Robust stability and robust performance of anaerobic digester for wastewater treatment via H_∞ control

R. Flores-Estrella[a], G. Quiroz[b], R. Femat*[a]
[a]IPICyT, División de Matemáticas Aplicadas
Camino a la presa de San José Apdo. Postal 2055, Col. Lomas 4a Sección,
San Luis Potosí, S.L.P., México.
[b]CIIDIT, Facultad de Ingeniería Mecánica y Eléctrica, UANL, Ciudad Universitaria
San Nicolás de los Garza, N.L., México.
rene.flores@ipicyt.edu.mx, griselda.quirozcm@uanl.edu.mx.
*Corresponding author: rfemat@ipicyt.edu.mx

Resumen— **A robust H_∞ controller was developed in order to regulate the COD in an anaerobic bioreactor from the winery industry. A sensitivity analysis allows the identification of the parameters set having the most significant effect on dynamic behavior of a nonlinear model. Such parameters were selected as uncertain. Controller performance was assessed in terms of weight functions in key signals. The H_∞ controller was obtained, thus arbitrary constant reference can be tracked while operation of bioreactor is guaranteed.**
Derecho reservado ©AMCA.

Keywords: Robust Control, Anaerobic Bioreactor, H_∞ control.

I. INTRODUCCIÓN

Anaerobic Digestion (AD) is a process that converts organic matter into a gaseous mixture mainly of methane and carbon dioxide through the action of a series of complex biological and enzymatic reactions. The AD process has been traditionally used for wastewater treatment but there is considerable interest in this field. Operating and controlling AD process is not a simple (Méndez-Acosta, 2008) (Méndez-Acosta, 2007): (i) the characteristic of wastewater varies continuously in quantity and composition, (ii) the biomass activity changes under the influence of internal and external factors, (iii) the lack of adequate sensors to measures on-line all the important variables in the process, and (iv) parametric uncertainty due to bacterial population distribution. For these reasons it is curtail to implement a control system in order to maintain the stability of the AD process. In the present work a controller is derived for an AD using classical H_∞ theory. In face to uncertainties but ensuring the robustness of closed loop control, the proposed scheme is designed to deal with modeling errors and attenuate disturbances in the inlet concentrations.

II. MODEL DESCRIPTION

The AD model (1) was proposed and experimentally validated in (Bernard, 2001) and it stands for a fixed bed anaerobic digester for vinasses treatment. Moreover, the model has been used to design and implement controllers (Méndez-Acosta, 2008), (Rodriguez, 2008). The dynamical model is the following:

$$\begin{aligned}
\dot{x_1} &= (\mu_1 - \alpha D)\, x_1 \\
\dot{x_2} &= (\mu_2 - \alpha D)\, x_2 \\
\dot{x_3} &= (x_{3,in} - x_3)\, D \\
\dot{x_4} &= (x_{4,in} - x_4)\, D - k_1 \mu_1 x_1 \\
\dot{x_5} &= (x_{5,in} - x_5)\, D + k_2 \mu_1 x_1 - k_3 \mu_2 x_2 \\
\dot{x_6} &= (x_{6,in} - x_5)\, D - q_{CO2} + k_4 \mu_1 x_1 + k_5 \mu_2 x_2
\end{aligned} \tag{1}$$

where the states variables respectively stand for: x_1, acidogenic bacteria concentration (g/l); x_2, methanogenic bacteria concentration (g/l); x_3, total alkalinity ($mmol/l$); x_4, Chemical Oxygen Demand (COD, g/l); x_5, volatile fatty acids (VFA, $mmol/l$); x_6, total inorganic carbon (TIC, $mmol/l$). The terms x_{in} for x_{3-6} represent the inlet compositions. The dilution rate, (D, hr^{-1}), is defined as the ratio of the inlet flow rate, Q_{in},(lhr^{-1}), divided by the reactor volume (l). The fraction of biomass in the liquid phase is given by the constant parameter α; which belongs to the interval $0 \leq \alpha \leq 1$ (where $\alpha = 0$ corresponds to an ideal fixed bed reactor, whereas $\alpha = 1$ corresponds to an ideal continuous-flow stirred tank reactor). The term q_c is related to mass transfer, denotes the molar flow rate of carbon dioxide.

