Self-Triggered Stabilization of Linear System:
Towards a Reduction of the Control Updates even in presence of Noise

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Abstract— **The self-triggered control paradigm allows to considerably reduce the frequency of events, even in the case of noise since it operates in open loop between two updates (and hence does not detect a variation of the system dynamics). However, the robustness is highly concerned due to this open-loop setup. In this paper, we propose an original alternative for the stabilization of a disturbed linear system in presence of noise: i) a self-triggered control strategy computes and updates the control signal using a copy of the undisturbed model and ii) an event-based corrector updates this copy when it deviates from the real measurement (that is when a disturbance occurs). Such a technique operates in closed loop and can immediately react in case of a perturbation. Therefore, combining together a self-triggered control with such an even-based correction yields a robust framework. The different proposals are tested in simulation and compared in terms of updates, robustness and frequency of events in presence of noise.**

INTRODUCTION

The classical (time-triggered) discrete time framework of controlled systems consists in sampling the system uniformly in time with a constant sampling period. Although periodicity simplifies the design and analysis, it results in a conservative usage of resources since the control law is computed and updated at the same rate regardless of whether is really required or not. In this context, some works addressed more recently resource-aware implementations of the control law, where the control law is event-driven (also called asynchronous).

Typical event-based detection mechanisms are functions of the state variation (or the output) of the system, like in (Årzén, 1999; Sandee *et al.*, 2005; Durand y Marchand, 2009; Sánchez *et al.*, 2009). Although the event-triggered control is well-motivated and allows to relax the periodicity of computations, only few works report theoretical results about the stability, convergence and performance. In (Åström y Bernhardsson, 2002) in particular, it is proved that such an approach reduces the number of sampling instants for the same final performance. Some stability and robustness proprieties are exploited in (Åström y Bernhardsson, 2002; Heemels *et al.*, 2009; Lunze y Lehmann, 2010; Donkers y Heemels, 2010; Eqtami *et al.*, 2010). An alternative approach consists in taking events related to the variation of a Lyapunov function – and consequently to the state too – between the current state and its value at the last sampling, like in (Velasco *et al.*, 2009), or in taking events related to the time derivative of the Lyapunov function, like in (Tabuada, 2007; Anta y Tabuada, 2008; Marchand *et al.*, 2011; Téllez-Guzmán *et al.*, 2012). In the two latter references (nonlinear proposal and its linear version respectively), the updates ensure the strict decrease of the Lyapunov function and thus the asymptotic stability of the closed-loop system.

On the other hand, a self-triggered implementation saves much more computing since it eliminates the resource utilization for continuously monitoring an event function. Such an original setup, which was firstly proposed in (Velasco *et al.*, 2003), consists in computing the next sampling time using the last state measurement (the last time an event occurred). Therefore, one does not require any knowledge of the current state anymore. Typical algorithms to calculate the next activation time are based on the emulation of the events generated by an event-based technique, like in (Mazo Jr. *et al.*, 2009; Mazo Jr. y Tabuada, 2009; Mazo Jr. *et al.*, 2010; Anta y Tabuada, 2010). In particular, the present work is based on (Durand *et al.*, 2012) which is the (linear) self-triggered version of the event-based strategies proposed in (Marchand *et al.*, 2011; Téllez-Guzmán *et al.*, 2012), and so is asymptotically stable the closed-loop system too. Such a scheme is also highly considered for networked control system purposes, like in (Mazo Jr. y Tabuada, 2008; Camacho *et al.*, 2010; Arajo *et al.*, 2011; Tiberi *et al.*, 2010; Tiberi *et al.*, 2011), since a constant monitoring means a continuously listening of the communication network and, consequently, a strong energy consumption.

