Congreso Nacional de Control Automático 2013
Ensenada, Baja California, Mexico, Octubre 16-18, 2013

Presence and elimination of chattering on fuzzy controllers designed following the fuzzy Lyapunov synthesis

Nohe R. Cazarez-Castro
and Selene L. Cardenas-Maciel
Instituto Tecnológico de Tijuana
Posgrado en Ciencias de la Ingeniería
Div. de Estudios de Posgrado e Investigación
Tijuana, B.C. 22500, México
nohe{selene}@ieee.org
Teléfono: (52)-664-6072942

Luis T. Aguilar
Instituto Politécnico Nacional
Tijuana, B.C. 22510, México
laguilarb@ipn.mx
Teléfono: (52)-664-6231344

Oscar Castillo
Instituto Tecnológico de Tijuana
Div. de Estudios de Posgrado e Investigación
Tijuana, B.C. 22500, México
ocastillo@tectijuana.mx
Teléfono: (52)-664-6827229

***Abstract*— This paper reports the presence of chattering in a closed-loop fuzzy control system where the controller was designed using the fuzzy Lyapunov synthesis. First, a fuzzy control system is directly designed following the fuzzy Lyapunov synthesis, where the existence of chattering effect is identified experimentally. Then, a chattering-free fuzzy control systems is proposed, where the fuzzy controller rule base structure is modified in order to eliminate the chattering effect while Lyapunov stability condition persists. The case of study is a servomechanism with nonlinear backlash.**

Keywords: Fuzzy control, Lyapunov stability, chattering, non-minimum phase.

I. Introduction

Fuzzy control is an excellent control strategy for nonlinear systems like robot manipulators, this technique that was and is hard criticized by traditional control experts because fuzzy control is widely considered a control strategy based on problem-solving approach while traditional control is considered a strategy based on stability testing approach. In (Margaliot y Langholz, 1999; Margaliot y Langholz, 2001) is reported the fuzzy Lyapunov synthesis as an alternative in the design process of fuzzy controllers, providing a qualitative analytical approach in the designing process. This approach was then reported in (Cazarez-Castro, 2009) and (Cazarez-Castro, 2011) for designing closed-loop fuzzy controllers for nonsmooth mechanical systems.

This paper is motivated by the robot manipulator depicted in Fig. 1, which is mainly applied in industrial process, (Cazarez-Castro, 2009) and (Cazarez-Castro, 2011). Robot manipulators usually have problems in its joints due to the presence of the backlash and friction. In this paper, we will focus in one joint of the robot manipulator of Fig. 1 only, and the problem is to solve the output regulation problem for the electrical actuator consisting of a motor part driven by a DC motor and a reducer part (load) operating under uncertainty conditions in the presence of nonlinear backlash effects. The objective is to drive the load to a desired position while providing the roundedness of the system motion and attenuating external disturbances. Because of practical requirements (see e.g., (Lagerberg y Egardt, 1999)), the motor's angular position is assumed to be the only information available for feedback.

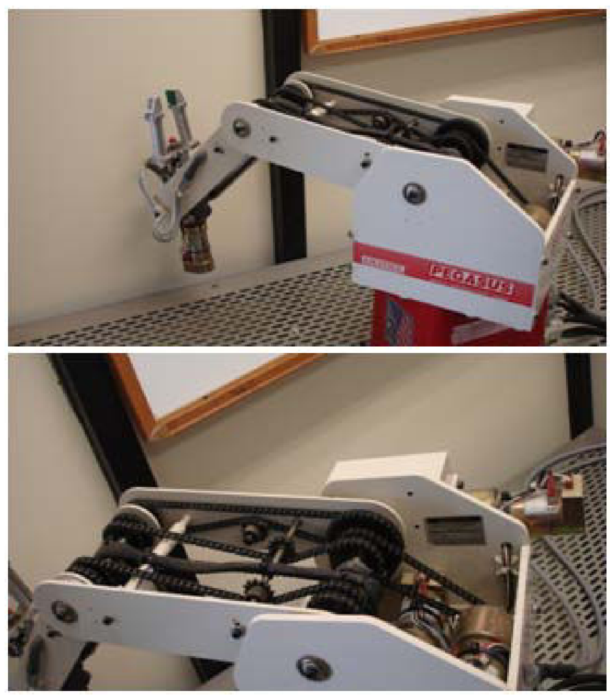

Figure 1. Robot Manipulator.