The AD model (1) includes the specific growth of acidogenic and methanogenic bacteria by using Monod law, denoted by $\mu_1(x_4)$, and Haldane-like kinetics, $\mu_2(x_5)$ (Rodriguez, 2008). Such kinetic expressions are respectively given by:

$$\mu_1(x_4) = \mu_{1\max}\frac{x_4}{x_4 + K_{S1}}$$
$$\mu_2(x_5) = \mu_{2\max}\frac{x_5}{x_5 + K_{S2} + (x_5/K_{I2})^2} \quad (2)$$

where $\mu_{1\max}$ (hr^{-1}), K_{S1} (g/l), $\mu_{2\max}$ (hr^{-1}), K_{S2} (g/l), K_{I2} (g/l) are constant, uncertain, and finite.
The AD model (1) was analyzed in order to derive the following equilibrium coordinates (Méndez-Acosta, 2005):

$$\begin{aligned}
x_1^* &= \frac{(x_{4,in} - x_4^*)D}{k_1\mu_1(x_4^*)} \\
x_2^* &= \frac{(x_{5,in} - x_{5-}^*)D + k_2\mu_1(x_4^*)x_1^*}{k_3\mu_2(x_{5-}^*)} \\
x_3^* &= x_{3,in} \\
x_4^* &= \frac{\alpha D K_{S1}}{(\mu_{1\max} - \alpha D)} \\
&\alpha D x_5^{*2} + x_5^*\left(\alpha D K_{I2}^2 - \mu_{2\max}K_{I2}^2\right) + \alpha D K_{S2}K_{I2}^2 = 0 \\
x_6^* &= \frac{x_{6,in}D - q_{CO2} + k_4\mu_1(x_4^*)x_1^* + k_5\mu_2(x_{5-}^*)x_2^*}{D}
\end{aligned} \quad (3)$$

where x_{5+}^* and x_{5-}^* are solutions of second order equation. The unique equilibrium point results in two operation condition. It is said that a digester is operating in washout condition as the biomass is inactive $(x_1, x_2 = 0)$ for all $t \geq 0$. Then, under such a condition the equilibrium values of the remaining states of AD model (1) are given by its inlet composition and the biomass remains inactive. It is said that a digester is operating in normal operating condition as the biomass remains active, which means that $(x_1, x_2 \neq 0)$ for all and any initial state.
The results on unique equilibrium point are summarized in the following propositions (Méndez-Acosta, 2005):
Proposition 1. Consider the anaerobic digester (1). Then, assuming that the inlet composition $x_{j,in}$ is piecewise constant, system (1) has a unique equilibrium point x^* for any constant pair α, D^* under normal operating conditions. In addition, such an equilibrium point is contained into the closed set $(x_1, x_2 \neq 0, D^* \in [\underline{D}, \overline{D}] \subset R_+)$ and $x_{j,in} > x_j$ for $j = 3 - 6$ which contains all normal operating conditions; where x_{max} is the concentration vector obtained as $\overline{D}$ is used whereas x_{min} is obtained for $\underline{D}$.
Proposition 2. Let $x^* \in \Omega$ the equilibrium point of the anaerobic digester (1) for any constant pair (α, D^*) such that $D^* \in [\underline{D}, \overline{D}]$ Then, under normal operating conditions such an equilibrium point is locally stable.

III. Problem formulation

The main objective of anaerobic digester for vinery wastewater treatment is to decompose the organic compound resulting in the different processes. These organics compounds are considered pollutants and the final concentration in the outlet stream must accomplish environmental and safety local regulations. These pollutants can be measured in terms of its COD. Then, one of the key control variables in the AD is the COD. As Méndez-Acosta *et al* showed (Méndez-Acosta, 2005), since the AD model (1) has an unique and locally stable equilibrium value for a given and the inlet compositions are bounded and uncertain $(x_{j,in})$ a controller is needed to lead trajectories to the (locally stable) equilibrium point. An alternative, dilution rate (D) has to be varied, obeying the controller in order to achieve the COD regulation at the desired value $(x_{4,ref})$. In other words, D is the manipulated variable represented by u (input variable), and the measure output variable COD (x_4) is represented by y.
The controller should accomplish certain requirements such that: nonlinearity of the process dynamics, robustness to modeling error, parametric uncertainess, internal instabilities due an inhibition due to high concentration of VFA, and restrictions in actuators (as constrained values in D). Therefore, the reference-tracking problem should take into account the presence of disturbances such that the concentration of wastes in the inlet streams. The measurement noise represents an unavoidable difficulty in this type of industrial environment with the measurement technology available.
Significant applications of robust control are reported in the area of chemical processes, nevertheless very few results are available for biological process. The classic H_∞ approach is very promising since it achieves the kind of requirements and difficulties posed for this class of systems. This work reports the application of the H_∞ approach to the anaerobic digester process.

