A coordination control strategy for a group of unicycle robots

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Abstract

Coordinated tracking control for a group of unicycle robots is investigated. To this end a nonlinear coordination controller and the subsequent Lyapunov asymptotic stability analysis are proposed. The results are validated in experiments using a group of 3 mobile robots.

Keywords: Nonlinear control, unicycle robots, coordination.

1 Introduction

It is not surprising that a group of robots can execute more tasks and increase the maneuverability than a sole element can do. For this reason, coordination control of a group of multiple vehicles is an intensive research field that has been developed in the last years. This encompasses usage of cooperative robots, mobile robots, spacecraft, aircraft, underwater vehicles, among others [1]-[5]. Particularly, for mobile robotics various applications can be encountered e.g. simultaneous localization and mapping, exploration of unknown environments, automated highway systems, among others, (see [6]-[8]). Formation control for a group of unicycle type robots has been studied in [9]-[15], and several approaches have been developed to solve problems that comprise formation control with saturations constraints in control signals (see [11], [14]), coordination control with collision avoidance (see [14], [16]), and also using a virtual structure approach ([12], [13], [15]). All these approaches are capable to keep the formation structure and/or also trajectory tracking of all robots. In some cases, some of them are capable to drive the formation to the desired trajectories even in case external disturbances are presented [15].

The main contribution in this paper is a control algorithm that ensures joint trajectory tracking for a group of unicycle robots. The proposed control guarantees asymptotic stability, which is proven via Lyapunov like method, and which is successfully validated in experiments. Different from the control proposed in [15] where the x, y, θ coordinates are coupled, the proposed controller only requires coupling through the x and y coordinates. Also, different from the control proposed in [14] the coupling errors are simplified.

This paper is organized as follows. Section 2 describes previous work and the problem state-

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ment. In Section 3 the main result is showed. In Section 4 simulations and experimental results that validate the proposed control algorithm are presented. And Section 5 provides some conclusions and future work.

2 Controller design

As in [14] and [15], we consider a group of unicycle mobile robots that have to mantain a desired shape meanwhile each robot follows a desired trajectory. To this end a nonlinear control approach, where all the robots are tracked to their desired trajectory, is considered, this allows to develop trajectories that fulfill the nonholonomic side-slip constraint that unicycle robots posses. The coupling between the robots is defined using their planar coordinates to develop different trajectories.

A group of n unicycles mobile robots is considered. The kinematic model of each robot is described by the following non-holomic model [17]

$$\begin{aligned} \dot{x}_i &= v_i cos\theta_i \\ \dot{y}_i &= v_i sin\theta_i \\ \dot{\theta}_i &= \omega_i \end{aligned} \quad (1)$$

for $i = 1, \ldots, n$, where x_i and y_i are the planar coordinates of the robot i, v_i and ω_i are the linear and angular velocities, respectively, and θ_i is the heading angle relative to the horizontal axis of the reference frame. System (1) possesses a non-holonomic, no-side-slip constraint $\dot{x}_i sin\theta_i - \dot{y}_i cos\theta_i = 0$. According to a reference frame and by considering a desired posture (see Figure 1), the tracking error is defined as follows:

$$\begin{aligned} x_{ei} &= cos\theta_i(x_{ri} - x_i) + sin\theta_i(y_{ri} - y_i) \\ y_{ei} &= -sin\theta_i(x_{ri} - x_i) + cos\theta_i(y_{ri} - y_i) \\ \theta_{ei} &= \theta_{ri} - \theta_i \end{aligned} \quad (2)$$

where the reference coordinates are x_{ri}, y_{ri}, and θ_{ri}.

