State and Sensor Fault Estimation using Proportional-Integral Observers

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***Abstract*— This paper presents a Proportional-Integral (PI) Observer for states estimation and sensor fault estimation. The main purpose of this manuscript is to construct a Fault Detection and Diagnosis (FDD) scheme, using the PI observer which provides a sensor fault estimation and states estimation. This observer reconstructs the sensor faults based on augmented state equations. This scheme reconstructs the sensor faults based on augmented state equations. An auxiliary state is assigned to represent the dynamic behavior of the fault. An illustrative example is presented in simulation.**

Keywords: Proportional-Integral Observer, Sensor Faults, Singular Systems, Linear Parameter Variant (LPV) Systems.

I. INTRODUCTION.

Recent researches in FDD design schemes, especially model-based FDD, are focused on systems modeled by coupled differential and algebraic equations (DAE's). Singular systems constitute an important class of systems of both theoretical and practical interest, which include, chemical and biological processes, interconnected large-scale systems, power and electrical circuits systems and robotic manipulators, among others. In this way, singular systems have attracted considerable attention in recent years. Much research has aimed at generalizing existing theories, especially in the time domain, from normal systems to singular systems, which include controllability and observability (Yip and Sincovec, 1981), feedback control (Junchao *et al.*, 2007; Jiang *et al.*, 2009), observer design (Dai, 1988; Darouach and Boutayeb, 1995; Koenig, 2006), optimal control (Zhong and Zhang, 2009) and robust control (Xu *et al.*, 2002; Yang *et al.*, 2006).

The purpose of detection is to generate an alarm to inform the operators that there is at least one fault in the system. This can be achieved from either the direct observation of the systems inputs and outputs or the use of certain types of redundant relations (i.e., model-based fault detection and diagnosis or analytical redundancy methods). The problem of fault diagnosis for LPV descriptor systems has been studied in manuscripts as (Astorga *et al.*, 2009), where the authors present a strategy for detection and isolation sensor faults using a bank of residuals within a Generalized Observer Scheme (GOS), but with this strategy is not possible the estimation fault and don't take account the effects of disturbances and other uncertain factors in the system. In (Hamdi *et al.*, 2009), a Polytopic Unknown Input Observer (PUIO) is applied to estimate the states of the system in the presence of unknown inputs and is dedicated to detect, isolate and estimate only an actuator fault using a bank of observers.

For LPV systems, the interpolation techniques present a good approach to get a polytopic structure. This structure is a set of linear model scheduled by weighting functions which represent polytopic LPV models (Rodrigues *et al.*, 2008). Taking this representation some authors as (Ichalal *et al.*, 2009), have developed a method for fault diagnosis using the polytopic models for nonlinear systems described by Takagi-Sugeno multiple models. Or as in (Hamdi *et al.*, 2009), that to represent the LPV descriptor system by a polytopic form and where observers are dedicated to detect, isolate and estimate actuator faults.

Realizations for singular systems are interesting tools for analysis and control of LPV systems. This comes from the fact that state space equations with dependency on the parameter can be transformed into a polytopic descriptor form as in (Grenaille *et al.*, 2008; Astorga *et al.*, 2009; Hamdi *et al.*, 2009). Starting from this transformation, the present paper proposes new sufficient conditions for the design of (PI) Observer for n multi-models LPV. The stability and the convergence properties are ensured by using Linear Matrix Inequality (LMI's). The method used is based in the approach proposed in (Park *et al.*, 1994), where the sensor faults are presented as pseudo-actuator faults based on augmenting the system equations by an auxiliary state that represents their dynamic, and in effect converts the sensor fault as an actuator fault.

This paper is organized of the following form. In the Section II the LPV Singular System representation is described. In the Section III a method for modeling sensor fault is presented. In the Section IV, the observer synthesis is presented. Finally in the Section V, the results of the observer implemented are depicted using an illustrative example.