A. Nominal model construction

Linear classical H_∞ controller synthesis requires a linear system model. The Jacobian linearization of the AD model (1) around equilibrium point x^* is the following:

$$J|_{x^*} = \begin{vmatrix} J_{1,1} & 0 & 0 & J_{1,4} & 0 & 0 \\ 0 & J_{2,2} & 0 & 0 & J_{2,5} & 0 \\ 0 & 0 & J_{3,3} & 0 & 0 & 0 \\ J_{4,1} & 0 & 0 & J_{4,4} & 0 & 0 \\ J_{5,1} & J_{5,2} & 0 & J_{5,4} & J_{5,5} & 0 \\ J_{6,1} & J_{6,2} & J_{6,3} & J_{6,4} & J_{6,5} & J_{6,6} \end{vmatrix} \quad (4)$$

where $J_{i,k} = \partial f_i / \partial x_k$ for $i, k = 1, 2, ..., 6$.
Considering D the input variable represented by u, and the measure output variable COD (x_4) represented by y then the nominal plant model is the following:

$$\begin{aligned} \dot{x} &= Ax + Bu \quad x(t_0) = x_0 \\ y &= Cx \end{aligned} \quad (5)$$

where A is given by (4), $B = [0, 0, x_{3,in}, x_{4,in}, x_{5,in}, x_{6,in}]^T$, and $C = [0, 0, 0, 1, 0, 0]$. For the following nominal parameters (Méndez-Acosta, 2005), (Rodriguez, 2008): $\alpha = 0.5, D = 0.02, k_1 = 42.14, k_2 = 116.5, k_3 = 268, k_4 =$

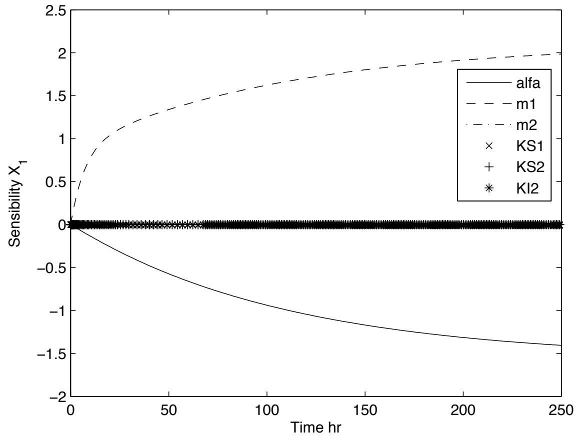

Figura 1. Sensitivity of acidogenic bacteria state.

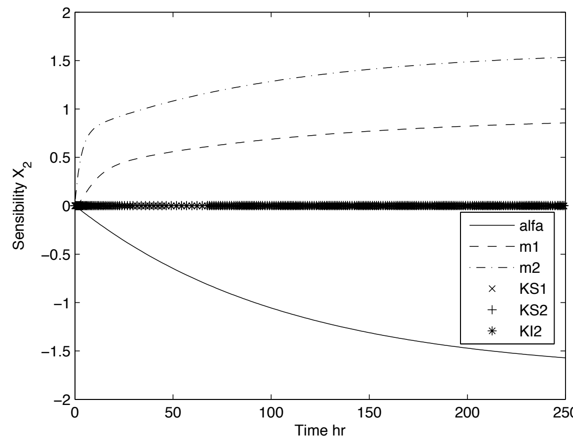

Figura 2. Sensitivity of metanogenic bacteria state.