Asynchronous paradigms allow to highly reduce the frequency of events, however, both have some disadvantages. On one hand, a self-triggered system operates in open loop between updates of the control law and robustness is therefore highly concerned. On the other hand, even if the control is updated less frequently than with a periodic scheme, the even-based scheme can behave as the classical time-triggered strategy in presence of noise whether the noised measurements always enforce events. This problem is targeted here in particular since we propose a solution to reduce the updates even in presence of noise. The idea is to have benefit of both techniques, combining together the self-triggered principle with the event-based scheme. The rest of the paper is organized as follows. In section I, an overview of the context is provided and the problem is stated. The system architecture is introduced in section II and the main contribution is detailed. The stability and robustness is also analyzed. Simulation results are provided in section III to highlight the capabilities of the proposed approach. Some discussions finally conclude the paper.

I. Context description

Let consider the linear time-invariant dynamical system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ \text{with} \quad x(0) &:= x_0 \end{aligned} \tag{1}$$

with $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ the state and input vectors.

A. Event-based control

By *event-based state-feedback* we mean a set of two functions:

i) an event function $\xi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, that indicates if one needs (when $\xi \leq 0$) or not (when $\xi > 0$) to recompute the control law;

ii) a state-feedback function $\mathbb{R}^n \to \mathbb{R}^m$ such that

$$u(t) = -Kx(t) \tag{2}$$

where K is the feedback matrix. The solution of (1) with an event-based state-feedback starting in x_0 at $t = 0$ is then defined as the solution of the differential system

$$\dot{x}(t) = Ax(t) - BKx(t_i) \qquad \forall t \in [t_i, t_{i+1}[\tag{3}$$

where the time instants t_i, with $i \in \mathbb{N}$ (determined when the event function ξ vanishes) are considered as *events* and $x(t_i)$ is the memory of the state value at the last event. In (Marchand *et al.*, 2011; Téllez-Guzmán *et al.*, 2012), it is proved that the linear system (1) can be asymptotically stabilized by means of a particular event-based state-feedback, defined by

$$u(t) = -Kx(t_i) \qquad \forall t \in [t_i, t_{i+1}[\tag{4}$$

$$\text{with} \quad K := 2\varepsilon B^T P \tag{5}$$

$$\begin{aligned} \xi\big(x(t), x(t_i)\big) = {} & (\sigma - 1)x(t)^T Q_1 x(t) \\ & -4\varepsilon x(t)^T Q_2 \big[\sigma x(t) - x(t_i)\big] \end{aligned} \tag{6}$$

$$\text{with} \quad \begin{aligned} Q_1 &:= PA + A^T P \\ Q_2 &:= PBB^T P \end{aligned}$$

where $\sigma \in]0, 1[$, $\varepsilon > 0$ are some tunable parameters, and P is a positive definite matrix solution of the Riccati equation

$$Q_1 - 4\varepsilon Q_2 = -Q \tag{7}$$

where Q is also positive definite. The idea behind the construction of the event function (6) is to compare the time derivative of the Lyapunov function (7) *i)* in the event-based case, that is applying $x(t_i)$ in the state-feedback control, like in (4), and *ii)* in the classical case, that is applying $x(t)$ instead of $x(t_i)$ in the feedback, like in (2). The event function is the weighted difference between both, where σ is the weighted value. By construction, an event is enforced when the event function vanishes to zero, that is when the stability of the event-based scheme does not behave as the one in the classical case. Also, ε changes how fast is the control signal (this parameter was identify as an event-based LQR parameter in (Téllez-Guzmán *et al.*, 2012)).

It is also proved in (Marchand *et al.*, 2011) that the feedback (4)-(6) is *uniformly MSI* (Minimal inter-Sampling Interval), that means it is a piecewise constant control with non zero sampling intervals (avoiding Zeno phenomena).

B. Self-triggered control

Event-based control allows for computational savings. It has notably been shown in (Åström y Bernhardsson, 2002) that the control law can be updated less frequently than with a periodic scheme while still ensuring the same performance. However, a self-triggered implementation saves much more computing since it eliminates the resource utilization for continuously monitoring an event function. This could be highly costly in some cases, especially when events are based on Lyapunov function as this is the case in (6). Typical self-triggered technique consists in the emulation of the event-based strategy in order to calculate the next sampling time. Therefore, one does not require any knowledge of the current state anymore. By analogy with the previous definition, by *self-triggered state-feedback* we mean a set of two functions:

i) a sampling function $\lambda : \mathbb{R}^n \to \mathbb{R}$ that calculates the next activation time (the next time the control law has to be computed and updated);

ii) a state-feedback function $\mathbb{R}^n \to \mathbb{R}^m$ like in (2).