II. LPV System Representation

Consider the continuous-time descriptor nonlinear system:

$$\begin{aligned} E\dot{x} &= F(x(t), u(t), d(t)) \\ y(t) &= \mathbf{C}x(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the input vector, $d(t) \in \mathbb{R}^w$ is a disturbance vector and $y(t) \in \mathbb{R}^m$ is the output vector. $F(\cdot)$ is continuous and indefinitely differentiable nonlinear function. $E \in \mathbb{R}^{qxn}$ is a singular matrix with constant parameters and $C \in \mathbb{R}^{mxn}$, is a constant matrix. The linearization of the function $F(\cdot)$ by Taylor series around ε operation points (x_i, u_i, d_i) gives a set local linear singular models (Hamdi *et al.*, 2010*a*). The representation of a LPV system is:

$$\begin{aligned} E\dot{x}(t) &= A(\rho(t))x(t) + B(\rho(t))u(t) + R(\rho(t))d(t) \\ \mathbf{y}(t) &= Cx(t) \end{aligned} \tag{2}$$

where $E, A \in \mathbb{R}^{qxn}, B \in \mathbb{R}^{nxp}, R \in \mathbb{R}^{nxw}, C \in \mathbb{R}^{mxn}$, are constant matrices.

Then, the new representation can defined as a multi-linear system where the matrices are set by known operation points (Rodrigues *et al.*, 2008). The first step is to represent the descriptor LPV system by a polytopic form when the parameters evolve in a polytopic domain. The vertices of this polytope are called the submodels of this representation, i.e., the parameter $\rho(t)$ varies in a convex polytope of vertices ρ_i such that $\rho(t) \in Co\{\rho_1; \rho_2, ..., \rho_M\}$. Those submodels are then combined by weighting functions to yield a global model. Then, the system (2) can be rewritten as a continuous-time polytopic descriptor LPV system of the form:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \left[A_i x(t) + B_i u(t) + R_i d(t)\right] \\ y(t) &= Cx(t) \end{aligned} \tag{3}$$

where A_i, B_i, R_i and C are known constant matrices defined above with $i = 1, \ldots, M$, and M is the total number of weighting functions $\varepsilon_i(\rho(t))$. And $\varepsilon_i(\rho(t))$ are defined as the relative contribution of each local model to build the global model and,

$$\sum_{i=1}^{M} \varepsilon_i(\rho(t)) = 1, \quad 0 \leq \varepsilon_i(\rho(t)) \leq 1 \tag{4}$$

The second step is devoted to the design of a global observer for the simultaneously estimation disturbance and fault detection in the new representation descriptor LPV system. Let us consider the following Descriptor LPV system in the presence of sensor fault $f_s(t)$ and the disturbances $d(t)$:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \left[A_i x(t) + B_i u(t) + R_i d(t) + \Delta x_i\right] \\ y(t) &= Cx(t) + Jf_s(t) \end{aligned} \tag{5}$$

where $f_s(t) \in \mathbb{R}^{n_f}$ is a fault vector and J represents the sensor fault distribution matrix. Δx_i is a vector that depends of the $i^{th}th$ operating point. To carry out appropriate state estimation, the following assumptions are established:

Assumption 1: The triplet (E, A_i, C) is R-observable $\forall i = 1, \ldots, M$ (Darouach *et al.*, 1996), i.e.:

$$\text{rank} \begin{bmatrix} sE - A_i \\ C \end{bmatrix} = n \qquad \forall s \in \mathcal{C} \text{ and}$$

where $\mathcal{C}$ is the set of complex numbers.

Assumption 2: The impulsive terms of the system are observable (the triplet (E, A_i, C) is Impulse-Observable) $\forall i = 1, \ldots, M$ (Darouach *et al.*, 1996), if:

$$\text{rank} \begin{bmatrix} E & A_i \\ 0 & E \\ 0 & C \end{bmatrix} = n + rank(E)$$

The next section is dedicated to the development of a method for detecting and reconstruct the sensor fault in order to provide an efficient monitoring tool in the operator's decision.

