Unknown Input Estimation for Linear Parameter Varying (LPV) Singular Systems: Application to a Binary Distillation Column

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***Abstract*— This paper presents an Unknown Input Observer (UIO) for states estimation and disturbances reconstruction. This observer provides a robust estimation against model uncertainties. The main purpose of this manuscript is the application of a full-order observer to a binary distillation column, using a strategy to simplify the nonlinear model with bounded unknown inputs. Then, the nonlinear system is presented as a Linear Parameter Variant (LPV) singular system. The applied method guarantees the regularity and the convergence of the observer.**

Keywords: Singular systems, multi-model, unknown inputs observer, binary distillation column.

I. INTRODUCTION.

Descriptions of singular dynamical systems arise naturally when they are formed from interconnected subsystems. In fact, any system can be considered as an interconnection of subsystems. Singular systems contain differential and algebraic equations, where algebraic equations represent the constraints to the solution of the differential part. Many practical processes, such as electronic circuit systems, constrained control problems, chemical, large scale systems, etc., can be modeled as singular systems because can preserve the structure of physical systems, describe non-dynamic constraints, and finite dynamic and impulsive behavior simultaneously, (Debeljkovic, 2004).

In the same way, LPV systems can be used to approximate and to derive nonlinear control laws for non-linear systems. From a practical point of view, LPV systems have at least two interesting interpretations: they can be viewed as Linear Time-Invariant (LTI) systems subject to time varying parametric uncertainty $\theta(t)$, or can be models of linear time-varying plants or models resulting from linearization of nonlinear plants along trajectories of the parameter θ. The idea is to represent the system as an interpolation of local models. The transformation is realized without loss of information and the obtained system has the same state trajectory as the original system.

In the diagnosis system, the parameters and state variables of the system must be estimated online. The design of the observer and the study of stability should allow to develop a scheme to isolate sensor and actuator faults adequately. However, few works used a model-based state-estimation algorithm to detect the changes in the correlation among the state-variables (Basseville, 1988). These results have been extended to nonlinear descriptor systems. For this reason, the design of observers for singular systems has received considerable attention such as full and reduced-order observers (Darouach and Boutayeb, 1995), Proportional-Integral (PI) observer for systems with unknown inputs (Koenig and Mammar, 2002), UIO for nonlinear singular systems (Koenig, 2006) and Luenberger observer with parametric approach (Duan *et al.*, 2007).

For this class of dynamic, the fault detection and diagnosis (FDD) has been of considerable interest, where most of the existing methods are based on the design of appropriate observer. Some authors have proposed fault diagnosis systems for singular systems, as (Yeu *et al.*, 2005) where an UIO is considered for fault detection, isolation and reconstruction of actuator and sensor faults, but not simultaneously. In (Hamdi *et al.*, 2009), the design of a polytopic UIO for LPV singular systems, is able to estimate the states of the system in spite of the presence of unknown inputs.

Fault Detection and Isolation (FDI) is the first step in fault accommodation to monitor the system and to determine the fault location, and the fault estimation is utilized to determine online the fault magnitude. Then, an active Fault Tolerant Control (FTC) system can be obtained by *fault accommodation*, which controls the faulty system, or by *system reconfiguration*, which controls the healthy part or to compensate the system (Noura *et al.*, 2009). Accordingly, it is very important to a proper FDI scheme in many practical systems, because in absence of any fault and/or disturbance, the reconfiguration loop is inactive and does not affect the performance of the nominal closed-loop system. In the presence of a failure, the FTC system should be provides a corrective term in order to compensate the effects of the failure signals, (Zhiqiang *et al.*, 2010).

The main objective of this work is to develop an UIO that makes possible to integrate a certain degree of robustness in the state estimation thanks to the integral action. This scheme is appropriate for an adequate estimation of states and unknown inputs (disturbances). This scheme is applied for to obtain tray compositions estimation for non-ideal mixtures in a binary distillation column.