$50.6, k_6 = 453, \mu_{1,\max} = 0.05, \mu_{2,\max} = 0.031, K_{S1} = 7.1, K_{S2} = 9.28, K_{I2} = 16$; and the inlet condition $x_{in} = [0, 0, 90, 16, 68.78, 67.1]$ used. Then the equilibrium is given by $x^* = [0.67, 0.77, 90, 1.77, 4.45, 104.95]$.

B. Uncertainty model construction

Uncertainty due to differences between an actual lineal nominal model could be related to variation in model parameters. A parametric sensitivity analysis was performed on the nonlinear model in order to determinate the terms most responsible for changes in the digester model (Khalil, 2001).

For a set of parameters:

$$\Pi = [\alpha, \mu_{1,\max}, \mu_{2,\max}, K_{S1}, K_{S2}, K_{I2}] \tag{6}$$

the sensitivity function for solutions is the following:

$$\dot{S}_f = A_{Sf} S_f + B_{Sf} \tag{7}$$

where $S_f \triangleq [\partial x / \partial \pi]_{\pi_0}$, $A_{Sf} \triangleq [\partial f(x) / \partial x]_{x*}$, $B_{Sf} \triangleq [\partial f(x) / \partial \pi]_{\pi_0} \in \Re^{6 \times 6}$, and the nominal parameter $\pi_0 \in \Pi$. The Fig. 1 - Fig. 4 shows solution of the sensitivity equation for states x_1, x_2, x_4 y x_5, respecting to parameter set Π.

The parameters $\alpha, \mu_{1,\max}$, and $\mu_{2,\max}$ were found to be the most sensitive to variations in the anaerobic digester model. In the absence of physical data from which to identify ranges for parametric variation, it was assume that $\pm 15\%$ parametric variability in $\alpha, \mu_{1,\max}$, and $\mu_{2,\max}$ represented a broad range of potential plants. As a subset of Π, the uncertainty was restricted to three most highly sensitive parameters. The nonlinear model was linearized around each of these parameter variations of nominal parameter value. Uncertainty in the frequency domain was manifested through parameter variations and was measured with respect

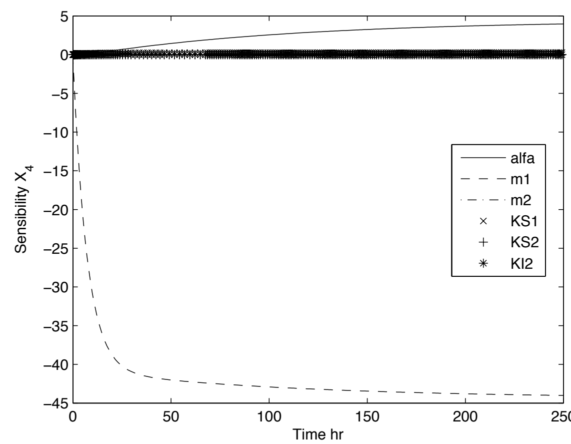

Figura 3. Sensitivity of Chemical Oxygen Demand state.

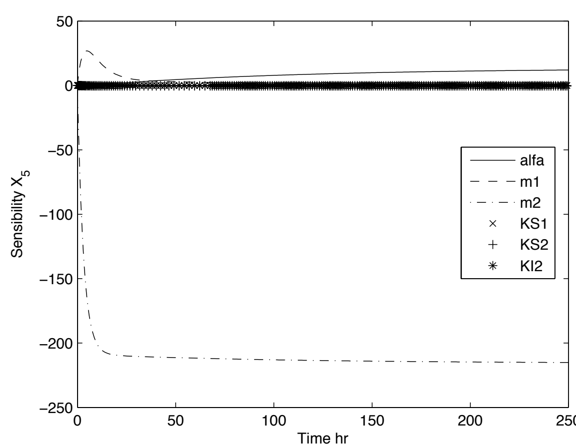

Figura 4. Sensitivity of volatile fatty acids state.