The rest of the paper is organized as follows. The dynamic model of the non-minimum phase servomechanism with nonlinear backlash and the problem statement are

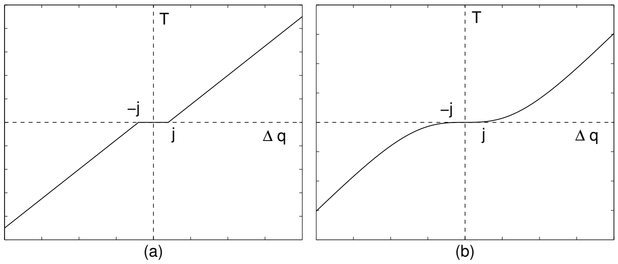

Figure 2. a) Dead-zone, b) monotonic approximation of backlash.

presented in Section II. Section III addresses fuzzy sets and systems theory. The design of Fuzzy Logic Controllers using the Fuzzy Lyapunov Synthesis is presented in Section IV including experimental results. A second, chattering free design including experimental results is presented in Section V. Conclusions are presented in Section VI.

II. Nonsmooth Mechanical System: A Servomechanism with Nonlinear Backlash

II-A. *Dynamic model*

The dynamic model of the angular position $q_i(t)$ of the DC motor and the $q_o(t)$ of the load are given according to

$$\begin{aligned} J_0N^{-1}\ddot{q}_0 + f_0N^{-1}\dot{q}_0 &= T + w_0, \\ J_i\ddot{q}_i + f_i\dot{q}_i + T &= \tau_m + w_i, \end{aligned} \quad (1)$$

hereafter, J_0, f_0, $\ddot{q}_0$ and $\dot{q}_0$ are, respectively, the inertia of the load and the reducer, the viscous output friction, the output acceleration, and the output velocity (Aguilar *et al.*, 2007). The inertia of the motor, the viscous motor friction, the motor acceleration, and the motor velocity are denoted by J_i, f_i, $\ddot{q}_i$ and $\dot{q}_i$, respectively. The input torque τ_m serves as a control action, and T stands for the transmitted torque. The external disturbances $w_i(t)$, $w_0(t)$ have been introduced into the driver equation (1) to account for destabilizing model discrepancies due to hard-to-model nonlinear phenomena, such as friction and backlash.

The transmitted torque T through a backlash with an amplitude j is typically modeled by a dead-zone characteristic (Nordin *et al.*, 2001, p. 7):

$$T(\Delta q) = \begin{cases} 0 & |\Delta q| \leq j \\ K\Delta q - Kj\mathrm{sign}(\Delta q) & \text{otherwise} \end{cases} \quad (2)$$

with

$$\Delta q = q_i - Nq_0, \quad (3)$$

where K is the stiffness, and N is the reducer ratio. Such a model is depicted in Fig. 2. Provided the servomotor position $q_i(t)$ is the only available measurement on the system, the above model (1)–(3) appears to be non-minimum phase because along with the origin the unforced system possesses a multivalued set of equilibria (q_i, q_0) with $q_i = 0$ and $q_0 \in [-j, j]$.

To avoid dealing with a non-minimum phase system, the backlash model (2) is replaced with its monotonic approximation:

$$T = K\Delta q - K\eta(\Delta q) \quad (4)$$

where

$$\eta = -2j\frac{1 - \exp\left\{-\frac{\Delta q}{j}\right\}}{1 + \exp\left\{-\frac{\Delta q}{j}\right\}}. \quad (5)$$

The present backlash approximation is inspired from (Merzouki *et al.*, 2004). Coupled to the drive system (1) subject to motor position measurements, it is subsequently shown to continue a minimum phase approximation of the underlying servomotor, operating under uncertainties $w_i(t)$, $w_0(t)$ to be attenuated. As a matter of fact, these uncertainties involve discrepancies between the physical backlash model (2) and its approximation (4) and (5).