This paper is organized of the following form. In the Section II, the observer synthesis is presented. In the Section III-A, the nonlinear distillation column model is presented and the singular model for the binary distillation column is proposed. Finally in the Section IV, the results of the observer implemented for estimate the liquid composition of the light component and the disturbance are showed.

II. PROPORTIONAL-INTEGRAL OBSERVER DESIGN.

For LPV systems, the interpolation techniques present a good approach to get a polytopic structure. This structure is a set of linear model scheduled by weighting functions which represent polytopic LPV models (Rodrigues *et al.*, 2008). Taking this representation some authors as (Ichalal *et al.*, 2009), have developed a method for fault diagnosis using the polytopic models for nonlinear systems described by Takagi-Sugeno multiple models. Or as in (Hamdi *et al.*, 2009), that to represent the LPV descriptor system by a polytopic form and where observers are dedicated to detect, isolate and estimate actuator faults.

Consider the representation of a singular linear system is:

$$\begin{aligned} &\mathbf{E}\dot{\mathbf{x}}(t) = Ax(t) + Bu(t) + Rd(t) \\ &y(t) = Cx(t) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^k$, $d \in \mathbb{R}^l$ and $y \in \mathbb{R}^p$ are the state vector, the input vector, the disturbance vector and the output vector respectively. $E \in \mathbb{R}^{m \times n}$ is a singular matrix with $rank\ E = r < n$. $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{m \times k}$, $R \in \mathbb{R}^{m \times l}$ and $C \in \mathbb{R}^{p \times n}$ are known constant matrices. To carry out appropriate state estimation, the following definitions are established.

Definition 1: A singular system is observable if and only if:

i). rank $\begin{bmatrix} sE - A \\ C \end{bmatrix} = n$ and

ii). rank $\begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix} = n + rank(E)$

Definition 2: A singular system is called regular if: $det(sE - A) \neq 0 \quad \forall s \in \mathcal{C}$. It is a well-known result that the necessary and sufficient condition for detectability of the additive faults, is that the fault event vector is detectable if and only if the pair (A, C) is observable.

In this case, the system (1) can be written as a continuous-time polytopic singular LPV system of the form:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{J} \varepsilon_i(\rho(t)) \left(A_i x(t) + B_i u(t) + R_i d(t)\right) \\ y(t) &= Cx(t) \end{aligned} \tag{2}$$

where

$$\sum_{i=1}^{J} \varepsilon_i(\rho(t)) = 1, \;\; 0 \leq \varepsilon_i(\rho(t)) \leq 1 \tag{3}$$

Considering a singular system represented by the polytopic LPV descriptor model (2), the equations that government the PI Observer are:

$$\begin{aligned} \dot{Z}(t) &= \sum_{i=1}^{N} \varepsilon_i(\rho(t)) \left[N_i Z(t) + G_i u(t) + L_i y(t) + H_i \hat{d}(t)\right] \\ \hat{x}(t) &= Z(t) + My(t) \\ \dot{\hat{d}}(t) &= \sum_{i=1}^{N} \varepsilon_i(\rho(t)) \Phi_i (y(t) - \hat{y}(t)) \end{aligned} \tag{4}$$

where $\hat{x}(t), z(t) \in \mathbb{R}^n$ and $\hat{d}(t) \in \mathbb{R}^p$ are the estimate state vector, state observer vector and estimate unknown input respectively. N_i, G_i, L_i, H_i, M and Φ_i are unknown matrices for the PI observer that should be calculated. The observer (4) has an estimation error definite by:

$$\begin{aligned} &e(t) = x(t) - \hat{x}(t) \\ &e(t) = (I_{n+1} - MC)x(t) - Z(t) \end{aligned} \tag{5}$$

where I_n represents the identity matrix of order n, then on can defined a real matrix $U \in \mathbb{R}^{n \times n}$ such that