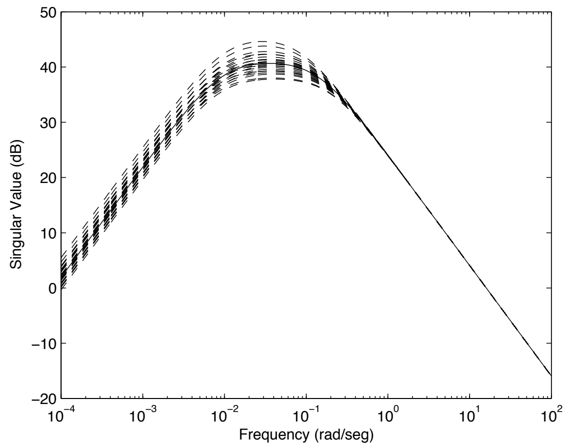

Figura 5. Frequency domain of nominal plant $G_{nom}(\omega)$ (−) and the set of perturbed models $G_p(\omega)$ (−−).

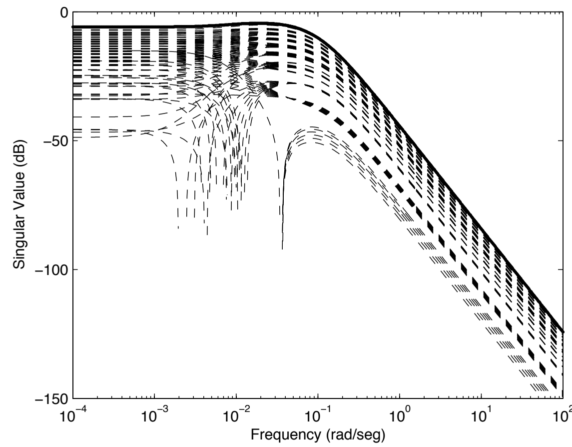

Figura 6. Frequency domain of $U_{rel}(\omega)$

to the nominal lineal model of AD over the frequency range of interest, $\omega = \left[1 \times 10^{-4}, 1 \times 10^{2}\right]$. This was represented as a relative uncertainty, with the following mathematical description:

$$U_{rel}(\omega) = \left| \frac{G_p(\omega) - G_{nom}(\omega)}{G_{nom}(\omega)} \right| \tag{8}$$

where $G_{nom}(\omega)$ represent the nominal model of AD, and $G_p(\omega)$ represent the set of perturbed models. The Fig. 5. show the perturbed models and the nominal plant frequency domain.

The 6 shows the upper bound of relative uncertainty as a function of the frequency. This bound was created by taking the maximum uncertainty magnitude of the set of relative uncertainty differences $U_{rel}(\omega)$. This bound was calculated by following expression:

$$\max_{\omega} \left| \frac{G_p(\omega) - G_{nom}(\omega)}{G_{nom}(\omega)} \right| \tag{9}$$

The resulted multiplicative uncertainty weight function is given by $W_p(s) = W_{p,N}(s)/W_{p,D}(s)$ such that: $W_{p,N}(s) = 8.128 \times 10^{-6}s^3 + 4.811 \times 10^{-4}s^2 + 6.126 \times 10^{-3}s + 5.006 \times 10^{-5}$, and $W_{p,D}(s) = s^3 + 0.207s^2 + 0.01141s + 9.892 \times 10^{-5}$.

IV. Robust Synthesis

The H_∞ control framework is well suited for AD regulation, due to the ability to tune the controller for robustness to uncertainty while to ensure internal stability and mathematically guaranteeing a certain degree of performance. In this case, it is important for a closed-loop controller to tolerate dynamic uncertainty while rapidly rejecting COD disturbances and tracking the constant COD reference. The controller allowed custom tuning to trade-off these potentially conflicting objectives.

The Fig. 7 shows the block diagram of the problem to be solved. In the sense of (5), since the AD model (1) has an unique and locally stable equilibrium point (x^*), it is possible to design a suboptimal control input (u). The control is synthesised under nominal parameter values ($\pi_0 \in \Pi$), which determines the coordinates of the equilibrium point, in face to disturbances at the inlet concentration ($x_{4,in}$).

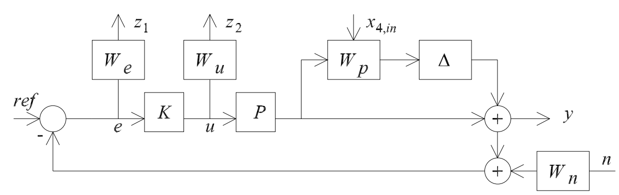

Figura 7. Block diagram for the control synthesis.

