for flow and implying some drawbacks for this model, whose author (Daganzo) is aware.**

**Real values of data has been obtained from two important streets of a central city of Mexico, including traffic light cycles, which had to be standardized and coordinated for taking them into account in simulations, which results are presented along with the respective analysis, which also has been used to calibrate the model to this network.**

**Final simulation results are compared with real data, discussed, and conclusions established.**

**Keywords: Cell transmission model, discon-tinuous flow, urban networks**

## I. INTRODUCTION

Almost two decades ago, Daganzo (1994, 1995) published his work about the cell transmission model (CTM), a simplification over the well-known hydrodynamic approximation.

Since then, Daganzo's model has been used in derivations or extensions of such a model (Muñoz et al., 2003), in simulations (Muñoz et al., 2004), in traffic parameter and variable estimation (Rosas-Jaimes and Alvarez, 2007) and in control design (Gomes, 2004). A common procedure to develop a control for any system begins with establishing a model to that system, to calibrate it and to validate it, and then to design a convenient scheme to regulate it. This procedure has been used for highways (Muñoz et al., 2004) (Gomes, 2004), looking at them as dynamic systems and using CTM as a base. Our main purpose in this work is to show the approximation of CTM to a real network, calibrate its parameters and compare simulation results with real data, settling a base for future developments in models and controls.

## II. CELL TRANSMISSION MODEL (CTM)

The original CTM formulation has the advantage to be quite simple but precise enough to represent quite well the reality in traffic. CTM departs from the density-flow fundamental diagram, a pictorial representation of the relations among data pairs of density $k$ and flow $q$ of moving vehicles in a road (Figure 1a).

His author, Daganzo (1994), has proposed this model trying to give a simple approximation to the density-flow fundamental diagram, with the selection among three straight lines of possible values of density $k$ and flow $q$ pairs (Figure 1b). It is then evident that density $k$ is a functional of flow $q$, related by an expression (1).

$$q_{i+1} = \min\left\{ v_i k_i,\ \ q_{\max},\ \ w\left(k_j - k_{i+1}\right)\right\} \tag{1}$$

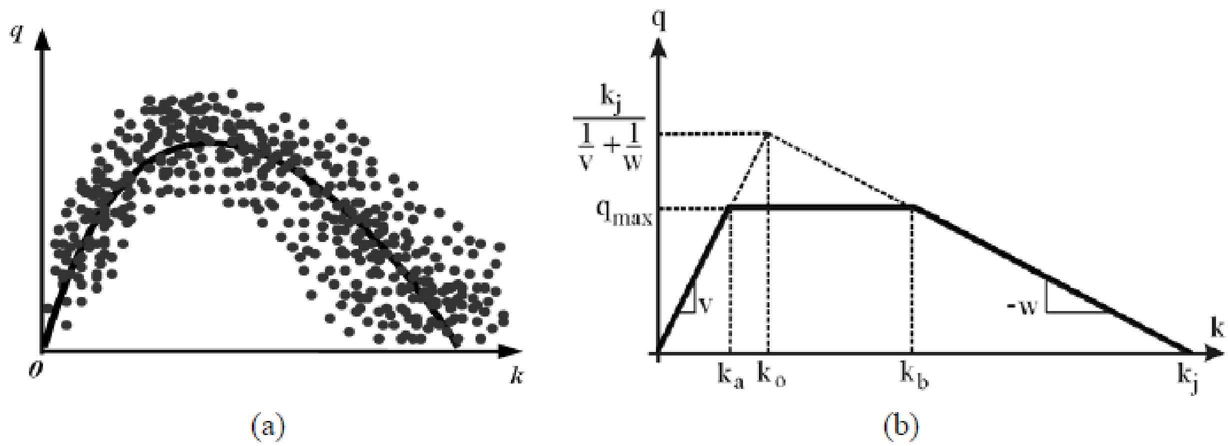


Figure 1: (a) Flow-density data set, adjusted to a functional *q(k)*. (b) Graphical representation of flow *q* as a function of density *k*, as proposed by CTM's approach.