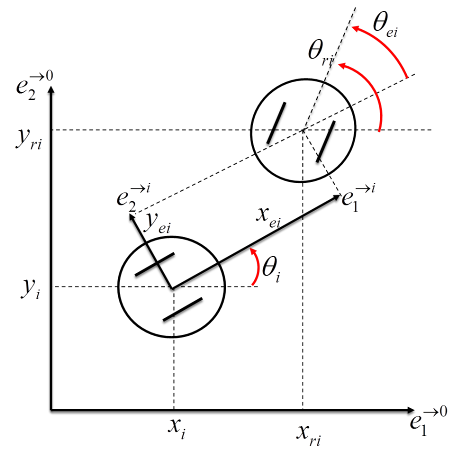

Figure 1: The unicycle actual and desired coordinates (1)

The error dynamics is given by:

$$\begin{aligned} \dot{x}_{ei} &= \omega_i y_{ei} - v_i + v_{ri} cos\theta_{ei} \\ \dot{y}_{ei} &= -\omega_i x_{ei} + v_{ri} sin\theta_{ei} \\ \dot{\theta}_{ei} &= \omega_{ri} - \omega_i \end{aligned} \quad (3)$$

Feasible desired trajectories for each robot i fulfill the nonholonomic constraint, *i.e.* $\dot{x}_i sin\theta_i - \dot{y}_i cos\theta_i = 0$, therefore for $\dot{x}_{ri} \neq 0$ and $\dot{y}_{ri} \neq 0$, the reference linear and angular velocities derived from the reference trajectory are:

$$\begin{aligned} v_{ri} &= \sqrt{\dot{x}_{ri}^2 + \dot{y}_{ri}^2} \\ \omega_{ri} &= \frac{\dot{x}_{ri}\ddot{y}_{ri} - \ddot{x}_{ri}\dot{y}_{ri}}{v_{ri}^2} \end{aligned} \quad (4)$$

The formation aim is defined as cooperative tracking, that is each robot tracks its own reference trajectory and by requiring that this is done in a balanced manner, the coupling errors are defined as:

$$\begin{pmatrix} x_{ei} - x_{ej} \\ y_{ei} - y_{ej} \end{pmatrix} \to \mathbf{0}, \quad \forall i, j \in (1, 2, \ldots, n), i \neq j \quad (5)$$

It is desired that each robot tracks its reference trajectory, i.e. $x_{ei}, y_{ei}, \theta_{ei} \to 0$, and complying to a defined shape, i.e. each trajectory is designed to keep an equal distance between the robots. To achieve above mentioned aims the following control laws for the linear and angular velocities are proposed.

$$\begin{aligned} v_i &= v_{ri} cos\theta_{ei} + \\ & C_{ix}\left[x_{ei} + \sum_{j=1, j\neq i}^{n} C_{ijx}(x_{ei} - x_{ej})\right] \\ \omega_i &= \omega_{ri} + C_{i\theta}\theta_{ei} + vr_i \frac{sin\theta_{ei}}{\theta_{ei}}\frac{K}{\alpha_i}C_{iy} \\ & \left[y_{ei} + \sum_{j=1, j\neq i}^{n} C_{ijy}(y_{ei} - y_{ej})\right] \end{aligned} \quad (6)$$

Where C_{ix}, C_{ijx}, C_{iy}, C_{ijy}, $C_{i\theta}$ and K are the control gains, and the term α_i is provided to bound the effect of the tracking and coupling errors and is defined as follows,

$$\alpha_i = \sqrt{K^2 + x_{ei}^2 + y_{ei}^2 + \beta_{ij}} \quad (7)$$

$$\beta_{ij} = \sum_{j=1, j\neq i}^{n} (x_{ei} - x_{ej})^2 + \sum_{j=1, j\neq i}^{n} (y_{ei} - y_{ej})^2 \quad (8)$$

Remark. It can be noticed that in the proposed controllers (see (6)), there is a compromise between the tracking reference trajectory errors $x_{ei}, y_{ei}, \theta_{ei}$ and the formation errors $[(x_{ei} - x_{ej}), (yei - y_{ej})]$, meaning that through the control gains $C_{ix}, C_{iy}, C_{i\theta}$ the tracking aim is accomplished, whereas the formation aim is affected for both gains, C_{ijx}, C_{ix} and C_{ijy}, C_{ixy}, for each x_i and y_i, respectively.