$UE = I_n - MC$, so for $\begin{bmatrix} E \\ C \end{bmatrix}$ is full rank column,

$$\begin{bmatrix} U & M \end{bmatrix} = \begin{bmatrix} E \\ C \end{bmatrix}^{+} \tag{6}$$

where the superscript $^{+}$ represents the inverse generalized matrix, and the estimation error can be rewritten as:

$$e(t) = UEx(t) - Z(t) \tag{7}$$

Is possible suppose that the unknown inputs are bounded and their dynamic is slow, i.e., $\dot{d}(t) \simeq 0$ not sensor bias faults under consideration. Then, for $\delta(t) = d(t) - \hat{d}(t)$, the unknown input derive is defined as:

$$\dot{\delta}(t) = -\dot{\hat{d}}(t) \tag{8}$$

The estimation error dynamic is written as:

$$\begin{aligned} \dot{e}(t) = \sum_{i=1}^{N} \varepsilon_i(\rho(t)) \left[(UA_i - L_i C - N_i UE)x(t)\right. \\ \left.(UB_i - G_i)u(t) + (UR_i - H_i)d(t) + H_i \rho(t) + N_i e(t)\right] \end{aligned} \tag{9}$$

where the following conditions can be defined:

$$UA_i = N_iUE - L_iC \quad (10)$$

$$G_i = UB_i \quad (11)$$

$$H_i = UR_i \quad (12)$$

$$I_{n+1} = UE + MC \quad (13)$$

From (4), (7) and (9), estimation error and the unknown input dynamic is:

$$\dot{e}(t) = \sum_{i=1}^{N} \varepsilon_i(\rho(t))(N_ie(t) + H_i\delta(t)) \quad (14)$$

$$\dot{\delta}(t) = \sum_{i=1}^{N} \varepsilon_i(\rho(t))(-\Phi_iC)e(t) \quad (15)$$

and the following function can be established:

$$\begin{bmatrix} \dot{e}(t) \\ \dot{\delta}(t) \end{bmatrix} = \sum_{i=1}^{N} \varepsilon_i(\rho(t)) \begin{bmatrix} N_i & H_i \\ -\Phi_iC & 0 \end{bmatrix} \begin{bmatrix} e(t) \\ \delta(t) \end{bmatrix} \quad (16)$$

Then, the state estimation error (16) converge asymptotically to zero if $Re\lambda_i \begin{bmatrix} N_i & H_i \\ -\Phi_iC & 0 \end{bmatrix} < 0$, i.e., are stables. Matrices N_i, L_i, G_i and H_i must be determined by the following equations:

$$UA_i = N_i(I_{n+1} - MC) + L_iC \quad (17)$$

$$N_i = UA_i - (L_i - N_iM)C \quad (18)$$

$$K_i = L_i - N_iM \quad (19)$$

$$N_i = UA_i - K_iC \quad (20)$$

and

$$L_i = K_i + N_iM \quad (21)$$

From (18) and (19) the state estimation error (16) can be rewritten as:

$$\begin{bmatrix} \dot{e}(t) \\ \dot{\delta}(t) \end{bmatrix} = \sum_{i=1}^{N} \varepsilon_i(\rho(t))(\bar{A}_i - \bar{K}_i\bar{C}) \begin{bmatrix} e(t) \\ \delta(t) \end{bmatrix} \quad (22)$$

where $\bar{A}_i = \begin{bmatrix} UA_i & H_i \\ 0 & 0 \end{bmatrix}$, $\bar{K}_i = \begin{bmatrix} K_i \\ \Phi_i \end{bmatrix}$ and $\bar{C} = [C \;\; 0]$.