Where *v* is the average maximum speed achieved by cars moving in the cell, and is coincident with the slope of the left portion of the diagram in Figure 1b, while *w* is another velocity, one related with a wave of congestion that goes in an opposite direction of that of the traffic. Lastly, $k_j$ is the maximum value of density that a cell can contain.

Flows calculated by (1) occur between two stretches of a road, or cells, where the name of the model is originated from (Figure 2).

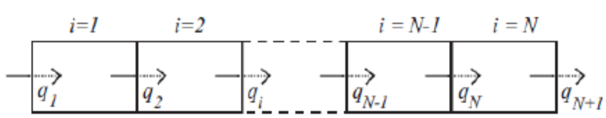


Figure 2: Portion of a road divided in cells

A more detailed inspection of equation (1) shows that flows like $q_1$ and $q_{N+1}$ need a special treatment due to the index of the respective densities to perform calculations are beyond the available values. This is easily solved by entering suitable and convenient functions for such flows, due to they can represent common behaviors of traffic in different free-flow or congestion situations. This fact also determines that free-flow traffic is affected by upstream conditions, but congested flow is influenced by downstream values.

Once flows $q_i$ are calculated, densities are achieved by the conservation hydrodynamic law (2), where $L_i$ is the length of each cell

$$\frac{dk_i}{dt} = \frac{1}{L_i}\left(q_i - q_{i+1}\right) \qquad (2)$$

Solving trajectories for Equation (2) can be achieved by numerical algorithms as, for example, an Euler-like approach

$$k_i(t+\Delta t) = k_i(t) + \frac{1}{L_i}\left[q_i(t) - q_{i+1}(t)\right]\Delta t \qquad (3)$$

Or a more precise Taylor-like approach

$$k_i(t+\Delta t) = k_i(t) + \frac{1}{L_i}\left[q_i(t) - q_{i+1}(t)\right]\Delta t + \\ + \frac{1}{L_i}\left[\frac{q_{i+1}(t) - q_{i+1}(t-1)}{\Delta t} - \frac{q_i(t) - q_i(t-1)}{\Delta t}\right]\left(\frac{\Delta t^2}{2}\right) \qquad (4)$$

Daganzo (1995) dedicates a special formulation for road networks through the development of cells that also takes into account convergent or divergent flows to simulate networks.

## III. URBAN TRAFFIC ROAD MODELING

Sets of flows and velocities data are available from six measuring stations placed in two main streets of the city of Toluca, in the State of Mexico, approximately 80 kilometers distant from Mexico City. One of such streets is J. M. Morelos Avenue and has a measuring station in cell 1, in the way we have divided and identified for our purposes; a second station at the middle of cell 22, and a last one near the exit of the last cell (Table 1).

Table 1. Example of flow and velocity data in three measuring stations at J. M. Morelos Avenue

| | Morelos Avenue | | | | | |
|---|---|---|---|---|---|---|
| Cell | 1 | | 22 | | 36 | |
| Hour dt=30s | *q [veh/s]* | *v [m/s]* | *q* | *v* | *q* | *v* |
| 10:00:00 | 0.57 | 8.91 | 0.43 | 11.03 | 0.40 | 11.41 |
| 10:00:30 | 0.03 | 6.81 | 0.03 | 9.03 | 0.13 | 7.72 |
| 10:01:00 | 0.43 | 6.24 | 0.60 | 8.11 | 0.03 | 9.83 |
| 10:01:30 | 0.80 | 11.22 | 0.77 | 8.47 | 0.53 | 9.20 |
| 10:02:00 | 0.50 | 9.61 | 0.40 | 7.97 | 0.57 | 12.03 |
| 10:02:30 | 0.07 | 6.50 | 0.27 | 6.19 | 0.17 | 10.32 |
| 10:03:00 | 0.27 | 6.03 | 0.57 | 6.39 | 0.20 | 9.44 |

A second group of sensors are in cell 59, cell 81 and near the end of cell 91 for M. Hidalgo Avenue. CTM is applied to a limited network.