Although the controller (6) is quite similar to that presented in [14], we want to highlight its main differences as follows:

- The coupling errors (see (5)) are not the same, here these are are defined as $[(x_{ei} - x_{ej}), (yei - y_{ej})]$ meanwhile in [14] they are defined as $\mathbf{R}^T(\theta_i + \theta_j)\varepsilon_{ij}$.
- We are not using saturation functions, and
- Finally , in the term α_i (see (8)), it can be appreciated that only the tracking error in the planar coordinates x_i, y_i is considered, and the coupling errors are not affected by any gain of the form $l_{i,j}$.

The design parameters C_{ix}, C_{ix} and $C_{i\theta}$ are the control gains that are responsible for tracking the reference trajectories of each robot. The terms C_{ijx} and C_{ijy} are the coupling gains, and they affect the coordination of the robots to the formation. The gain K normalizes the term $\frac{K}{\alpha_i}$ to 1 when the tracking trajectory error and the coupling error are zero, otherwise, bounds the term $[y_i + \sum_{j=1, j\neq i}^{n} C_{ijy}(y_{ei} - y_{ej})]$ in the computation of the angular velocity ω_i in (6).

3 Main result

The stability properties of control law (6) in closed-loop with system (1) are provided through Lyapunov stability analysis in the following theorem.

Theorem 3.1 *(Formation control). Consider the system described by (1) with $i = 1, \dots, n$ and the control (6). Assuming that the desired trajectories are provided by (4) that fulfill the non-holonomic constraint $\dot{x}_i sin\theta_i - \dot{y}_i cos\theta_i = 0$. Moreover, assume that the control parameters fulfill:*

$$C_{ix}, C_{ix}, C_{i\theta}, C_{ijx}, C_{ijy}, K \in \mathbb{R}_+ \quad (9)$$

Then, the tracking error dynamics (3) for each robot $i = 1, \dots, n$ are global asymptotically stable at zero. As a consecuence the coupling errors (5) converge to **0** *as well.*

Proof: Consider the positive definite and proper Lyapunov function candidate as:

$$V = \frac{1}{2}\left[\sum_{i=1}^{n} x_{ei}^2 + \sum_{i=1}^{n} y_{ei}^2 + \sum_{i=1}^{n} \theta_{ei}^2\right] \quad (10)$$

We obtain the time derivative of the proposed Lyapunov (10) function along the solutions of the closed-loop system (3)-(6),

$$\dot{V} = -[\mathbf{C}^{\theta}\theta_e^T\theta_e + \mathbf{X}_e\mathbf{C}^x x_e + \mathbf{\Gamma}_{v_r sin\theta}\mathbf{C}^y y_e] \leq 0 \quad (11)$$

where $x_e = [x_{e1}, \ldots, x_{en}]^T$, $y_e = [y_{e1}, \ldots, y_{en}]^T$, and $\theta_e = [\theta_{e1}, \ldots, \theta_{en}]^T$, are the error vectors, meanwhile $\mathbf{X}_e = \text{diag}[x_{e1}, \ldots, x_{en}]$. $\mathbf{C}^{\theta} = \text{diag}[C_{1\theta}, \ldots, C_{\theta n}]$, $\mathbf{\Gamma}_{v_r sin\theta} = \text{diag}[v_{r1} sin\theta_{e1}\frac{K}{\alpha_1}, \cdots, v_{rn} sin\theta_{en}\frac{K}{\alpha_n}]$ and matrices $\mathbf{C}^x$, $\mathbf{C}^y$ are shown in equations (18)-(20).

From (11) we have the following facts:

- $\mathbf{\Gamma}_{vr_i sin\theta}$ vanishes along the trajectories because
$$\lim_{t\to 0} v_{ri} sin\theta_{ei}\frac{K}{\alpha_i} = 0$$
- $\mathbf{C}^x$ is a positive definite matrix, due the fact that C_{ix} and C_{ijx} are definite positive with dominant terms in the diagonal, which by definition are positive, moreover, $\pi_{ix} > C_{ix}C_{ijx}$, therefore, $\mathbf{C}^x > 0$. For this reason the term $C_x(\pi_{ix})x_e^T x_e \geq \mathbf{X}e\mathbf{C}^x x_e$, with $C_x(\pi_{ix}) = diag(\pi_{1x}, \ldots, \pi_{nx})$. Moreover, if all the coupling gains and the tracking gains are chosen to be equal for the n robots, i.e., $C_{ijx} = C_{ijy}$ and $C_{ix} = C_{iy}$ with $i, j = 1, \ldots, n$ and $j \neq i$ it is ensured that matrix $\mathbf{C}^x$ is symmetric and therefore positive definite due to the dominant diagonal terms. In contrary case we always can ensure that the diagonal terms are higher than the others.
- The remaining term in (11), $\mathbf{C}^{\theta}\theta_e^T\theta_e$ is definite positive.