III. Representation of the Sensor Faults

Consider the descriptor LPV system with sensor fault described by (5). As proposed by (Park *et al.*, 1994), an auxiliary state describing the sensor fault can be considered. Consequently, the descriptor LPV system is defined as follows:

$$\begin{aligned} \bar{E}\dot{\bar{x}}(t) &= \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \left[\bar{A}_i \bar{x}(t) + \bar{B}_i \bar{u}(t) + \bar{R}_i d(t) + \Delta \bar{x}_i\right] \\ y(t) &= \bar{C}\bar{x}(t) \end{aligned} \tag{6}$$

where $\bar{x}(t) \in \mathbb{R}^{n+1}$ is the augmented state vector and is defined as $\bar{x}(t) = \begin{bmatrix} x(t) \\ f_s(t) \end{bmatrix}$, the control vector is $\bar{u} = \begin{bmatrix} u & \xi \end{bmatrix}$ and the matrices are given by:

$$\bar{E} = \begin{bmatrix} E & 0 \\ 0 & 1 \end{bmatrix} \quad \bar{C} = \begin{bmatrix} C & J \end{bmatrix} \quad \bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & \alpha \end{bmatrix}$$

with a sensor input defined as: $\xi = \dot{f}_s - \alpha f_s$, and

$$\bar{B}_i = \begin{bmatrix} B_i & 0 \\ 0 & 1 \end{bmatrix} \quad \bar{R}_i = \begin{bmatrix} R_i \\ 0 \end{bmatrix} \quad \Delta \bar{x}_i = \begin{bmatrix} \Delta x_i \\ 0 \end{bmatrix}$$

The term α can be considered as an additional degree of freedom in the observer design. The fault can be modeled by a linear system of arbitrary order, but this fact depends of the characteristics of the fault (Park *et al.*, 1994).

Also, a necessary and sufficient condition for sensor fault detectability is: the pair $(\bar{A}_i, \bar{C})$ is observable if and only if (A_i, C) of the system (5) is observable.

IV. Proportional integral observer (PIO)

The fault reconstruction is achieved considering a descriptor system represented by the polytopic descriptor LPV model (6), the equations of the PIO are (Hamdi *et al.*, 2010*b*):

$$\begin{aligned} \dot{Z}(t) &= \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \left[N_i Z(t) + G_i \bar{u}(t) + L_i y(t) + H_i \hat{d}(t) + \Delta Z_i\right] \\ \hat{\bar{x}}(t) &= Z(t) + \tilde{M} y(t) \\ \dot{\hat{d}}(t) &= \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \Phi_i (y(t) - \hat{y}(t)) \end{aligned} \tag{7}$$

where $\hat{\bar{x}}(t) \in \mathbb{R}^{n+1}$, $z(t) \in \mathbb{R}^{n+1}$ and $\hat{d}(t) \in \mathbb{R}^p$ are the estimated state vector, observer state vector and estimated unknown input respectively. $N_i, G_i, L_i, H_i, \Delta Z_i, \tilde{M}$ and Φ_i are unknown matrices for the PI observer that should be calculated. From (6) with $y(t) = \bar{C}\bar{x}(t)$ the observer system (7) has an estimation error by:

$$\begin{aligned} \bar{e}(t) &= \bar{x}(t) - \hat{\bar{x}}(t) \\ \bar{e}(t) &= (I_{n+1} - \tilde{M}\bar{C})\bar{x}(t) - Z(t) \end{aligned} \tag{8}$$

where I_{n+1} represents the identity matrix of order $n+1$, then is possible define a real matrix $U \in \mathbb{R}^{(n+1)\times(n+1)}$ such that

$U\bar{E} = I_{n+1} - \tilde{M}\bar{C}$, so for $\begin{bmatrix} \bar{E} \\ \bar{C} \end{bmatrix}$ is column full rank ,

$$\begin{bmatrix} U & \tilde{M} \end{bmatrix} = \begin{bmatrix} \bar{E} \\ \bar{C} \end{bmatrix}^{+} \tag{9}$$

where the superscript $^+$ represents the inverse generalized matrix, and the estimation error can be rewritten as:

$$\bar{e}(t) = U\bar{E}\bar{x}(t) - Z(t) \tag{10}$$

Assuming that the unknown inputs are bounded and their dynamic is slow, i.e., $\dot{d}(t) \simeq 0$. Then, for $\delta(t) = d(t) - \hat{d}(t)$, the unknown input derive is defined as:

$$\dot{\delta}(t) = -\dot{\hat{d}}(t) \tag{11}$$

The estimation error dynamic is written as:

$$\begin{aligned} \dot{\bar{e}}(t) = \sum_{i=1}^{M} \varepsilon_i(\rho(t)) &\left[(U\bar{A}_i - L_i\bar{C} - N_i U\bar{E})\bar{x}(t)\right. \\ &(U\bar{B}_i - G_i)\bar{u}(t) + (U\bar{R}_i - H_i)d(t) + \\ &\left.(U\Delta\bar{x}_i - \Delta Z_i) + H_i\rho(t) + N_i\bar{e}(t)\right] \end{aligned} \tag{12}$$

where the following conditions can be defined:

$$U\bar{A}_i = N_i U\bar{E} - L_i\bar{C} \tag{13}$$

$$G_i = U\bar{B}_i \tag{14}$$

$$H_i = U\bar{R}_i \tag{15}$$

$$\Delta Z_i = U\Delta\bar{x}_i \tag{16}$$

$$I_{n+1} = U\bar{E} + \tilde{M}\bar{C} \tag{17}$$

From (3), (11) and (12), the estimation error and the unknown input dynamic is:

$$\dot{\bar{e}}(t) = \textstyle\sum_{i=1}^{M} \varepsilon_i(\rho(t))(N_i\bar{e}(t) + H_i\delta(t)) \tag{18}$$

$$\dot{\delta}(t) = \textstyle\sum_{i=1}^{M} \varepsilon_i(\rho(t))(-\Phi_i\bar{C})\bar{e}(t) \tag{19}$$

and the following function can be established:

$$\begin{bmatrix} \dot{\bar{e}}(t) \\ \dot{\delta}(t) \end{bmatrix} = \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \begin{bmatrix} N_i & H_i \\ -\Phi_i\bar{C} & 0 \end{bmatrix} \begin{bmatrix} \bar{e}(t) \\ \delta(t) \end{bmatrix} \tag{20}$$

Then, the state estimation error (20) converge asymptotically to zero if the real part of the eigenvalues of $\begin{bmatrix} N_i & H_i \\ -\Phi_i\bar{C} & 0 \end{bmatrix} < 0$, i.e., are stables. The matrices N_i, L_i, G_i and H_i must be determined by the following equations:

$$N_i = U\bar{A}_i - (L_i - N_i\tilde{M})\bar{C} \tag{21}$$

$$K_i = L_i - N_i\tilde{M} \tag{22}$$

$$N_i = U\bar{A}_i - K_i\bar{C} \tag{23}$$

and

$$L_i = K_i + N_i\tilde{M} \tag{24}$$

From (22) and (23) the state estimation error (20) can be rewritten as:

$$\begin{bmatrix} \dot{\bar{e}}(t) \\ \dot{\delta}(t) \end{bmatrix} = \sum_{i=1}^{M} \varepsilon_i(\rho(t))(\breve{A}_i - \breve{K}_i\breve{C}) \begin{bmatrix} \bar{e}(t) \\ \delta(t) \end{bmatrix} \tag{25}$$

where $\breve{A}_i = \begin{bmatrix} U\bar{A}_i & H_i \\ 0 & 0 \end{bmatrix}$, $\breve{K}_i = \begin{bmatrix} K_i \\ \Phi_i \end{bmatrix}$ and $\breve{C} = [\bar{C} \ \ 0]$.