These corridors cover a length of about 3.6 km in both directions, with 36 and 32 intersections crossing perpendicular streets respectively, 40 of them with traffic lights. These are two important streets in this city, and many other roads are related to them, giving as a consequence that thousands of private and public passengers circulate through them daily on any weekday (Fonseca, 2008).

As we have already mentioned, we have divided these avenues into cells, taking every intersection as the boundaries of each one of those cells. The rest of the related perpendicular streets between them are taken just for their average flows, i.e. we have considered their behavior is steady, assuming accuracy of such an assumption is enough for our purposes. For that reason, the flows of different directions converging or diverging in such intersections must be the proportional average transit that can be registered in each one of such nodes and we take the net entering flow of such a calculation. Figure 3 shows a scheme of one of these avenues.

Table 2. Relation of sample time to traffic light cycles

| | # Street Name | | |
|---|---|---|---|
| Time = t | 1.Motolinia | 2.A Venegas | 3.F villanueva |
| 09:50:00 | 0 | 1 | 0 |
| 09:50:30 | 1 | 1 green | 1 |
| 09:51:00 | 0 | 1 | 1 |
| 09:51:30 | 1 | 0 | 0 |
| 09:52:00 | 1 | 1 | 0 red |
| 09:52:30 | 1 | 1 | 1 |
| 09:53:00 | 0 | 1 | 1 |
| 09:53:30 | 1 | 0 | 0 |
| 09:54:00 | 1 | 1 | 0 |
| 09:54:30 | 1 | 1 | 1 |
| 09:55:00 | 0 | 1 | 1 |

These stations collected data during a week in 24-hours periods (Fonseca, 2008). A sample for Tuesday is shown in Figure 4 where we have marked an interval, from approximately 9:50 to 16:45 of that day showing a minimum variation. We choose this profile because it is close to a steady state, the same assumption we made for the rest of the streets and because this pattern repeats almost consistently at other days.

Due to the vehicular traffic in this corridor is affected by traffic lights in 40 of the 68 intersections, it was necessary to perform the register of the green-yellow-red cycle of each light. In order to simplify this process, and due to practical considerations, yellow interval has been considered as part of the green interval, so a more simple green-red cycle is registered. Table 2 shows the pattern approximation for the traffic lights in 30-second samples, found at intersection 1, between cells 1 and 2 of J.M. Morelos Avenue.

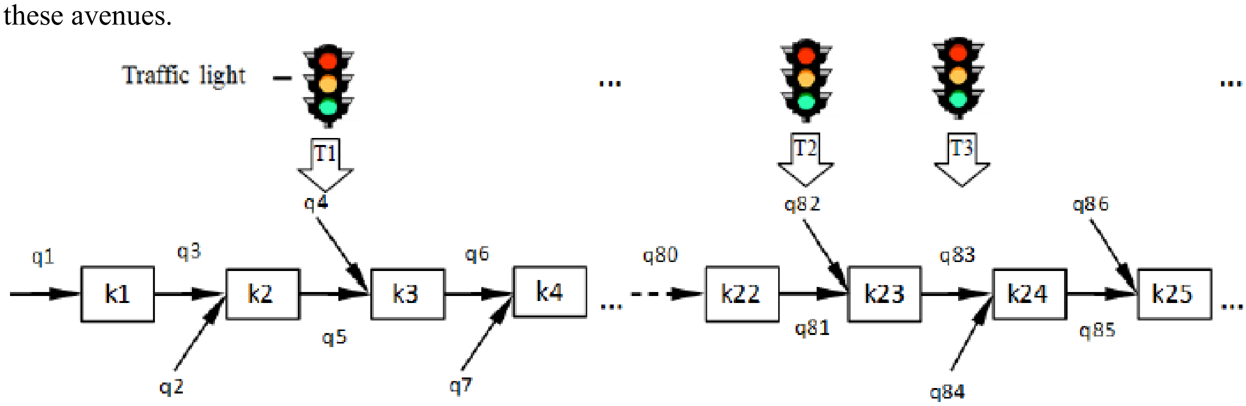

Figure 3: Network scheme used with net flows for J.M. Morelos Avenue.