The Fig. 7 shows the most important elements of the problem control. The input (u) is D, noise n, and the COD disturbance $x_{4,in}$. The outputs are: the error signal z_1, control signal input z_2, and plant measurement COD y. The generalized plant was derived from the input/output relations: $z = \left[\begin{array}{cc|c} z_1 & z_2 & e \end{array}\right]^T$ and $d = \left[\begin{array}{c|c} n & u \end{array}\right]^T$. Then $G(s)$ is given by the following equation:

$$G(s) = \left[\begin{array}{c|c} W_e W_n & -W_e P_{nom} \\ 0 & W_u \\ \hline -W_n & -P_{nom} \end{array}\right] \tag{10}$$

A classical H_∞ controller, if one exits, minimizes:

$$\begin{array}{l} \|T_{zd}\|_\infty = \max_\omega \sigma\left[\Im_l(P,K)\right] \\ \Im_l(P,K) = P_{1,1} + P_{1,2}K\left(I - P_{2,2}K\right)^{-1}P_{2,1} \\ \left[\begin{array}{c} \left[\begin{array}{c} z_1 \\ z_2 \end{array}\right] \\ e \end{array}\right] = \left[\begin{array}{cc} P_{1,1} & P_{1,2} \\ P_{2,1} & P_{2,2} \end{array}\right] \left[\begin{array}{c} [n] \\ u \end{array}\right] \end{array} \tag{11}$$

A. Weighting function design

The control design was completed by constructing the weighting functions in Fig. 7. The input weight W_u, error performance weight W_e, and noise weight A typical mechanical pump inlet in AD would be subject to the magnitude constraint. In addition, saturation in control signal must be avoided. In order to achieve the previous issues of error, a performance weight restriction was included in the following form:

$$\begin{aligned} &\left|W_e\left(1+PK\right)^{-1}\right| \leq 1 \\ &\left|{}^{1}\!/_{W_e}\right| \leq (1+PK) \\ &W_e = \frac{{}^{s}\!/_{M_e} + \omega_b^*}{s+\omega_b^*\varepsilon_e} \\ &W_e = \frac{{}^{s}\!/_{10} + 100}{s+100\times 0.011} \end{aligned} \tag{12}$$

The weight sensitivity function was considered a good measure of the closed-loop performance in disturbance rejection scenarios. Disturbance rejection at steady state is governed by $\varepsilon_e = 0.011$ and $M_e = 10$ is taken as the least upper bound on the disturbance sensitivity. This corresponds to a steady-state tracking error, because $W_e^{-1}(0) = 0.01$, and amplification of the high-frequency disturbances by a factor $W_e^{-1}(s\to\infty) = 10$. The frequency range in which the sensitivity function is forced to be small is up to $\omega_b^* = 100 rad/seg$, which includes the range of expected disturbances.

The weight sensitivity function W_u is related with the control effort and saturation limit in low-frequencies, and high-frequency noises rejection. The magnitude of $\left|K\left(1+PK\right)^{-1}\right|$ in the low-frequency range is essentially limited by the allowable cost of control effort and saturation limit of the actuators, in this case is limited by upper bound in low-frequencies $M_u = 0.1$, which is related with the maximum dilution rate $D = 0.1hr^{-1}$. The high-frequency noises are attenuated upper bandwidth $\omega_b^* = 100 rad/seg$ and is related with the high-frequency gain $\varepsilon_u = 0.05$.

$$\begin{aligned} &\left|W_uK\left(1+PK\right)^{-1}\right| \leq 1 \\ &\left|{}^{1}\!/_{W_u}\right| \leq \left|K\left(1+PK\right)^{-1}\right| \\ &W_u = \frac{s+{}^{\omega_b^*}\!/_{M_u}}{\varepsilon_u s+\omega_b^*} \\ &W_u = \frac{s+{}^{100}\!/_{0.1}}{0.05s+100} \end{aligned} \tag{13}$$

The noise weight function was considered in order to account any measurement effects noise. The choice of include frequency domain was to include high and low frequency. A low frequency noise is related with the effect of slow measurement equipment, i.e. COD (ε_n= 0.01). A high frequency noise effects describe the typical electrical noise ($M_n = 0.1$). The transition of two effects is related with $\omega_b^* = 1000 rad/seg$.