Then, the PI observer (7) for a descriptor LPV system with inputs unknown (3) exists and their estimation error converge asymptotically to zero, if and only if, the pairs $(\breve{A}_i, \breve{C})$ are detectable $\forall i = 1, 2, ..., M$. This observer is asymptotically stable if exists a positive definite symmetric matrix P and matrices $W_i = P\breve{K}_i$ such that the following LMI holds:

$$(\breve{A}_i^T P + P\breve{A}_i - \breve{C}^T W_i^T - W_i\breve{C}) < 0, \ \ \forall i \in 1, 2, ..., j. \tag{26}$$

Observer gains can be calculated from $\breve{K}_i = P^{-1}W_i$. For ensuring the stability and convergence of the observation error, it is possible to define in the left part of the complex plane a bounded area S with a line of abscissa $(-\sigma)$ where $\sigma \in \mathbb{R}^+$, and the LMI's defined in (26) must be replaced by the following inequalities:

$$\begin{aligned} &(\breve{A}_i^T P + P\breve{A}_i - \breve{C}^T W_i^T - W_i\breve{C}) + 2\sigma P < 0, \\ &\forall i \in 1, 2, ..., M. \end{aligned} \tag{27}$$

then, consequently $\hat{\bar{x}}(t)$ will asymptotically converge to $\bar{x}(t)$ and $\hat{d}(t)$ to $d(t)$.

V. ILLUSTRATIVE EXAMPLE

In order to illustrate the efficiency of the presented method, is considered the continuous-time descriptor LPV nonlinear system (1) (Hamdi *et al.*, 2010*a*), described by:

$$\begin{array}{l} \dot{x}_1(t) = -1.5x_1^2(t) + 0.2x_3(t)x_4(t) \\ \dot{x}_2(t) = -u_1(t)x_1^2(t) - x_4(t)x_3^2(t) - 0.5x_2(t) + d(t) \\ 0 = 0.5x_2(t) - x_3(t) + 0.2x_4(t) \\ 0 = -x_2^2(t) + x_3^2(t) - 2x_4(t) + u_2(t) \\ y(t) = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t) \end{array} \tag{28}$$

where $u_1(t)$ and $u_2(t)$ are constant signals of magnitude 10 and 7 respectively. $d(t)$ is a step signal. LPV multi-model representation of the nonlinear dynamic system is given by the follow set of matrices:

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathbf{R_i} = \mathbf{R} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A_1} = \begin{bmatrix} -0.8775 & 0 & 0.526 & -0.0274 \\ -5.8500 & -0.5 & 0.1481 & 0.0026 \\ 0 & 0.5 & -1 & 0.2 \\ 0 & 2.6522 & -0.274 & -2 \end{bmatrix}, \mathbf{B_1} = \begin{bmatrix} 0 & 0 \\ -0.0856 & 0.01 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{A_2} = \begin{bmatrix} -0.6375 & 0 & 0.6166 & 0.0307 \\ -4.2500 & -0.5 & 0.2176 & -0.0036 \\ 0 & 0.5 & -1 & 0.2 \\ 0 & 1.8526 & 0.3068 & -2 \end{bmatrix}, \mathbf{B_2} = \begin{bmatrix} 0 & 0 \\ -0.0452 & 0.01 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{A_3} = \begin{bmatrix} -0.6357 & 0 & 0.6047 & 0.0226 \\ -4.2380 & -0.5 & 0.1162 & -0.0015 \\ 0 & 0.5 & -1 & 0.2 \\ 0 & 1.9660 & 0.2264 & -2 \end{bmatrix}, \mathbf{B_3} = \begin{bmatrix} 0 & 0 \\ -0.0449 & 0.01 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{\Delta x_1} = \begin{bmatrix} 0.2004 \\ 1.6753 \\ 0 \\ 1.7397 \end{bmatrix}, \mathbf{\Delta x_2} = \begin{bmatrix} -0.0268 \\ 0.8083 \\ 0 \\ 0.8346 \end{bmatrix}, \mathbf{\Delta x_3} = \begin{bmatrix} -0.0011 \\ 0.8505 \\ 0 \\ 0.9535 \end{bmatrix}$$

The weighting functions $\mu_i(\rho(t))$ characterize the dynamic behavior of the descriptor nonlinear system and its evolution depends of parameters that are functions of the state variables as:

$$\rho_i(t)(x_3(t)) = \frac{\mu_i(x_3(t))}{\sum_{i=1}^{3} \mu_i(x_3(t))} \tag{29}$$

the parameters trajectory is determined by the behavior of the system variables as:

$$\begin{array}{l} \mu_1(x_3(t)) = exp(-1/2(\frac{x_3+5}{2})^2) \\ \mu_2(x_3(t)) = exp(-1/2(\frac{x_3}{2})^2) \\ \mu_3(x_3(t)) = exp(-1/2(\frac{x_3-5}{2})^2) \end{array} \tag{30}$$

V-A. Fault-free case.