II-B. *Problem Statement*

To formally state the problem, let us introduce the state deviation vector $x = [x_1, x_2, x_3, x_4]^T$ with

$$\begin{aligned} x_1 &= q_0 - q_d, \\ x_2 &= \dot{q}_0, \\ x_3 &= q_i - Nq_d, \\ x_4 &= \dot{q}_i, \end{aligned}$$

where x_1 is the load position error, x_2 is the load velocity, x_3 is the motor position deviation from its nominal value, and x_4 is the motor velocity. The nominal motor position Nq_d has been pre-specified in such a way to guarantee that $\Delta q = \Delta x$, where

$$\Delta x = x_3 - Nx_1.$$

Then, system (1)–(5), represented in terms of the deviation vector x, takes the form

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= J_0^{-1}[KNx_3 - KN^2x_1 - f_0x_2 + KN\eta(\Delta q) + w_o], \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= J_i^{-1}[\tau_m + KNx_1 - Kx_3 - f_ix_4 + K\eta(\Delta q) + w_i]. \end{aligned} \quad (6)$$

The zero dynamics

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= J_0^{-1}[-KN^2x_1 - f_0x_2 + KN\eta(-Nx_1)], \end{aligned} \quad (7)$$

of the undisturbed version of system (6) with respect to the output

$$y = x_3 \quad (8)$$

is formally obtained (see (Isidori, 1995) for details) by specifying the control law that maintains the output identically to zero. The following result, extracted from (Orlov, 2005), guarantees that the error system (6) and (8) is globally minimum phase.

The objective of the Fuzzy Control output regulation of the nonlinear driver system (1) with backlash (4) and (5), is thus to design a Fuzzy Controller so as to obtain the closed-loop system in which all these trajectories are bounded and the output $q_0(t)$ asymptotically decays to a desired position q_d as $t \to \infty$ while also attenuating the influence of the external disturbances $w_i(t)$ and $w_0(t)$.

III. Fuzzy Sets and Systems

A Type-1 Fuzzy Set (T1FS), denoted A is characterized by a Type-1 membership function (T1MF) $\mu_A(z)$ (Castillo y Melin, 2008), where $z \in Z$, and Z is the domain of definition of the variable, i.e,

$$A = \{(z, \mu(z)) | \forall z \in Z\} \tag{9}$$

where $\mu(z)$ is called a Type-1 *membership function* of the T1FS A. The T1MF maps each element of Z to a membership grade (or membership value) between 0 and 1.

Type-1 Fuzzy Logic Systems (T1FLS) - also called Type-1 Fuzzy Inference Systems (T1FIS)-, are both intuitive and numerical systems that map crisp inputs into a crisp output. Every T1FIS is associated with a set of rules with meaningful linguistic interpretations, such as:

$$R^l : \text{IF } y \text{ is } A_1^l \text{ AND } \dot{y} \text{ is } A_2^l \text{ THEN } u \text{ is } B_1^l, \tag{10}$$

which can be obtained either from numerical data, or from experts familiar with the problem at hand. In particular (10) is in the form of Mamdani fuzzy rules (Mamdani y Assilian, 1975)-(Mamdani, 1976). Based on this kind of statements, actions are combined with rules in an antecedent/consequent format, and then aggregated according to approximate reasoning theory to produce a nonlinear mapping from input space $U = U_1 \times U_2 \times \cdots \times U_n$ to output space W, where $A_k^l \subset U_k$, $k = 1, 2, \ldots, n$, and the output linguistic variable is denoted by τ_m.

A T1FIS consists of four basic elements (see Fig. 3): the *Type-1 fuzzifier*, the *Type-1 fuzzy rule-base*, the *Type-1 inference engine*, and the *Type-1 defuzzifier*. The *Type-1 fuzzy rule-base* is a collection of rules in the form of (10), which are combined in the *Type-1 inference engine*, to produce a fuzzy output. The *Type-1 fuzzifier* maps the crisp input into a T1FS, which are subsequently used as inputs to the *Type-1 inference engine*, whereas the *Type-1 defuzzifier* maps the T1FSs produced by the *Type-1 inference engine* into crisp numbers.