Then, the PI observer (4) for a singular system LPV with inputs unknown (2) exists and their estimation error converge asymptotically to zero, if only if, the pairs $(\bar{A}_i, \bar{C})$ are detectable $\forall i = 1, 2, ..., N$. This observer is asymptotically stable if exists a positive definite symmetric matrix P and matrices $W_i = P\bar{K}_i$ such that the following the Linear Matrix Inequality (LMI's) holds:

$$(\bar{A}_i^T P + P\bar{A}_i - \bar{C}^T W_i^T - W_i\bar{C}) < 0, \;\; \forall i \in 1, 2, ..., N. \quad (23)$$

The observer gains can be calculated from $\bar{K}_i = P^{-1}W_i$. For ensuring the stability and convergence of the observation error, is possible to define in the left part of the complex plan a bounded area S with a line of abscissa $(-\sigma)$ where $\sigma \in \mathbb{R}^+$, and LMI's defined in (23) must be replaced by the following inequalities:

$$\begin{aligned} &(\bar{A}_i^T P + P\bar{A}_i - \bar{C}^T W_i^T - W_i\bar{C}) + 2\sigma P < 0, \\ &\forall i \in 1, 2, ..., N. \end{aligned} \quad (24)$$

then, consequently $\hat{x}(t)$ will asymptotically converge to $x(t)$ and $\hat{d}(t)$ to $d(t)$.

III. Binary distillation column model.

To understand the dynamic properties of a distillation column, it is necessary to have a good appreciation of its steady-state behavior. To carry out the distillation operation, it is important to known the vapor-liquid equilibrium correlations or properly estimate them. In most cases these relationships are a critical factor because they are nonlinear functions of temperature, pressure and composition. For this reasons the following assumptions are considered in the model formulation (Luyben, 1992):

(A1) Constant pressure.
(A2) Ideal liquid-vapor equilibrium.
(A3) Liquid properties behave as a non-ideal mixture.
(A4) Negligible molar vapor holdup compared to the molar liquid holdup.
(A5) Boiler as a theoretical tray.
(A6) Total condenser.
(A7) Constant liquid volumetric hold up.

The nonlinear characteristic (more common in chemical systems at low pressure) is the non-ideality of the liquid phase. In consequence, specially designed models are used to represent these non-idealities. For low pressure systems, the equation that represents the vapor composition of the desired component is:

$$y_pP_T = P_i^{sat}x_p\gamma_i \quad (25)$$

where x_p and y_p are the liquid and vapor compositions in the tray p respectively. P_i^{sat} is the vapor partial pressure, P_T is the total pressure and γ_i is the activity coefficient of each component. This is a correction factor highly dependent on the concentration and one method to determine this coefficient uses the Van Laar equation:

$$\begin{cases} ln\gamma_1 = A_{12}\left(\dfrac{A_{21}(1-x)}{A_{12}x + A_{21}(1-x)}\right)^2 \\ ln\gamma_2 = A_{21}\left(\dfrac{A_{12}(1-x)}{A_{12}x + A_{21}(1-x)}\right)^2 \end{cases} \quad (26)$$

where A_{12} and A_{21} are two interaction parameters established for a binary mixture and x is the liquid composition. Their values for the ethanol-water mixture can be found in (Perry, 1999).

The column model is divided into four basic models, which represent: the condenser x_1, a tray x_p, the feeding tray x_f and the boiler x_N. The enthalpies of the process are considered constant, therefore, the energy balance is not taken into account in this model. The dynamic model of a binary distillation column obtained from the given assumptions results in the following system of differential equations:

$$\begin{cases} M_1 \dfrac{d(x_1)}{dt} = V_2 y_2 - L_1 x_1 - D x_1 \\ M_p \dfrac{d(x_p)}{dt} = V_R(y_{p+1} - y_p) + L_R(x_{p-1} - x_p) \\ with\ p = 2, ..., f-1 \\ M_f \dfrac{d(x_f)}{dt} = V_R(y_{f+1} - y_f) + L_R(x_{f-1} - x_f) + F(z_F) \\ M_p \dfrac{d(x_p)}{dt} = V_S(y_{p+1} - y_p) + L_S(x_{p-1} - x_p) \\ with\ p = f+1, ..., N-1 \\ M_N \dfrac{d(x_N)}{dt} = L_S x_{N-1} - V_S y_N - B x_N \end{cases} \tag{27}$$

where F and z_F are the molar feeding flow and molar composition. D and B are the distilled and bottom product respectively. V_R, L_R and V_S and L_S are the vapor and liquid molar flow on the rectifying and the stripping section. M_p is the molar hold-up on each tray and N is the total number of trays.