Due to the facts above it can be deduced that the Lyapunov function (10) is positive definite and radially unbounded, and its time derivative (11) negative semi-definite along the trajectories as:

$$\dot{V} \leq -\left[\mathbf{C}^{\theta}\theta_e^T\theta_e + C_x(\pi_{ix})x_e^T x_e\right] \leq 0 \quad (12)$$

From the inequality (12) we have that

$$0 \geq \int_o^{\infty} dV(t) \geq -\Big\{\int_0^{\infty} \mathbf{C}^{\theta}\theta_e^T\theta_e dt + \int_0^{\infty} C_x(\pi_{ix})x_e^T x_e dt\Big\} \quad (13)$$

Since $V > 0$ outside $x = \mathbf{0}$, and $\mathbf{C}^{\theta}\theta_e^T\theta_e$, $C_x(\pi_{ix})x_e^T x_e$ are positive functions, and also considering that system (3) is uniformly continuous in $[0, \infty)$, with Barbalat's lemma we get:

$$\lim_{t\to 0}\left(\mathbf{C}^{\theta}\theta_e^T\theta_e + C_x(\pi_{ix})x_e^T x_e\right) = 0 \quad (14)$$

which implies that

$$\lim_{t\to 0}\Big(||\theta_e|| + ||x_e||\Big) = 0 \quad (15)$$

From the closed-loop dynamics of $\dot{\theta}_e$ with controller (6) we have that:

$$\dot{\theta}_e = -\mathbf{C}^{\theta}\theta_e - \Gamma\mathbf{C}^y y_e \quad (16)$$

where $\mathbf{\Gamma} = \text{diag}[v_{r1}\frac{sin\theta_{e1}}{\theta_{e1}}, \ldots, v_{rn}\frac{sin\theta_{en}}{\theta_{en}}]$. Therefore, since from (15) $\theta_e = [\mathbf{0}]$ and by (16) we have that

$$\lim_{t\to 0}\Gamma\mathbf{C}^y y_e \to 0 \quad (17)$$

The only possible solutions implies that $y_e \to 0$, then, it can be concluded that the control law (6) in closed loop with the error dynamic system (3) drives the trajectory errors (2) to zero. Hence, the original system composed by n robots (1) reaches the desired trajectories and the coupling error ε_{ij} is driven to $\mathbf{0}$, where each robots keeps a desired shape, therefore, the origin is asymptotically stable. ■

4 Experimental results

To see the performance of controller (6) an experiment was developed with three e-puck mobile robots [18]. The e-puck robot has two driven wheels, which are individualy actuated by means of stepper motors. Two cameras are used for acquiring position and orientation of all robots and a PC. The PC generates the robot trajectories, process images to get the actual pose of the robots, and runs control laws for all the robots. The length, width and height of the arena are 2.2, 3.2 and 2.3 meters, respectively. The control velocities are sent from the PC to the robots via BlueTooth protocol. The control gains used are $C_{ijx} = C_{jix} = C_{ijy} = C_{jiy} = 1$, $C_{ix} = 2$, $C_{iy} = 100$ and $C_{i\theta} = 0.5$. In Figure 2 the shape of a circular trajectory of radius $r = 0.5$ meters for all the robots is presented and a disturbance is made in robot $i = 2$ which is moved away from its trajectory. It can be seen that despite this disturbance, all robots recover their desired trajectories. The forward and angular velocities, v_i and ω_i, respectively, which are the control inputs are shown in Figures 3-4, whereas the tracking errors are depicted in Figure 5.

