A flatness approach for the control of vibratory systems: a case study of an elastic robot

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***Abstract*— We study flatness based control of vibratory systems. A summary of the theory of differential flat systems is presented, including results concerning linearization and controllability. A case of study is analyzed, and for it, an explicit flat input is calculated. Some numerical simulations are carried out regarding the state variables and the flat input.**

Index terms: Vibratory systems, flatness based control, differential algebra.

I. INTRODUCTION

According to most textbooks in theoretical mechanics, a *vibration* is defined as an oscillatory motion of a mechanical system about its equilibrium. The simplest case of a vibratory system is the so-called *one degree of freedom linear vibratory system* (ODF), that includes an spring coupled with a damper, as in figure 1.

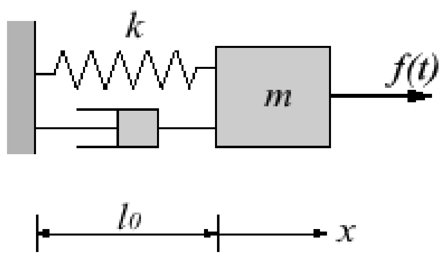

Fig. 1. Linear vibration of the ODF system

For this elementary vibratory system the mathematical model is provided by the linear non-homogeneous differential equation of second order:

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = f(t), \tag{1}$$

where ω is the natural frequency of the undamped system, ζ is the damping factor and $f(t)$ external force, which is usully taken, in the control theory viewpoint, as the control parameter.

It is well known that this model can be extended to the case of a system with a finite number of degrees of freedom; a natural presentation of such a system consists of a finite number of springs coupled with a finite number of damping devices, see for instance (J.M. Krodkiewski, 2008).

From the practical viewpoint, it is natural to pursue the control of the vibrations, since they can produce undesirable behavior of the mechanisms. Generally speaking there are, at least, two approaches for tackling the vibratory phenomena: the passive approach based on techniques of the vibro-isolation and the active one, that pursues an on-line control of the vibration, see (M. Barcik et al., 1998).

In this paper we follow the active approach, considering the vibratory system as a closed loop non-linear control system, and applying for it the so-called flatness based control. We analyze a case of study of a vibratory system which is inspired in a robotic mechanism consisting of a prismatic pair coupled with a revolute and containing an oscillatory element in the end-effector.

Apart from this introduction the paper contains seven sections, in section II we present a general scheme for vibratory system under a control theoretic point of view, including the general description of the so-called *flatness based control* approach. In section III the general framework for flat differential systems is presented, including some elements of differential algebra which is part of the mathematical background of the theory. Section IV is devoted to the description of equivalence of differential systems including a summary of results concerning linearization an controllability. In section V a case study for a vibratory system is introduced, the corresponding Euler-Lagrange equations are written as a non-linear control system. Section VI analyzes the flatness of the system, an explicit calculation of a flat input is carried out. Section VII provides some numerical simulations regarding the state variables and the input, and finally in section VIII some conclusions and perspectives for future work are presented.

II. INTRODUCTION TO THE CONTROL OF VIBRATORY SYSTEMS

A vibratory system can be written, in general, as the higher dimensional generalization of equation (1), as follows

$$M\ddot{x} + Bx + Kx = P. \tag{2}$$

Where the state variable x belongs to a smooth manifold, and matrices M, B and K have the appropriate dimensions.

It is assumed that the damping takes place within certain rank, in such a way that matrix B can be considered to be independent of vibration. Furthermore, if the frequency of input P is equal to one of the natural frequencies of the system, then small input amplitudes cause resonance that should be incorporated to the model. For tackling the vibration phenomena two problems should be addressed: decreasing the input forces, (change P), increasing natural vibration frequency (choose appropriate M and K), see (M. Barcik et al., 1998)

If a harmonic force $F(\omega)$ is added to an oscillator, then the mass oscillates at the same the same frequency of the applied force but with a certain phase shift. The amplitude of the vibration $X(\omega)$ can be written in terms of $F(\omega)$ and the constant of the spring. Furthermore, the vibration problem can be stated as an open loop system where the force is the input and the vibration the output. By representing the force and vibration in the frequency domain (magnitude and phase), the system can be described as: $X(\omega) = H(\omega) \cdot F(\omega)$, where $H(\omega)$ is the so-called the frequency response function, also referred to as the transfer function.