In this paper, to get the crisp output of Fig. 3, a Centroid of Area (COA) (Castillo y Melin, 2008) is computed as a *Type-1 defuzzifier*. The COA is defined as follows:

$$\tau_m = u_{COA} = \frac{\int_u \mu_A(u) u du}{\int_u \mu_A(u) du} \tag{11}$$

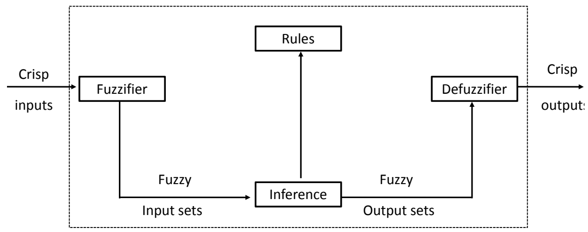

Figure 3. Type-1 Fuzzy Inference System.

where $\mu_A(u)$ is the aggregated output T1MF. This is the most widely adopted defuzzification strategy, which is reminiscent of the calculation of expected values of probability distributions.

IV. Direct fuzzy controller via fuzzy Lyapunov synthesis

IV-A. *First design*

To apply the Fuzzy Lyapunov Synthesis, the following is assumed:

1. The system may have really two degrees of freedom referred to as x_1 and x_2, respectively. Hence by (6), $\dot{x}_1 = x_2$.
2. The states x_1 and x_2 are the only measurable variables.
3. The exact equations (1)–(5) are not necessarily known.
4. The angular acceleration $\dot{x}_2$ is proportional to τ_m, that is, when τ_m increases (decreases) $\dot{x}_2$ increases (decreases).
5. The initial conditions $x(0) \in \mathbf{R}^2$ belong to the set $\mathcal{N} = \{x \in \mathbf{R}^2 : \|x - x^*\| \leq \varepsilon\}$ where x^* is the equilibrium point.

The *control objective* is to design the rule-base as a fuzzy controller $\tau_m = \tau_m(x_1, x_2)$ to stabilize the system (1)–(5).

Theorem 1 that follows establish conditions that allows the design of the fuzzy controller ensuring asymptotic stability. The proof can be found in (Khalil, 2002).

Theorem 1 (Asymptotic stability (Khalil, 2002)): Consider the nonlinear system (1)–(5) with an equilibrium point at the origin, i.e., $f(0) = 0 \in \mathbf{R}^4$, and let $x \in \mathcal{N}$, then the origin is asymptotically stable if there exists a scalar Lyapunov function $V(x)$ with continuous partial derivatives such that

- $V(x)$ is positive definite
- $\dot{V}(x)$ is negative definite.

The fuzzy controller design proceeds as follows. Let us introduce the Lyapunov function candidate

$$V(x_1, x_2) = \frac{1}{2}\left(x_1^2 + x_2^2\right), \tag{12}$$

TABLE I

FIRST DESIGN FUZZY IF-THEN RULES

No.	error	change of error	control
1	positive	positive	negative big
2	negative	negative	positive big
3	positive	negative	zero
4	negative	positive	zero

which is positive-definite and radially unbounded function. The time derivative of $V(x_1, x_2)$ results in:

$$\dot{V}(x_1, x_2) = x_1\dot{x}_1 + x_2\dot{x}_2 = x_1x_2 + x_2\dot{x}_2, \qquad (13)$$

To guarantee stability of the equilibrium point $(x_1^*, x_2^*)^T = (0, 0)^T$ it is necessary to have:

$$x_1x_2 + x_2\dot{x}_2 \leq 0. \qquad (14)$$

Now, we can now derive sufficient conditions so that inequality (14) holds: If x_1 and x_2 have opposite signs, then $x_1x_2 < 0$ and (14) will hold if $\dot{x}_2 = 0$; if x_1 and x_2 are both positive, then (14) will hold if $\dot{x}_2 < -x_1$; if x_1 and x_2 are both negative, then (14) will hold if $\dot{x}_2 > -x_1$.