In order to show the effectiveness of the proposed modeling method, the nonlinear state and the multi-model approximation are depicted in the Figs. 1-2 in fault free case (for space reasons, only the states x_1 and x_3 are presented). The LPV multi-model represents an approximation of the nonlinear system. This substitution consists in finding a set of submodels with a simple linear structure and a set of appropriated weighting functions in order to combine these submodels to constitute the global model. It is possible to see that this representation of the nonlinear system can be used as design tool of the observer.

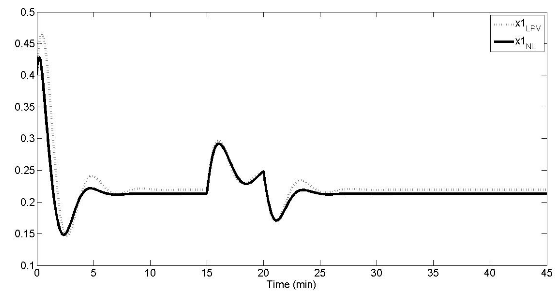

Figure 1. Dynamic behavior of the x1 nonlinear and LPV multi-model.

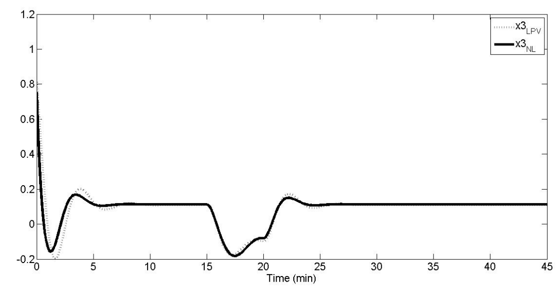

Figure 2. Dynamic behavior of the x3 nonlinear and LPV multi-model.

V-B. Sensor-fault case.

For the descriptor LPV system (5) the vector $f_s(t)$ is a sensor fault modeled as a sinusoidal signal $(0.2sin(5t-10))$ applied for $25 \leq t \leq 32$. To design the observer for the descriptor LPV system (5) the augmented system should be constructed as (6). Unknown input is modeled as a step signal of magnitude 0.15 applied for $15 \leq t \leq 20$. Then, the matrices U and $\tilde{M}$ can be calculated by (9) such that $U\bar{E} + \tilde{M}\bar{C} = I_{n+1}$. Before, the matrices G_i, H_i and ΔZ_i can be calculated from (14)-(16).

Using the Yalmip Toolbox (Lofberg, 2004) for solving the LMIs (27), it is possible to find a feasible solution that allows to determine the proportional gains K_i and Φ_i:

$$\mathbf{K_1} = \begin{bmatrix} -0.0578 & 0.1073 & 0.0603 \\ -0.3369 & 0.5628 & 0.2782 \\ 0.0077 & -0.0129 & -0.0042 \\ 0.5332 & -0.8861 & -0.4372 \\ -0.6026 & 0.9787 & 0.4665 \end{bmatrix}$$

$$\mathbf{K_2} = \begin{bmatrix} -0.0479 & 0.1062 & 0.0666 \\ -0.2875 & 0.5686 & 0.3139 \\ 0.0065 & -0.0134 & -0.0052 \\ 0.4558 & -0.8945 & -0.4927 \\ -0.5185 & 0.9917 & 0.5282 \end{bmatrix}$$

$$\mathbf{K_3} = \begin{bmatrix} -0.0491 & 0.1054 & 0.0638 \\ -0.2934 & 0.5645 & 0.3001 \\ 0.0067 & -0.0133 & -0.0049 \\ 0.4652 & -0.8883 & -0.4711 \\ -0.5286 & 0.9849 & 0.5047 \end{bmatrix}$$