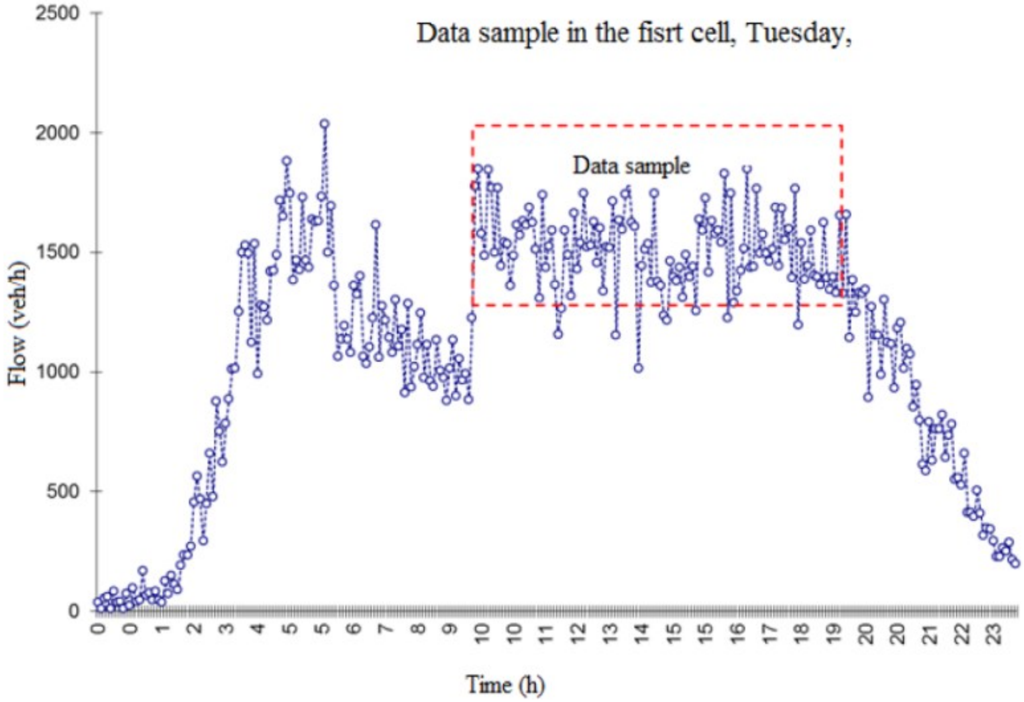


Figure 4: Data sample in the first cell, flow $q$ versus time $t$ during a 24-hour period. An interval spanning from 9:50 to 16:45 is indicated.

Due to flow and velocity data has been registered in 30-second samplings, this green-red pattern has been converted to a series of ones and zeros in order to synchronize the traffic lights behavior to the sampling sequence of the data, a feature that will be necessary to perform simulations.

The existence of traffic lights gives, as a consequence, discontinuities in flow and density. This fact must be taken into account when dealing with a continuous model like the cell transmission model. Suppose that a traffic light is placed at exactly a cell boundary. When traffic flow is interrupted due to a red cycle starts, flow facing traffic light is zero ($q_1 = 0$), but those cars that are leaving that point continue moving and giving a non-zero value to the flow that is passing the next cell boundary ($q_{i+1} \neq 0$). If this situation is carried over conservation law (2) is possible to observe that negative values of the density temporal derivative begin to appear, and if restrictions are not taken even flow and density became negative.

Fortunately, as simulations show up, relatively short times of traffic light cycles result in very small periods of such effects.