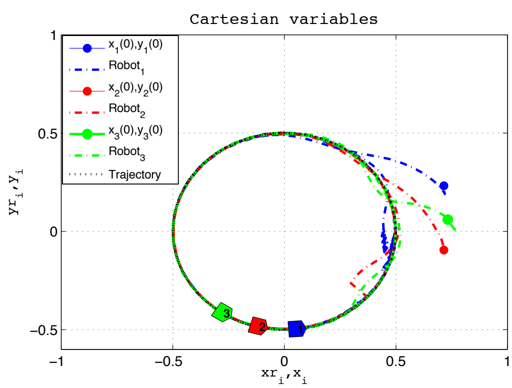

Figure 2: Formation and tracking trajectory for $i = 3$ using control(6).

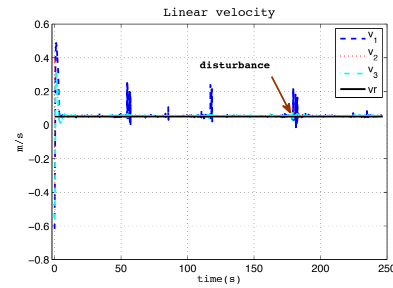

Figure 3: Linear velocity v_i.

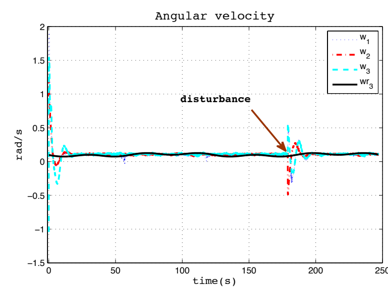

Figure 4: Angular velocity ω_i.

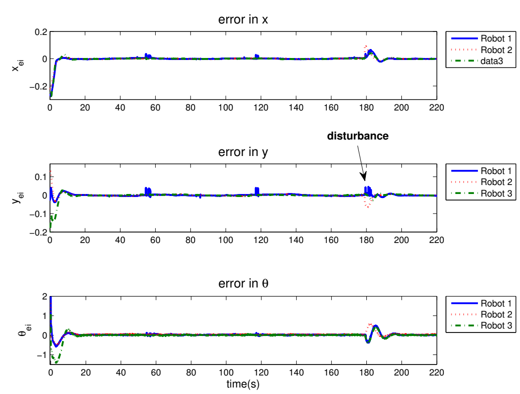

Figure 5: Tracking errors.

5 Conclusions

A nonlinear coordination controller for a group of unicycle robots is presented and its performance is demonstrated by experiments. Global asymptotic stability of the closed loop system with the proposed controller is proven by using Lyapunov methods. As future work the incorporation of additional robots to the formation can be considered.

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$$\mathbf{C}^x = \begin{pmatrix} \pi_{1x} & -C_{1x}C_{1jx} & \cdots & -C_{1x}C_{1jx} \\ -C_{2x}C_{2jx} & \pi_{2x} & \cdots & -C_{2x}C_{2jx} \\ \vdots & \vdots & \ddots & 0 \\ -C_{nx}C_{njx} & \cdots & -C_{nx}C_{njx} & \pi_{nx} \end{pmatrix} \quad (18)$$

$$\mathbf{C}^y = \begin{pmatrix} \pi_{1y} & -C_{1y}C_{1jy} & \cdots & -C_{1y}C_{1jy} \\ -C_{2y}C_{2jy} & \pi_{2y} & \cdots & -C_{2y}C_{2jy} \\ \vdots & \vdots & \ddots & \vdots \\ -C_{ny}C_{njy} & \cdots & -C_{ny}C_{njy} & \pi_{ny} \end{pmatrix} \quad (19)$$

with

$$\begin{aligned} \pi_{ix} &= (1+(n-1)C_{ijx})C_{ix}, \\ \pi_{iy} &= (1+(n-1)C_{ijy})C_{iy} - \frac{\alpha_i}{KC_{iy}} \\ for\ i,j &= 1,\ldots,n \quad j \neq i. \end{aligned} \quad (20)$$