The solution of (1) with a self-triggered state-feedback starting in x_0 at $t = 0$ is defined as previously as the solution of the differential system (3), but this is now followed by the computation of the next instant time at which the control law has to be updated, that is

$$t_{i+1} := \lambda\big(x(t_i)\big) + t_i \tag{8}$$

where t_{i+1} denotes the next sampling time. Such a method was proposed in (Durand *et al.*, 2012) for the (4)-(6) feedback case (where the asymptotic stability and uniformly MSI properties hence remain). The principle consists in approximating the system trajectory used in the event function ξ. In (Velasco *et al.*, 2008; Durand *et al.*, 2012), the solution of the closed-loop system (3) is defined as follows

$$\begin{aligned} x(t) &= x(t_i) + \Psi(t - t_i)Lx(t_i), \quad \forall t \geq t_i \\ \text{with} \quad L &:= A - BK \end{aligned} \tag{9}$$

where K is given in (5) and $\Psi : \mathbb{R}^{n\times n} \times \mathbb{R} \to \mathbb{R}^{n\times n}$ is defined as a power series $\Psi(t) = \sum_{k=1}^{\infty} \frac{A^{k-1}t^k}{k!}$, which can be approximated to then simplify the problem. For instance, an efficient (first-order) Taylor approximation of the next sampling time t_{i+1} is the smallest positive zero of the event function (6) such that

$$\xi\big(x(t_i) + (t_{i+1} - t_i)Lx(t_i), x(t_i)\big) = 0 \tag{10}$$

In the present case, this yields

$$t_{i+1} = \arg\min_{t>t_i} \Big\{ t = \lambda\big(x(t_i)\big) + t_i \Big\} \tag{11}$$

$$\text{with} \quad \lambda(z) := \frac{-\beta_z \pm \sqrt{\beta_z^2 - 4\alpha_z\gamma_z}}{2\gamma_z} \tag{12}$$

$$\begin{aligned} \text{and} \quad & \alpha_z := (\sigma-1)z^T\big[Q_1 - 4\varepsilon Q_2\big]z \\ & \beta_z := (\sigma-1)z^T L^T\big[Q_1 - 4\varepsilon Q_2\big]z \\ & \qquad + z^T\big[(\sigma-1)Q_1 - 4\sigma\varepsilon Q_2\big]Lz \\ & \gamma_z := z^T L^T\big[(\sigma-1)Q_1 - 4\sigma\varepsilon Q_2\big]Lz \end{aligned}$$

where Q_1, Q_2 and L are defined in (6) and (9). One can refer to (Durand *et al.*, 2012) for further details on this solution and a more general formula for other orders of approximation.

C. How to reduce events even in case of noise ?

The problem which is targeted in this paper is that an event-driven controlled system can behave as a continuous-time one in presence of noise whether the noised measurements always enforce events. For this reason, we propose to add an *event-based corrector* mechanism. A similar technique is suggested in (Lehmann y Lunze, 2011) for a classical (time-triggered) state-feedback control strategy for networked controlled systems under disturbances (in order to stabilize a disturbed system over a communication link while reducing the sending of measurement information). It is adapted here for the particular event-based feedback (4)-(6) case as well as for the self-triggered scheme (without network), both in presence of disturbance. Moreover, whereas the original setup studies the impact of communication delays, the present one is dedicated to the noise problem.

The idea is to make a copy of the model of system (1) without disturbance nor noise. This copy is then used to compute the event function and the control law, and corrected when it deviates too much from the real value (that is notably when a disturbance occurs), and so is finally reduced the number of events even in presence of noise (since the control is based on a model without noise).