$$\begin{aligned} &W_n = \frac{s+\omega_b^*\varepsilon_n}{{}^{s}\!/_{M_n}+\omega_b^*} \\ &W_n = \frac{s+1000\times 0.01}{{}^{s}\!/_{0.1}+\omega_b^*} \end{aligned} \tag{14}$$

V. RESULTS

The suboptimal control problem was numerically solved by means of robust Control Toolbox by Matlab® using standard Riccati solution. The approximated stabilizing controller $K(s)$ was derived after iterative numerical process (*hinfsyn* command), ensuring robust internal stability and robust performance with $\gamma = 0.9178$ (Glover, 1988). Stabilizing full order controller is a 8ht order, and is given by $K_{8th}(s) = K_{8th,N}(s)/K_{8th,D}(s)$, such that: $K_{8th,N}(s) = 0.466s^7 + 974.446s^6 + 8.174\times 10^4 s^5 + 2.896\times 10^4 s^4 + 3533s^3 + 167.3s^2 + 2.384s + 0.0103$ and $K_{8th,D}(s) = s^8 + 1028s^7 + 2.869\times 10^4 s^6 + 3.36\times 10^5 s^5 + 3.996\times 10^5 s^4 + 7.235\times 10^4 s^3 + 3824s^2 + 33.67s + 0.00829$. The Hankel´s values from full order controller is given: $\sigma_{8th} = [0.603,\ 0.159,\ 0.111,\ 0.029,\ 1.956\times 10^{-4}, 3.187\times 10^{-18}, 4.403\times 10^{-19}, 1.059\times 10^{-19}]$.

In order to seek for a low order controller, a balanced model truncation via square root method was applied to full order controller and the following controller resulted: $K_{4th}(s) = K_{4th,N}(s)/K_{4th,D}(s)$, such that: $K_{4th,N}(s) = 0.857s^3 + 83.44s^2 + 12.56s + 0.107$ and $K_{4th,D}(s) = s^4 + 28.69s^3 + 337.3s^2 + 340.6s + 0.0862$. The Hankel´s values from reduced order controller is given: $\sigma_{4th} = [0.603, 0.159, 0.111, 0.029]$.

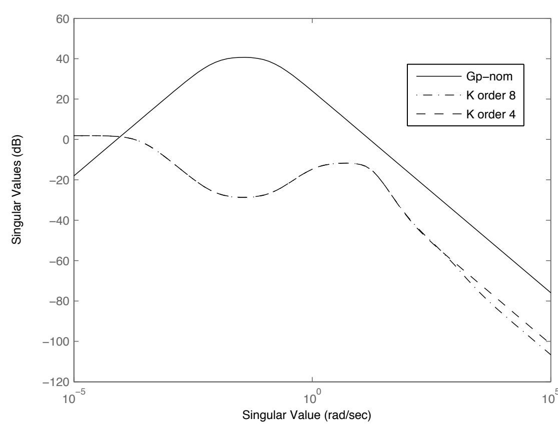

Figura 8. Frequency response for $K_{8th}(s)$ and $K_{4th}(s)$ controllers and nominal plant $G_p(\omega)$.

The Fig. 8 shows very similar frequency response for both controllers (K_{8th} and K_{4th}). Numerical evaluations of closed loop AD model (1) were performed for two cases:

regulation and tracking control problem. The simulations platform were programmed in Simulink by Matlab® and the result are show in the Fig. 9. Disturbances around nominal COD value $x_{4,in} = 16$ where attenuate in the regulation and tracking control problem.

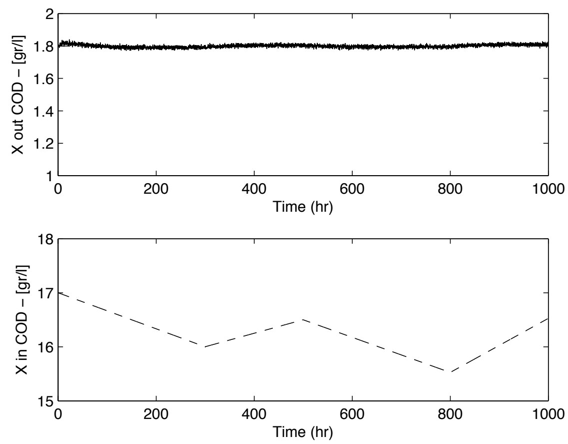

Figura 9. Simulation of regulation problem in AD using H_∞ controller K_{4th}. Upper graph shows outlet concentration COD in AD ($-$) and the reference ($--$), and bottom shows inlet disturbance signal in $x_{4,in}$ ($--$).