$$\mathbf{\Phi_1} = \begin{bmatrix} -0.8103 & 1.4680 & 0.7921 \end{bmatrix}$$

$$\mathbf{\Phi_2} = \begin{bmatrix} -0.6773 & 1.4643 & 0.8812 \end{bmatrix}$$

$$\mathbf{\Phi_3} = \begin{bmatrix} -0.6934 & 1.4534 & 0.8444 \end{bmatrix}$$

From the procedure of augmented system, given in the Section III, is possible to use the Proportional-Integral Observer for estimate the dynamic behavior of the sensor fault as an additional state. Fig. 3 presents the outputs of the nonlinear system with their estimation. Fig. 4 shows the signal of the sensor fault and their estimate. The estimation of the unknown input is illustrated in Fig. 5.

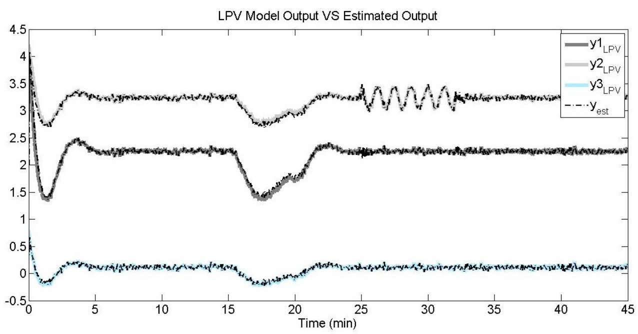

Figure 3. Output nonlinear system and output estimated.

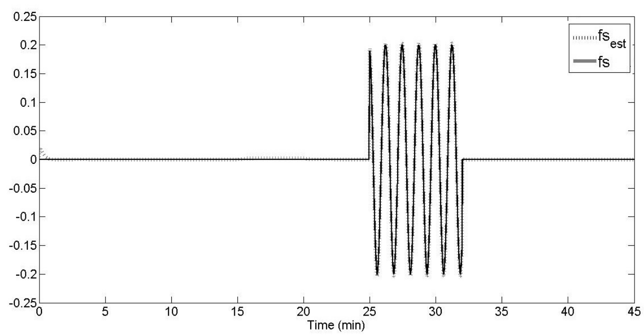

Figure 4. Sensor fault and corresponding estimated.

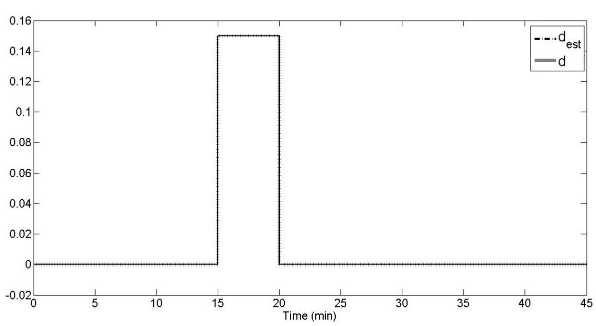

Figure 5. Unknown input estimated (disturbance).

To evaluate the performance of the PI-observer for the unknown inputs and state estimation, the results are presented with the same activation functions considered in the descriptor model. In this simulations a Gaussian white noise with variance center 0.01 is considered.

TABLE I

INCIDENCE MATRIX

Fault	F_1	F_2	F_3
$\|\|y-\hat{y}\|\|$	1	1	1
$\|\|\zeta_1-\hat{\zeta}_1\|\|$	0	1	1
$\|\|\zeta_2-\hat{\zeta}_2\|\|$	1	0	1
$\|\|\zeta_3-\hat{\zeta}_3\|\|$	1	1	0

For the purpose of fault diagnosis, a bank of observers is built as a Generalized Descriptor LPV Observer Scheme (based on GOS (Frank, 1994)), that provides an estimator dedicated to a certain sensor is driven by all outputs except that of the respective sensor, i.e., where each one of the them is driven by all inputs and all outputs except the k^{th} measurement variable. The measure y_k is not used in the k observer due to the fact that y_k is assumed corrupted. This scheme allows one to detect and isolate only a single fault in any of the sensors, however, with increased robustness with respect to unknown inputs (Frank, 1990).