For industrial columns the dimension of the above dynamic model is generally large. Even the most simple dynamic model of the distillation column proved to be complex due to the fact that the system of nonlinear differential equations has to equal the number of column plates. But, it can be reduced by time-scale considerations. We propose to simplify the nonlinear model as a singular model of a binary distillation column, keeping the dynamics and physical properties.

III-A. Binary distillation column singular model

Some developments based on singular models provide powerful tools to incorporate nonlinear aspects to design strategies for fault diagnosis and control, as in (Yang *et al.*, 2006). In this perspective a simplified representation of binary distillation column is proposed, maintaining the dynamics and physical properties of the plant. Taking into account the assumptions mentioned previously and the dynamics of the plant shown in Fig. 1, the equations that describe the balance for each component are shown in (27).

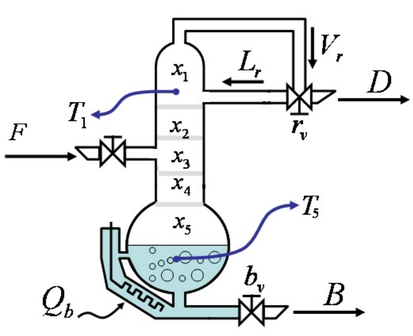

Figure 1. Scheme of a distillation column of five trays.

This system can be rewritten as $\dot{x} = f(\mathbf{x}, \mathbf{L}, \mathbf{V}, \mathbf{F}, \mathbf{z_F})$ where:

$$\begin{cases} f_1(x, L, V, F, z_F) = \dfrac{Vy_2 - Vx_1}{M_1} \\ f_p(x, L, V, F, z_F) = \dfrac{Vy_{p+1} - Vy_p + Lx_{p-1} - Lx_p}{M_p} \\ for \quad p = 2 \\ f_{p_f}(x, L, V, F, z_F) = \dfrac{Vy_{p+1} - Vy_p + Lx_{p-1} - (L+F)x_{p_f} + Fz_F}{M_{p_f}} \\ for \quad p_f = 3 \\ f_p(x, L, V, F, z_F) = \dfrac{Vy_{p+1} - Vy_p + Lx_{p-1} - (L+F)x_p}{M_p} \\ for \quad p = 4 \\ f_N(x, L, V, F, z_F) = \dfrac{(L+F)x_{N-1} - (L+F-V)x_N - Vy_N}{M_N} \end{cases} \tag{28}$$

According to (Levine and Rouchon, 1991), f is linear with respect to $\mathbf{L}, \mathbf{V}, \mathbf{F}$ and $\mathbf{z_F}$, so it is assumed that $\mathbf{L}, \mathbf{V}, \mathbf{F}$ y $\mathbf{z_F}$ are continuous in time, i.e., $t \in [0, +\infty)$ s.t. $\forall t, \mathbf{L}(t) < \mathbf{V}(t) < (\mathbf{L}(t) + \mathbf{F}(t))$. Therefore, it is possible to establish the following theorem:

Theorem 1 (Levine and Rouchon, 1991):