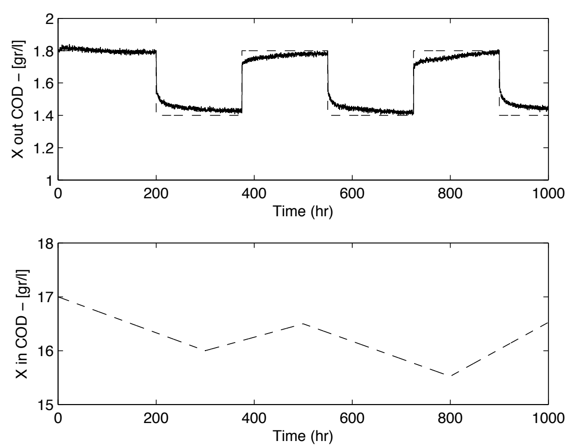

Figura 10. Simulation of tracking problem in AD using H_∞ controller K_{4th}. Upper graph shows outlet concentration COD in AD ($-$) and the reference ($--$), and bottom shows inlet disturbance signal in $x_{4,in}$ ($--$).

VI. Concluding Remarks

A classical H_∞ control design of an AD is proposed. Uncertainty from parameter variation within the nonlinear AD model representation was characterized using relative uncertainty. Robust performance criterions were assessed using weight functions in specific signals. The H_∞ controller is capable to attenuate load disturbances in the inlet concentrations and noisy measurements.

The H_∞ controller achieves the COD regulation a desired value of COD ($x_{4,ref}$). Despite of fact of the nature of linear controller, the H_∞ controller is able to: (i) Modeling errors: the control approaches are robust against uncertainties in the kinetic functions ($\mu_1(x_4),\ \mu_2(x_5)$) and hydrodynamic regimen (α). (ii) Load disturbances: in spite of H_∞ controller uses a nominal value of the COD inlet composition, the performance criterion in the control scheme results robust against variations in the inlet composition. (iii) Constraints in the control input: a performance criterion in the control scheme is proposes in the dilution rate in order to restricted and avoid possible deterioration of the controller closed-loop performance.

VII. Acknowledgements

R. A. Flores-Estrella thanks to CONACyT by financial support under scholarship grant No. 160117 and FONCI-CYT by financial support under BITA consortio (S-3146). G. Quiroz thanks to SEP by financial support under grant PROMEP/103.5/10/3889.

Referencias

Méndez-Acosta H.O., Palacios-Ruiz B., Alcaráz-Gonzalez V., Steyer J.P., González-Álvarez V., Latrille E. (2008). Robust Control of Volatile Fatty Acids in Anaerobic Digestion Processes. *Industrial and Engineering Chemistry Research* **47**, 7715–7720.

Méndez-Acosta H.O., Femat R., González Álvarez V. (2007). Selected Topics in Dynamics and Control of Chemical and Biological Processes. *Lecture Notes in Control and Information Sciences, Springer, Alemania* ISBN 01708643.

Bernard, O., Hadj-Sadok, Z., Dochain, D., Genovesi, A., Steyer, J.P., (2001). Dynamical model development and parameter identification for anaerobic wastewater treatment process. *Biotechnol Bioeng* **75**, 424–438.

Rodriguez J., Roca E., Lema J.M., Bernard O. (2008). Determination of the adequate minimum model complexity required in anaerobic bioprocesses using experimental data. *Journal of Chemical Thechnology and Biotechnology* **83**, 1694–1702.

Mendez-Acosta H.O., Campos-Delgado D.U., Femat R., González-Álvarez V. (2005). A robust feedforward/feedback control for an anaerobic digester. *Computers and Chemical Engineering* **29**, 1613–1623.

Khalil H.K. (2001). Nonlinear Systems. *Prentice Hall* ISBN 0130673897.

Glover K., Doyle J.C. (1988). A robust control scheme to improve the stability of anaerobic digestion processes. *Systems and Control Letters* **11**, 167–172.