For the bank of observers, the following descriptor LPV system in considered:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \left(A_i x(t) + B_i u(t) + R_i d(t) + \Delta x_i\right) \\ \zeta_k(t) &= \tilde{C}_j x(t) + \tilde{J}_j f_s(t) \end{aligned} \tag{31}$$

with $\tilde{C}_j$ and $\tilde{J}_j$ are the matrix and sensor fault distribution vector respectively, without the k^{th} row. The bank of observers generates an incidence matrix (Table I.) where a signal that is obtained from the residuals defining the effects associated with the fault.

The bank generates residuals different to zero, otherwise, only the observer which is insensitive to a sensor fault F_k effects generates a unique residual with a media zero. Thus, the fault is easily isolated using the GOS structure. Figs. 6-8, shows the results of the bank of observers according to the incidence matrix, only the observer designed insensitive to a sensor fault provides a residual vector equal to zero means.

This results are very effective in detecting and isolating sensor faults. Under unknown inputs, the residual generator possess robustness properties that allow adequate isolation of faults. This PIO insensitive to unknown inputs allows both estimate states and unknown inputs. However, it also can be used to detect, isolate and estimate sensor faults adequately.

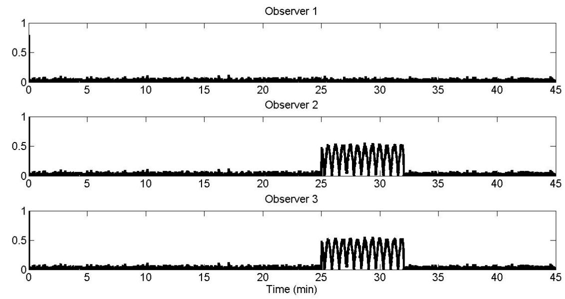

Figure 6. Residual vector norms with the first sensor out of order.

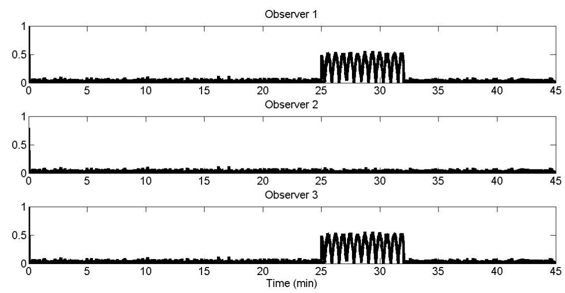

Figure 7. Residual vector norms with the second sensor out of order.

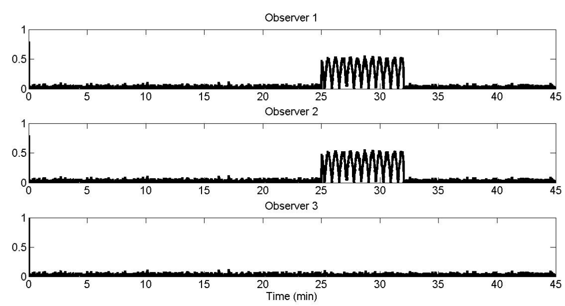

Figure 8. Residual vector norms with the third sensor out of order.

VI. CONCLUSIONS

This paper has presented a sensor fault diagnosis method to isolate sensor faults in descriptor LPV systems. Through proportional-integral observer approach, the error dynamics and the output error dynamics are converged to zero, as well as the sensor faults $f_s(t)$ are reconstructed. The observer designs is a extension of a PI observer for a special class of multi-output polytopic descriptor LPV systems presented in (Hamdi *et al.*, 2010*b*). The main advantage of these models is the ability to extend the tools designed in the linear system framework to descriptor nonlinear systems. Conditions to ensure the existence and the stability of the proposed scheme by using a Lyapunov analysis based on LMI formulation were established. The observer proposed is designed for to estimate simultaneously the sensor fault and the unknown input and also, is used for to build a bank of observers that permits to detect and isolate adequately the sensor faults. The effectiveness of this algorithm is evaluated via simulations using a numerical example.

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