Contributions of the paper

In this paper, we propose a setup based on *i) a self-triggered state-feedback controller* (for a low computational cost) and *ii) an event-based corrector* (for robustness and reduced noise). We prove that such a proposal makes the control loop stable. We also show that the impact of the noise in the frequency of events is reduced with both asynchronous frameworks.

One could note that, whereas the event-based/self-triggered control strategy is dedicated to some previous works in (Marchand *et al.*, 2011; Téllez-Guzmán *et al.*, 2012; Durand *et al.*, 2012), the proposal can be easily generalized to other strategies.

II. Main results

The system architecture is presented in Fig. 1. On one hand, a self-triggered technique computes and updates the control signal in order to minimize the computational cost. On the other hand, an event-based corrector allows to correct the dynamical model used by the controller. The different events can occur at any time and independently, consequently, one needs to mark the time variable $t \in \mathbb{R}^+$ with respect to the source of events in order to formalize such a framework next. Two indexes are used herein:

- t_i denotes the time when an event is enforced for control, afterwards called *control's event*, with $i \in \mathbb{N}$;
- t_j denotes the *correction's event* time, with $j \in \mathbb{N}$.

Remember that both indexes are completely independent, and so are the marked time variables (there is no chronological relation between t_i and t_j).

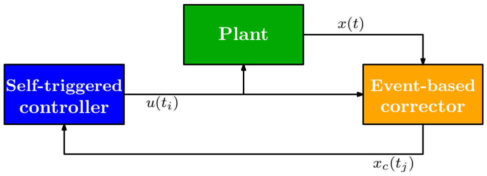


Fig. 1. System architecture.

A. System with noise

The plant is described by a perturbed linear model

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ed(t) \\ \text{with} \quad & x(0) := x_0 \end{aligned} \tag{13}$$

where $d \in \mathbb{R}^n$ is the disturbance. Several conditions are assumed in the sequel:

i) the dynamics of the plant as well as the initial conditions are accurately known (A, B and x_0 are known);
ii) the state x is measurable;
iii) the disturbance is bounded by

$$\|d(t)\| \leq d_{max} \tag{14}$$

Then, considering the system (13) and applying the event-based state-feedback (4)-(6), the continuous-time closed-loop system becomes

$$\begin{aligned} \dot{\breve{x}}(t) &= A\breve{x}(t) - BK\breve{x}(t_i) + Ed(t) \quad \forall t \in [t_i, t_{i+1}[\\ \text{with} \quad & \breve{x}(0) := x_0 \end{aligned} \tag{15}$$

where $\breve{x} \in \mathbb{R}^n$ is the uncorrected closed-loop event-based control state (whereas, in the present paper, a correction is then applied), K is defined in (5).

B. Event-based corrector

The corrector runs a copy of the closed-loop system model (15) without disturbance, that is

$$\begin{aligned} \dot{x}_c(t) &= Ax_c(t) + Bu(t_i) \quad \forall t \in [t_i, t_{i+1}[\\ \text{with} \quad & x_c(0) := x_0 \end{aligned} \tag{16}$$

where $x_c \in \mathbb{R}^n$ is the state of the event-based corrector. This model requires the control signal $u(t_i)$ which is applied to the real system (13) each time an event is enforced from the controller, it is directly obtained from the controller.

An event is generated for correction when the difference between the (real) perturbed system state $x(t)$ in (13) and the state of the event generator $x_c(t)$ in (16) reaches a given threshold $\bar{e}$, that is when

$$\|x(t_j) - x_c(t_j^-)\| = \bar{e} \tag{17}$$

where t_j^- is the time just before the event, and so is corrected the value of the event generator state such that

$$x_c(t_j^+) = x(t_j) \tag{18}$$

where t_j^+ is the time just after the event. This defines the correction's event instant t_j.