The above conditions can be translated into the following fuzzy rules:

- If x_1 is *positive* and x_2 is *positive* then $\dot{x}_2$ must be *negative big*.
- If x_1 is *negative* and x_2 is *negative* then $\dot{x}_2$ must be *positive big*.
- If x_1 is *positive* and x_2 is *negative* then $\dot{x}_2$ must be *zero*.
- If x_1 is *negative* and x_2 is *positive* then $\dot{x}_2$ must be *zero*.

However, using our knowledge that $\dot{x}_2$ is proportional to u, each $\dot{x}_2$ can be replaced with u to obtain the following fuzzy rule-base for the stabilizing controller:

- If x_1 is *positive* and x_2 is *positive* then u must be *negative big*.
- If x_1 is *negative* and x_2 is *negative* then u must be *positive big*.
- If x_1 is *positive* and x_2 is *negative* then u must be *zero*.
- If x_1 is *negative* and x_2 is *positive* then u must be *zero*.

This fuzzy rule-base can be represented as in Table I.

It is interesting to note that the fuzzy partitions for x_1, x_2, and u follow elegantly from expression (13). Because $\dot{V} = x_2(x_1 + \dot{x}_2)$, and since is required that $\dot{V}$ be negative, it is natural to examine the signs of x_1 and x_2; hence, the obvious fuzzy partition is *positive*, *negative*. The partition for $\dot{x}_2$, namely *negative big*, *zero*, *positive big* is obtained similarly when plug the linguistic values *positive*, *negative* for x_1 and x_2 in (13). To ensure that $\dot{x}_2 < -x_1$ ($\dot{x}_2 > -x_1$) is satisfied even though do not know x_1's exact magnitude, only that it is *positive* (*negative*), must set $\dot{x}_2$ to *negative big* (*positive big*). Obviously, it is also possible to start with a given, pre-defined, partition for the variables and then plug each value in the expression for $\dot{V}$ to find the rules. Nevertheless, regardless of what comes first, see that fuzzy Lyapunov synthesis transforms classical Lyapunov synthesis from the world of exact mathematical quantities to the world of computing with words (Zadeh, 1996), (Mendel, 2007).

To complete the controller's design, must model the linguistic terms in the rule-base using fuzzy membership functions and determine an inference method. Following (Castillo *et al.*, 2008), characterize the linguistic terms *positive*, *negative*, *negative big*, *zero* and *positive big*. The T1MFs are depicted in Fig. 4. To this end, had systematically designed a FLC following the Lyapunov stability criterion.

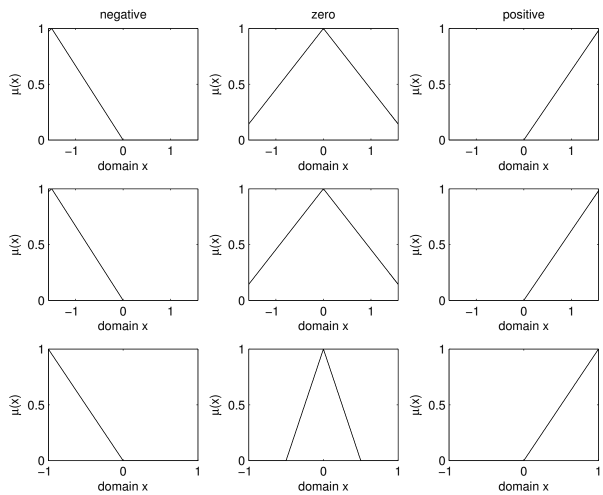

Figure 4. Set of type-1 membership functions (x_1, x_2 and u).

TABLE II

NOMINAL PARAMETERS.

Description	Notation	Value	Units
Motor inertia	J_i	2.8×10^{-6}	Kg-m^2
Load inertia	J_o	1.07	Kg-m^2
Motor viscous friction	f_i	7.6×10^{-7}	N-m-s/rad
Load viscous friction	f_o	1.73	N-m-s/rad

IV-B. *First results*

To perform experiments is used the dynamical model (1)–(5), which involves a DC motor linked to a mechanical load through an imperfect contact gear train (Aguilar *et al.*, 2007). The parameters of the dynamical model (1)–(5) are in Table II, while $N = 3$, $j = 0{,}2$ rad, and $K = 5$ N-m/rad.