Looking again at Equation (1), it is evident that suitable values for velocities $v$ and $w$ are needed. Due to these values are directly related to the slopes of the lines that appear in the formulation of CTM (see Figure 1 and Equation (1)), an approximation by a curve fitting of the data available is carried on. More specifically, Figure 5 shows a plot of measured data pairs of flow $q$ and density $k$ obtained from the measuring station at cell 1 for Thursday. A least-square fitting is performed in order to match a straight line that best approximates such a data set. Besides, we have calculated the corresponding jam density $k_j$ for this cell by Equation (5), corresponding to this case with $k_j = 0.5333$ *[veh/m]*

$$k_j = \frac{r}{c+g} \tag{5}$$

Where:
$c$ = Average vehicle length (6.1 m)
$g$ = Average gap between two consecutive vehicles (1.5 - 2.0 m)
$r$ = Number of lanes

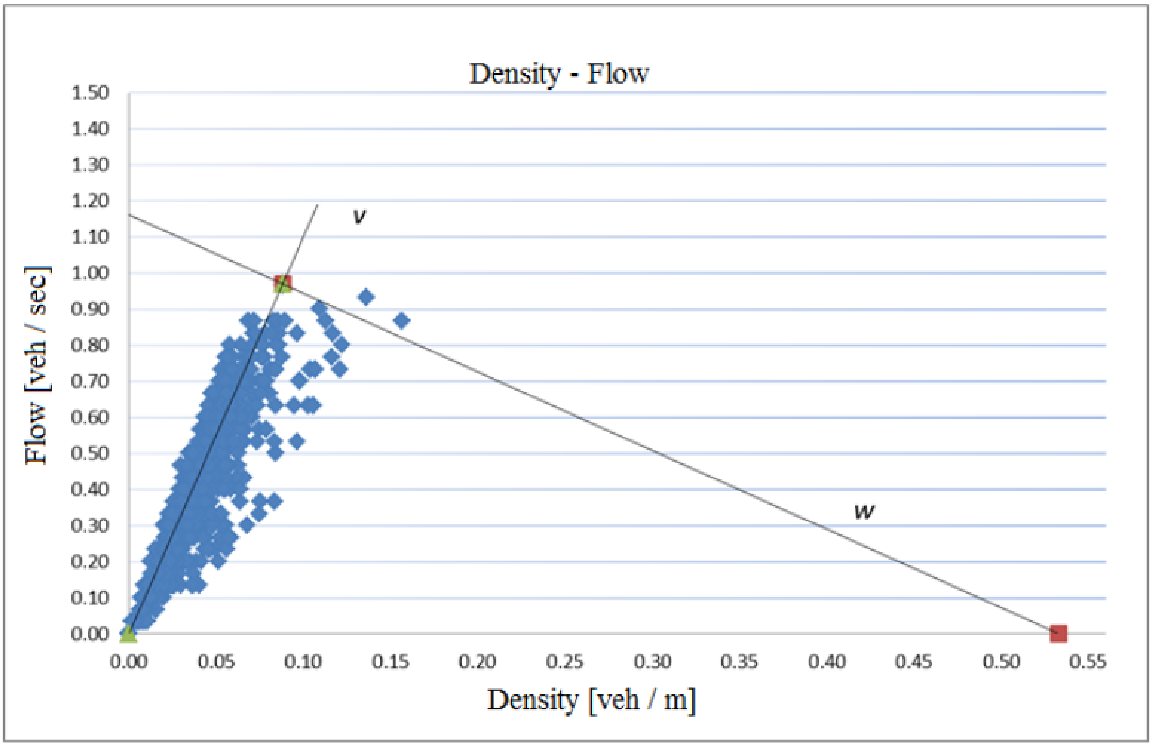


Figure 5: Calibration performed to obtain free-flow velocity v and congestion wave velocity w.

The maximum flow value $q_{max}$ have been assumed from the maximum registered value, $q_{max}$ = 1.07 *[veh/s]* in this case. With $q_{max}$ and $k_j$ values it is possible to draw a negative slope straight line corresponding with the congested representation of this fundamental diagram. Finally, we are neglecting the zero-slope straight line that Daganzo takes into account in his original model, due to several experiences (Muñoz, 2004) have shown there is no negative effect in this model's calibration.

## IV. RESULTS

We are aware that many assumptions, simplifications and approximations have been added. We wanted to know how much of such a set of inaccuracies would invalidate our CTM urban application. In this way, a code with all these data sets, parameter values and expressions was prepared using MatLab's editor ®.

Figure 6 shows comparisons among those flows obtained by measuring and those obtained by means of simulation. It is possible to see that behavior of the calculated values (black-dotted lines) tends to reproduce that of real ones (red lines) quite well despite of all the missing and simplified information. However, error magnitudes can reach more than 30 % in some instants.