C. Event-based controller

In fact, the event-based feedback (4)-(6) is not directly computed for the system (13) to control, but for the copy of the model available in the controller node, that is the corrector model (16). The control's event instant t_i is hence determined by the vanishing of the event function (6) applied to x_c, that is when

$$\xi\big(x_c(t), x_c(t_i)\big) \leq 0 \tag{19}$$

Also, the control law (4) becomes

$$u(t) = -Kx_c(t_i) \quad \forall t \in [t_i, t_{i+1}[\tag{20}$$

where K is defined in (5). The control signal $u(t_i)$ is then applied to both the plant and the corrector, and so it is available in (16).

We insist here on the fact that the event detection mechanism (19) is function of the state variation of the corrector x_c (and not the system x itself). As a result, the control signal will be updated less often because of noise. Indeed, no event will be enforced while the condition (17) is not satisfied, that is while the error between the noised and unnoised signals is lower than the threshold $\bar{e}$, and so has to be tuned this parameter in consequence.

D. Self-triggered controller

As already explained, the principle of the self-triggered technique consists in predicting the next time the control law has to be updated and applying the new control signal at this given time. The strategy used in this paper is a version based on the event-based feedback (19)-(20). Both strategies have the same performance and stability properties, this was demonstrated in (Durand *et al.*, 2012). The event-based behavior presented above hence remains identical. However, an update of the control signal is now followed by the computation of the next time at which the control law has to be updated. This next activation time is expressed in (11) for the undisturbed case. It is simply applied to the corrector model, that yields

$$t_{i+1} = \arg\min_{t>t_i} \Big\{ t = \lambda\big(x_c(t_i)\big) + t_i \Big\} \tag{21}$$

where λ is defined in (12). Also, a difference exists between both techniques concerning the robustness to some perturbations since the self-triggered control operates in open loop between events. This problem is solved here thanks to the corrector. Thus, if a correction of the model (16), i.e. at time t_j, appears before the next (predicted) sampling time, then the next activation time t_{i+1} has to be re-computed. In this case, the predicted time becomes

$$t_{i+1} = \arg\min_{t>t_j} \Big\{ t = \lambda\big(x_c(t_j)\big) + t_j \Big\} \tag{22}$$

Each time a sampling instant is computed, either from (21) or (22), then the predicted time t_{i+1} as well as the value of the control signal at this time, i.e. $u(t_{i+1})$, are applied to the controller. As previously, the impact of noise on the frequency of updates is reduced. This self-triggered control with event-based corrector proposal is hence a very low cost strategy.

E. Stability analysis

Let first recall some definitions from (Khalil, 2002).

Definition 2.1: The solution $x(t)$ of a continuous-time system is *Globally Uniformly Ultimately Bounded (GUUB)* if for every initial condition $x(0) \in \mathbb{R}^n$ there exists a positive constant μ and time ν such that $\|x(t)\| \leq \mu \; \forall t \geq \nu$.

Definition 2.2: The solution of a disturbed system when applying a continuous-time state-feedback (2), that is

$$\dot{x}(t) = Lx(t) + Ed(t)$$

where L is the closed-loop matrix defined in (9), is GUUB if the feedback matrix K renders the undisturbed system (3) stable and the disturbance $d(t)$ is bounded.

The stability of the proposed event-based/self-triggered state-feedback control strategies with event-based correction then naturally follows.

Theorem 2.3 (Stability of the event-based framework): Consider the event-based corrector (16)-(18). Consider the event-based state-feedback (19)-(20). Then, the state-feedback control loop for the disturbed linear system (13), which disturbance is bounded by (14), is uniformly MSI and GUUB.

Proof: We know that the uncorrected event-based state-feedback (4)-(6) renders the undisturbed linear system (3) asymptotically stable for a given feedback matrix K defined in (5). This was proved in (Marchand *et al.*, 2011; Téllez-Guzmán *et al.*, 2012). From Definition 2.2, one can hence say that the uncorrected continuous-time state-feedback system (15) is GUUB for a bounded disturbance (14) and the stabilizing feedback (4)-(6).