Applying this fuzzy controller to the proposed problem, through experiments is obtained the system's response trajectories of Figs. 5 and 6, that is, $q_0 \to q_d$ while $x_1 \to 0$. Fig. 7 shows the control signal provided by the fuzzy controller, it is important to note that this signal have the chattering effect.

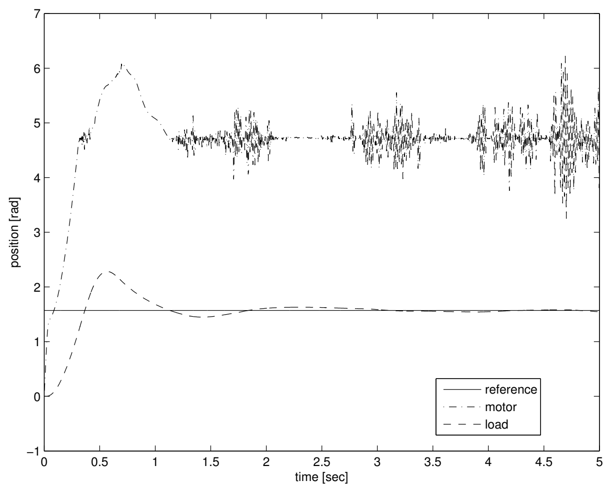

Figure 5. First system's response for the experiment.

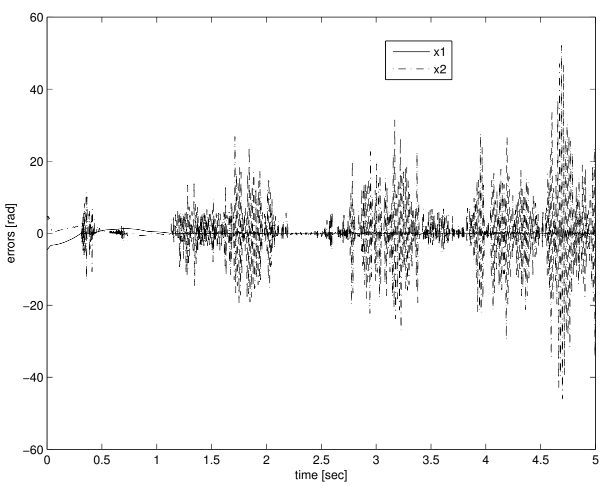

Figure 6. First x_1 and x_2 trajectories.

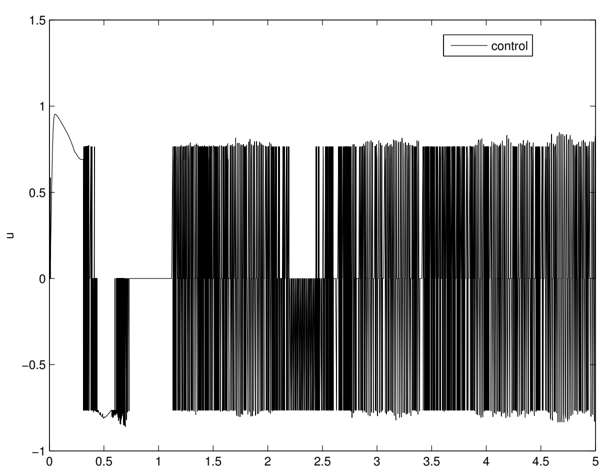

Figure 7. First control signal for the experiment with $j = 0{,}2$ and N=3.

TABLE III

SECOND DESIGN FUZZY IF-THEN RULES

No.	error	change of error	control
1	positive	positive	negative big
2	negative	negative	positive big
3	positive	negative	zero
4	negative	positive	zero
5	negative	zero	positive
6	positive	zero	negative

V. CHATTERING-FREE FUZZY LOGIC CONTROLLER

V-A. *Second design*

Figure 7 confirms the chattering effect is present in the first closed loop system designed following the fuzzy Lyapunov synthesis, and it is important to make and extension of the first design in order to avoid the chattering effect while the closed loop system guarantee stability.