Figure 7 shows density profiles for those cells related to the places where measuring stations were located. This set of plots also depicts how simulations performed by the use of CTM (blue-dotted lines) tend to approximate those measured data (red lines). However, we are aware that non-trivial error magnitudes are present. Similar results can be achieved for other cells where measuring stations are located, for this road and for M. Hidalgo Avenue.

These results show that CTM is quite accurate to simulate urban networks, but this accuracy can be improved if:

- Data sampling times tend to be shorter.
- Involved variables are effectively and directly measured.
- All data sets are standardized in the most convenient way.
- Discontinuities have a small effect relative to continuous behavior due to the system responses.

These results show that a model like CTM can be used in an urban network like that where data has been obtained, provided that all the points listed in the last paragraph are improved. Even though CTM has a more and natural approach to continuous-type systems, under enough tolerance could be used to simulate, design and analyze networks like that presented here.

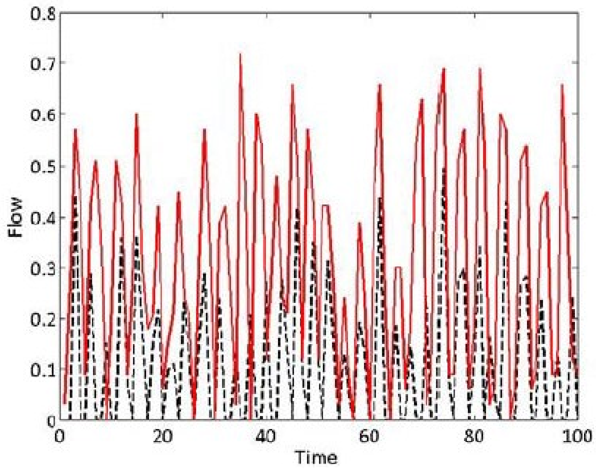


Figure 6: Portion of flow profiles for J. M. Morelos Avenue (cell 59). Red trajectories are measured data; black-dotted ones are calculated.

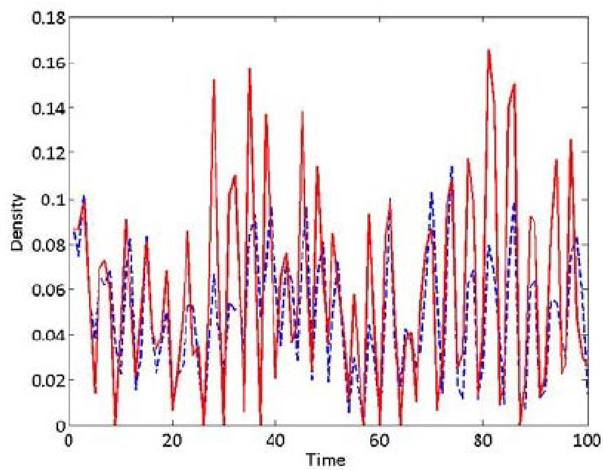


Figure 7: Portion of density profiles for J. M. Morelos Avenue (cell 59) Red trajectories are measured data; blue-dotted ones are calculated.

## V. CONCLUSIONS

CTM presents a good accuracy and simplicity balance, being used extensively since its publication. In the case presented here, we are exploring the aspects of its application to an urban network with discontinuities originated by traffic lights in a city of the central part of Mexico.

Classification by means of cells, as stated by such a model, together with calibration of the parameters needed, is performed in accordance with traffic theory and the real situation of this network. Provided measuring data are treated as inputs for the dynamic model CTM, which is coded in order to performed calculations which has an enough approximation to observe these results can be improved if some features are taken into account.

These results are very valuable to our research group, because integrates several lines of research relative to transport engineering, system dynamics and simulation. Future work shall be leaded to increase the number of measuring stations, with shorter sampling times, to enhance the ways data are obtained. Once succeeded in such goals, simulations will be performed to emergency scenarios.

## VI. ACKNOWLEDGEMENTS

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