On the other hand, let

$$e(t) := x(t) - \breve{x}(t) \tag{23}$$

be the difference between *i)* the state $x(t)$ of the closed-loop system of the present study case, i.e. (13)-(14), (16)-(18), (19)-(20) and *ii)* the state $\breve{x}(t)$ of the uncorrected closed-loop system (15). The derivative of $e(t)$ gives

$$\begin{aligned}\dot{e}(t) &= \dot{x}(t) - \dot{\breve{x}}(t)\\ &= Ax(t) - BKx_c(t_i) + Ed(t)\\ &\qquad -A\breve{x}(t) + BK\breve{x}(t_i) - Ed(t)\\ &= Ae(t) - BKe(t_i) + BKx_\Delta(t_i) \qquad (24)\\ \text{with} \quad & e(0) = 0\\ \text{and} \quad & x_\Delta(t) := x(t) - x_c(t) \qquad (25)\end{aligned}$$

This yields the upper bound of the error e as follows

$$\|e(t)\| \leq \int_0^\infty \|e^{As}BK\| ds\, d_{max} \qquad (26)$$

where d_{max} is the disturbance bound defined in (14), since the feedback matrix K defined in (5) renders the "undisturbed" approximation error dynamics (24) asymptotically stable (where x_Δ can be seen as the disturbance), and so becomes null the first right-hand term in (24). Since the uncorrected continuous-time state-feedback system (15) is GUUB and $\|e(t)\|$ in (26) is upper-bounded, one can conclude the proposed event-based state-feedback control with event-based correction is GUUB.

Also, the MSI property was demonstrated in (Marchand *et al.*, 2011) for the control case. Let calculate the minimal sampling time for the corrector case. From (13), (16), (20), the dynamics of the difference between the real (disturbed) system and the undisturbed one, previously denoted x_Δ in (25), is

$$\dot{x}_\Delta(t) = Ax_\Delta(t) + Ed(t)$$

which solution on the time interval $t \in [t_j, t_{j+1}[$ is

$$x_\Delta(t) = e^{A(t-t_j)}x_\Delta(t_j) + \int_{t_j}^t e^{A(t-s)}Ed(s)ds$$

Then, as no correction's event should be enforced according to (17), the inequality $\|x_\Delta(t)\| < \bar{e}$ has to hold for all $t \in [t_j, t_{j+1}[$. This yields

$$\left\| \int_{t_j}^t e^{A(t-s)}Ed(s)ds \right\| < \bar{e}$$

Then, an upper bound of the inter-sampling interval for which this inequality is satisfied is easily determined by

$$\begin{aligned}& t_{j+1} - t_j \geq \bar{\tau}_j\\ \text{with} \quad & \bar{\tau}_j = \arg\min_{\tau>0} \left\{ \int_0^\tau \left\|e^{As}E\right\| ds\, d_{max} = \bar{e} \right\} \qquad (27)\end{aligned}$$

This ends the proof. ■

Theorem 2.4 (Self-triggered framework's stability): Consider the event-based corrector (16)-(18). Consider the self-triggered feedback (20)-(22). Then, the state-feedback control loop for the disturbed system (13), which disturbance is bounded by (14), is uniformly MSI and GUUB.

Proof: We previously proved that the event-based state-feedback control with event-based correction and communication delays is GUUB. Also, we know that the control signal (20) is applied to the plant at the predicted activation time (21)-(22) in the self-triggered case. As a result, one can say the self-triggered controller operates like the event-based controller. Consequently, the proposed self-triggered state-feedback control with event-based correction is GUUB. The MSI property is trivial from the previous proof. ■

III. Simulation results

In this section, we test the proposal in simulation, using the *Matlab/Simulink* environment. The system is a simple double integrator, which matrices in (13) are given by

$$A = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, \quad B = \begin{bmatrix}0\\ 1\end{bmatrix}, \quad E = \begin{bmatrix}1\\ 0\end{bmatrix}, \quad x_0 = \begin{bmatrix}1\\ -3\end{bmatrix}$$