i. For each initial condition x^0 in the closed subset $[0, 1]$, the maximal solution of $\dot{x} = f(\mathbf{x}, \mathbf{L}, \mathbf{V}, \mathbf{F}, \mathbf{z_F})$ is defined for every $t \in [0, +\infty)$ and satisfies $x(t) \in [0, 1] \forall t \in [0, +\infty)$.
ii. For each $\mathbf{L}, \mathbf{V}, \mathbf{F}$ y $\mathbf{z_F}$ there exists a unique steady-state $\bar{x}(t) \in [0, 1]$ namely a unique solution of $f(\bar{\mathbf{x}}, \mathbf{L}, \mathbf{V}, \mathbf{F}, \mathbf{z_F}) = 0$, moreover, if γ_i satisfies $\gamma_i x > x \quad \forall x \in [0, 1]$, then $\bar{x}$ satisfies $1 > \bar{x}_1 > \bar{x}_2 > ... > \bar{x}_{N-1} > \bar{x}_N$
iii. If $\mathbf{L}, \mathbf{V}, \mathbf{F}$ and $\mathbf{z_F}$ are constant and if $x^0 \in [0, 1]$, then the system is Lyapunov-stable and its solution converges to the unique steady state associated to $\mathbf{L}, \mathbf{V}, \mathbf{F}$ and $\mathbf{z_F}$. Moreover, for every $\mathbf{L}, \mathbf{V}, \mathbf{F}$ and $\mathbf{z_F}$ the Jacobian matrix has real, distinct and negative eigenvalues.

The system can be described as a combination of slow and fast dynamics that under appropriate considerations, can be reduced to a slow dynamics only, using the Tikhonov theorem described in (Levine and Rouchon, 1991). For physical reasons, the behavior of every tray is similar to any other, the resident time in one tray is much shorter than the resident in the condenser or boiler. According to this, the reduced model is described by the differential-algebraic system:

$$\begin{cases} M_1 \dot{x}_1 = V(y_2 - x_1) \\ \quad 0 = V(y_3 - y_2) + L(x_1 - x_2) \\ M_3 \dot{x}_3 = V(y_4 - y_3) + Lx_2 - (L+F)x_3 + Fz_F \\ \quad 0 = V(y_5 - y_4) + (L+F)(x_3 - x_4) \\ M_5 \dot{x}_5 = (L+F)x_4 - (L+F-V)x_5 - Vx_5 \end{cases} \tag{29}$$

The substitution of the algebraic equations into the three differential equations, preserves the *tridiagonal structure* of the original system (27) and it also keeps the open loop behavior of the model physical. The system defined in (29) can be represented as a singular model, which is rewritten in the following compact form:

$$\left\{ \begin{array}{l} \mathbf{E}\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t) + \mathbf{R}d(t) \\ y(t) = \mathbf{C}x(t) \end{array} \right. \tag{30}$$

where $x = [x_1 x_2 ... x_5]^T$ is the state vector, $u = [LV]^T$ is the input vector and $d = [F z_F]^T$ is the disturbance vector. A nonlinear system that is linearized along a trajectories, can be reformulated as a LPV system with parametric dependence.

Then, the new model can be viewed as a multi-linear system in which the system matrices are set by known operation points (Rodrigues *et al.*, 2008). In this case, the system (30) can be written as a continuous-time polytopic singular LPV system of the form (2) and the parameter $\rho(t)$ varies in a convex polytope of vertices ρ_i such that $\rho(t) \in Co\{\rho_1; \rho_2, ..., \rho_J\}$.

IV. Simulations results.

Let us consider the model of a binary distillation column, taking into account four operation points that are achieved by manipulating the reflux valve, where of the molar flows variations are produced. These operations points represents different ethanol concentrations that can be achieved during the process distillation. The observer was synthesized using the model that describes the vapor-liquid equilibrium of the binary mixture ethanol-water, which estimates the ethanol composition at every tray.

Multi-model LPV representation of the nonlinear dynamic system is given by the follow set of matrices:

$$E = \begin{bmatrix} 0.073 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.69 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 189.4 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -1.9354 & 2.2216 & 0 & 0 & 0 \\ 0.9468 & -3.1684 & 2.4797 & 0 & 0 \\ 0 & 0.9468 & -4.4265 & 2.7913 & 0 \\ 0 & 0 & 1.9468 & -4.7381 & 4.2455 \\ 0 & 0 & 0 & 1.9468 & -4.2569 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0.0903 & -0.0442 \\ 0.0980 & -0.0261 \\ 0.0753 & -0.0757 \\ 0.1842 & -0.3018 \end{bmatrix} \quad R_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.0424 & 1 \\ 0.0753 & 0 \\ 0.1842 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1.5668 & 1.7686 & 0 & 0 & 0 \\ 0.9468 & -2.7154 & 1.9655 & 0 & 0 \\ 0 & 0.9468 & -3.9123 & 2.1737 & 0 \\ 0 & 0 & 1.9468 & -4.1205 & 3.4688 \\ 0 & 0 & 0 & 1.9468 & -3.8488 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0.0812 & -0.0491 \\ 0.1023 & -0.0247 \\ 0.0684 & -0.0850 \\ 0.2100 & -0.3032 \end{bmatrix} \quad R_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.0582 & 1 \\ 0.0684 & 0 \\ 0.2100 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1.5668 & 1.7225 & 0 & 0 & 0 \\ 1.3154 & -3.0379 & 1.8830 & 0 & 0 \\ 0 & 1.3154 & -4.1984 & 2.0484 & 0 \\ 0 & 0 & 2.3154 & -4.3638 & 3.4532 \\ 0 & 0 & 0 & 2.3154 & -4.2018 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0 & 0 \\ 0.0663 & -0.0557 \\ 0.1032 & -0.0266 \\ 0.0659 & -0.0974 \\ 0.2468 & -0.3025 \end{bmatrix} \quad R_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.0940 & 1 \\ 0.0659 & 0 \\ 0.2468 & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -1.9354 & 2.1583 & 0 & 0 & 0 \\ 1.3154 & -3.4737 & 2.3722 & 0 & 0 \\ 0 & 1.3154 & -4.6876 & 2.6292 & 0 \\ 0 & 0 & 2.3154 & -4.9446 & 4.2669 \\ 0 & 0 & 0 & 2.3154 & -4.6469 \end{bmatrix}$$

$$B_4 = \begin{bmatrix} 0 & 0 \\ 0.0745 & -0.0506 \\ 0.0996 & -0.0280 \\ 0.0740 & -0.0986 \\ 0.2217 & -0.3026 \end{bmatrix} \quad R_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.0769 & 1 \\ 0.0740 & 0 \\ 0.2217 & 0 \end{bmatrix}$$

Inputs (molar flows L and V respectively) are presented in Fig. 2 and changes in the operating point were made. The weighting functions $\varepsilon_i(\rho(t))$, that characterize the dynamic behavior of the nonlinear system and its evolution is illustrated.

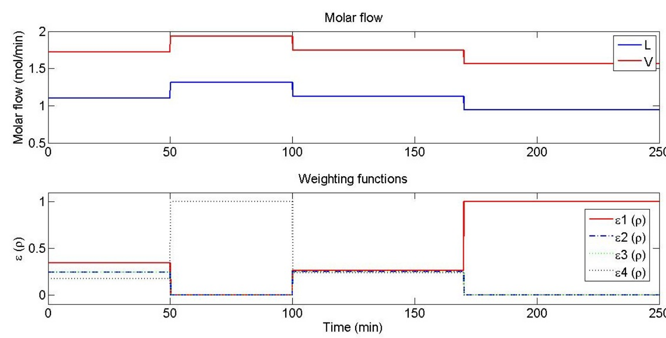

Figure 2. Inputs and weighting functions.

In order to show the effectiveness of the used modeling method, the nonlinear state and the multi-model approximation are depicted in the Fig. 3 in open-loop and without noise. This figure shows a comparison between the LPV singular system and nonlinear system. For reasons of space, only states corresponding to the condenser, feed tray and boiler are presented in this article.