The controller proposed in Section IV guarantees that (13) is semi-negative definite, but the designed fuzzy logic controller solve the problem under chattering effect, this is because there is no junction between the *negative* and *positive* membership functions for the variable x_2. Now it is necessary to solve this problem without altering the fact that (13) is negative semidefinite in (14), and to do this we include the following rules:

- If x_1 is *negative* and x_2 is *zero* then u must be *positive*.
- If x_1 is *positive* and x_2 is *zero* then u must be *negative*.

This rules added to the fuzzy rule-base of Table I, result on the fuzzy rule-base of Table III, and all considerations exposed in Section IV remains, and the set of membership functions remains like those shown on Fig. 4.

V-B. *Second results*

Experiments was performed as in Section IV. Applying this fuzzy controller to the proposed problem, experiments gives the system's response trajectories of Figs. 8 and 9, from experiments, that is, $q_0 \to q_d$ while $x_1 \to 0$. Fig. 10 shows the control signal provided by the fuzzy controller, here it is important to note that this signal do not presents the chattering effect.

VI. CONCLUSION

The main goal of this paper was to exhibit the presence of chattering on fuzzy controllers designed using the fuzzy Lyapunov synthesis. A change in the systematic methodology to design the fuzzy logic controller avoid the presence of chattering at the output of a servomechanism with nonlinear backlash. The proposed design strategy results in controllers that guarantee that the load reaches the desired position while the control inputs do not present chattering. The regulation problem was solved as was predicted, this affirmation is supported with experiments.

ACKNOWLEDGMENTS

Authors would like to thank the support given by project TIJ-ING-2012-120 and DGEST 2012 "Observación y control de sistemas no lineales".

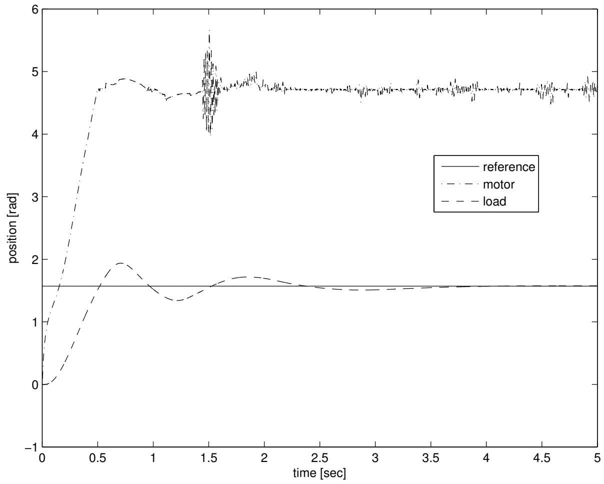

Figure 8. Second system's response for the experiment.

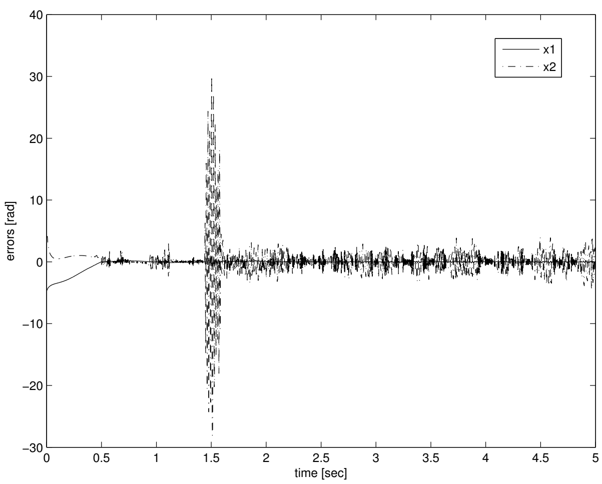

Figure 9. Second x_1 and x_2 trajectories for the experiment.