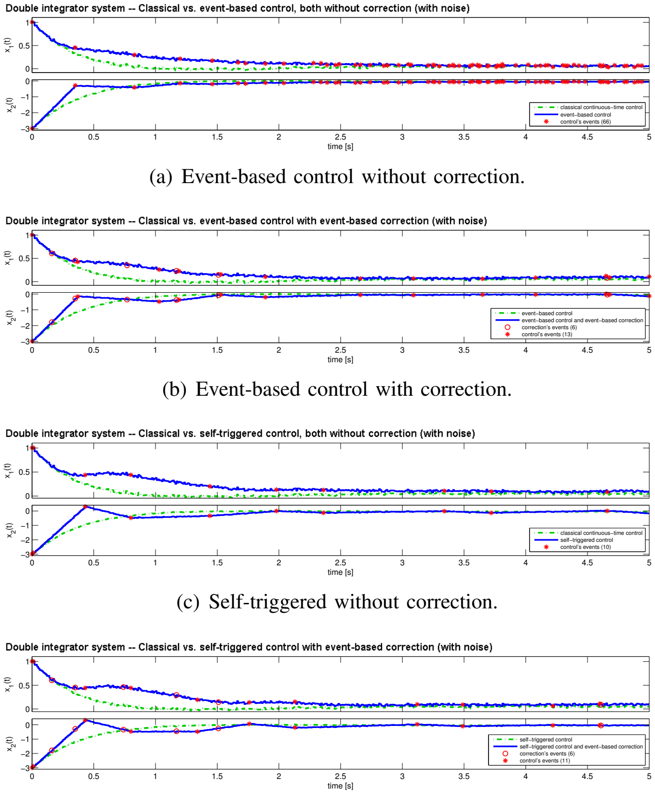
The control parameters to calculate K in (5) are

$$Q = \begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}, \quad \varepsilon = 1 \quad \text{and} \quad \sigma = 0.8$$

and the corrector parameter is $\bar{e} = 0.1$. Also, we consider a (randomly) varying disturbance which maximum value is $d_{max} = 0.1$.

Some simulation results of the system without correction and without noise can be found in (Durand *et al.*, 2012). To summarize, whereas the control signal is continuously updated in the classical setup, only few updates allow to stabilize the system in both event-based and self-triggered versions. The convergence rate is different but this can be tuned with the σ parameter in the event function (but this is not the aim of the current paper) and the difference between both asynchronous responses is due to the first-order approximation in the system trajectory (this can also be tuned applying a higher order of approximation in the system trajectory, but one has then to expect a higher computational cost of the sampling function in the self-triggered scheme in return).

Finally, we study the behavior of the asynchronous proposals with the event-based corrector in presence of noise. The system responses are depicted in Fig. 2. In particular, a reduction of about $75\,\%$ of updates is achieved in the event-based case when applying the event-based corrector (19 events which 6 for the correction mechanism, against 66 without correction). On the other hand, the self-triggered setup does not reduce the number of events in presence of noise (because this open-loop strategy does not detect anything between two events) but, at least, it now operates in closed loop thanks to the added event-based corrector (which is quite important for a disturbed system like in the present case). Moreover, the computational cost of the self-triggered technique is lower than the event-based one (by construction).



(a) Event-based control without correction.

(b) Event-based control with correction.

(c) Self-triggered without correction.

(d) Self-triggered with correction.

Fig. 2. Simulation results of the double integrator: event-based and self-triggered state-feedback control in presence of noise, with and without event-based correction.

Conclusion and future work

In this paper, we proposed to combine together i) an event-based/self-triggered control technique and ii) an event-based corrector for the stabilization of a disturbed linear system. We proved this framework is stable. Some simulation results were provided. They notably highlighted the low cost and robust properties of the proposals. They also show that the impact of noise in the frequency of updates is highly reduced (notably in the event-based case). Future work is to consider a Networked Control System, where communication delays have to be taken into account, and so is really interesting an asynchronous control to reduce the communications.

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