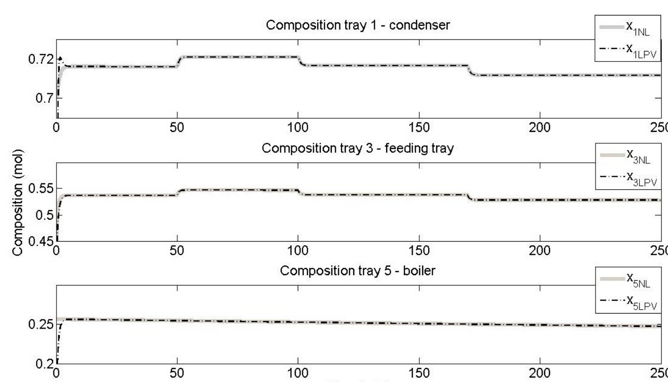

Figure 3. States of nonlinear system vs LPV singular model.

The effectiveness of the proposed observer scheme is illustrated in Fig. 4, with the system studied in open-loop. Here, the states of the nonlinear system with their estimation is showed. The estimation error are depicted in Fig. 5.

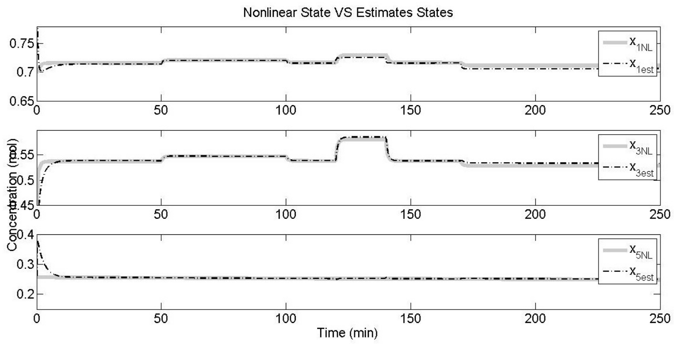

Figure 4. States of nonlinear system and estimated states.

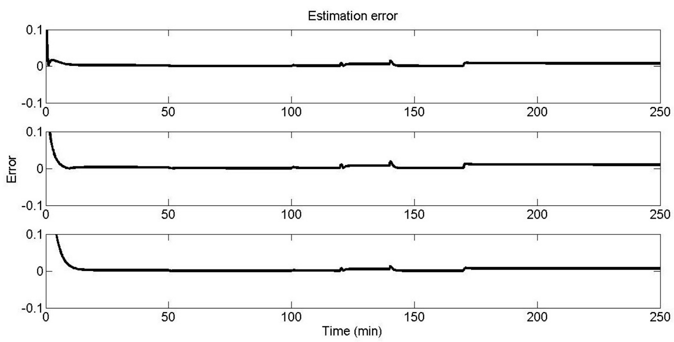

Figure 5. Estimation error $e = \mid x - \hat{x} \mid$.

For the descriptor LPV system (2) the unknown input is modeled as an impulse of magnitude 0.08 applied for $120 \leq t \leq 140$. The estimation of the unknown input is illustrated in Fig. 6.

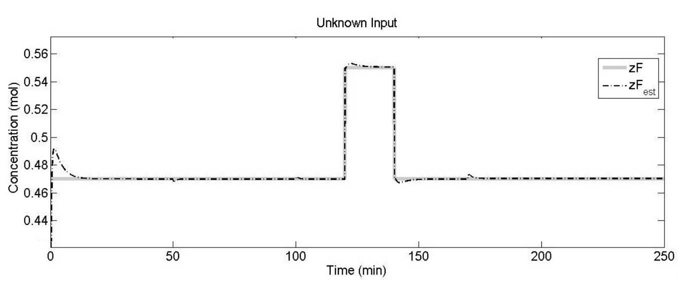

Figure 6. Unknown input and corresponding estimated.

V. CONCLUSION.

In this paper the application of a PI observer for singular systems has been presented. The proposed observer is evaluated using a simplification of a nonlinear model for a binary distillation column which can be seen as LPV singular system. The observer proposed is designed to estimate the states and the unknown input of the system. The effectiveness of this algorithm is evaluated via simulations using the Yalmip Toolbox (Lofberg, 2004) for solving the LMI's of Ec. (24). This strategy is applied for a nonlinear model of a binary distillation column and to estimate the tray compositions for non-ideal mixtures.

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