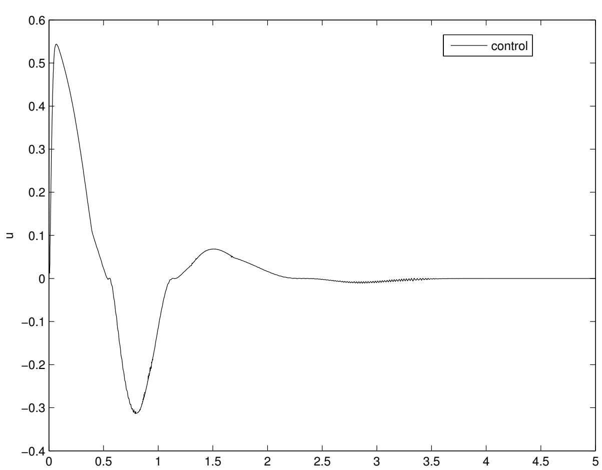

Figure 10. Second control signal for the experiment.

REFERENCES

Aguilar, L. T., Y. Orlov, J. C. Cadiou y R. Merzouki (2007). Nonlinear H_∞-output regulation of a nonminimum phase servomechanism with backlash. *J. Dyn. Syst. Meas. Contr.* **129**(4), 544–549.

Castillo, O. y P. Melin (2008). *Type-2 Fuzzy Logic: Theory and Applications*. Vol. 223. 1st ed. Springer-Verlag. Heidelberg, Germany. Book (ISBN: 978-3-540-76283-6).

Castillo, Oscar, Luis Aguilar, Nohe Cazarez y Selene Cardenas (2008). Systematic design of a stable type-2 fuzzy logic controller. *Applied Soft Computing* **8**(3), 1274 – 1279.

Cazarez-Castro, NR., Aguilar Bustos LT. Castillo Lopez O. (2011). Designing type-1 fuzzy logic controllers via fuzzy lyaupnov synthesis for nonsmooth mechanical systems: The perturbed case. *Computacion y sistemas* **14**(3), 283–293.

Cazarez-Castro, NR., Aguilar Bustos LT. Castillo Lopez O., Ed. (2009). *Designing Type-1 Fuzzy Logic Controllers via Fuzzy Lyapunov Synthesis for Nonsmooth Mechanical Systems*. AMCA.

Isidori, Alberto (1995). *Nonlinear Control Systems*. Springer-Verlag New York, Inc.. Secaucus, NJ, USA.

Khalil, Hassan K. (2002). *Nonlinear Systems*. 3th ed. Prentice Hall.

Lagerberg, A. y B. Egardt (1999). Estimation of backlash with application to automotive powertrains. *Proc. of the 42th Conf. on Decision and Control* pp. 135–147.

Mamdani, E.H. (1976). Advances in the linguistic synthesis of fuzzy controllers. *J. Man. Mach. Stud.* **8**, 669–678.

Mamdani, E.H. y S. Assilian (1975). An experiment in linguistic synthesis with fuzzy logic controller. *J. Man. Mach. Stud.* **7**(1), 1–13.

Margaliot, Michael y Gideon Langholz (1999). Fuzzy lyapunov-based approach to the design of fuzzy controllers. *Fuzzy Sets and Systems* **106**(1), 49 – 59. ¡ce:title¿Fuzzy Modeling and Dynamics¡/ce:title¿.

Margaliot, Michael y Gideon Langholz (2001). Design and analysis of fuzzy schedulers using fuzzy lyapunov synthesis. *Engineering Applications of Artificial Intelligence* **14**(2), 183 – 188.

Mendel, J.M. (2007). Computing with words: Zadeh, turing, popper and occam. *IEEE Comput. Intell. Mag.* **2**(4), 10–17.

Merzouki, R., J.C. Cadiou y N.K. M'Sirdi (2004). Compensation of friction and backlash effects in an electrical actuator. *J. Syst. and Cont. Eng.* **218**(12), 75–84.

Nordin, M., P. Bodin y P.O. Gutman (2001). *Adaptive Control of Nonsmooth Dynamic Systems*. Cap. New Models and Identification Methods for Backlash and Gear Play, pp. 1–30. Springer. Berlin.

Orlov, Y. (2005). Finite time stability and quasihomogeneous control synthesis of uncertain switched systems with application to underactuated manipulators. En: *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC '05. 44th IEEE Conference on*. pp. 4566–4571.

Zadeh, L.A. (1996). Fuzzy logic = computing with words. *IEEE Trans. Fuzzy Syst.* **4**(2), 103–111.