There is an extensive literature on mechanical vibrations, we refer the reader to the classical book (J.P. Hartog, 1956), and for a more modern treatment the volume (B.H. Tongue, 2002).

A. *Flatness based control of vibrations*

Vibration control is a relevant problem for both theoretical and experimental viewpoints, it is natural to pursue the suppression of vibration or to control it in order to allow the mechanism to behave within an acceptable regime.

Various techniques have been applied for tackling the vibratory phenomena, in general two approaches can be distinguished: the active and the passive. The passive vibration control is based mainly on isolation of the causes of vibration (vibro-isolation), parametric modifications, structural modifications, and vibration damping. Whereas the active vibration control leads to structural and/or parametric modification using additional energy sources under a feedback-feedforward scheme.

Flatness based control, FBC allows to design the control as a combination of a feedforward control and a stabilizing control. Roughly speaking a system $\dot{x} = f(x,u)$ is flat, if there is a function $h = (h_1, \ldots, h_{\dim u})$ depending on the state x and on a finite number of derivatives of the control parameter u, such that the solution (x, u) of the system $\dot{x} = f(x,u), \quad y = h(x, \dot{u}, \ddot{u}, \ldots)$, does not involve any differential equation at all and can be written explicitly as $x = \phi(\dot{y}, \ddot{y}, \ldots), u = \psi(\dot{y}, \ddot{y}, \ldots)$, for certain smooth functions ϕ and ψ, the variable y is called the *flat-output* of the system, see (P. Rouchon, 2005). In the next section we shall go into details on differential flat systems. In conclusion, in the FBC technique one provides a nominal input trajectory $u^*(t)$ which forces the system to a desired trajectory $y^*(t)$ as illustrated in figure 2

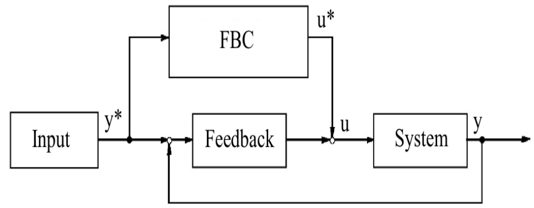

Fig. 2. Flatness based control

In order to apply the flatness technique it is assumed that a desired trajectory can be calculated off-line under consideration of limitations of input variables and states. Moreover, it is assumed that simple and standard controller design, for instance pole placement, are possible, taking into consideration nonlinearities, stabilization of disturbances and model errors , see (A. Ast and P. Eberhard, 2006)

III. Differentially flat systems

The concept of differential flat systems finds its mathematical foundations in David Hilbert's 22th problem about the uniformization of analytic relations by means of meromorphic functions (D. Hilbert, 1902). Although the main ideas were set in the so-called *equivalence method* for differential systems of E. Cartan (E. Cartan, 1914), which is a technique in differential geometry for determining whether two geometrical structures are the same up to a diffeomorphism.

The equivalence method is an essentially algorithmic procedure that has been successfully applied in problem in differential geometry and geometric control theory, see (R. Gardner, 1989) and (R. Gardner and W. Shadwick, 1990) . By contrast, it should be said that the problem of flatness characterization, in general an open mathematical problem, is not at all of algorithmic nature. The problem of flatness is similar to the problem characterization of hamiltonian systems: within the class of under-determined ODE's, flat systems play very much the same role that integrable systems play within the class of determined ODE's , (P. Rouchon, 2005).

The theory of differentially flat systems was introduced by M. Fliess and collaborators based in the mathematical formalism of differential algebras, (M. Fliess et al., 1992). Since then, there have been and increasing growth in the literature and a complete recollection of it goes beyond of this manuscript. We refer the reader to (M. Fliess et al., 1995) where the concept of defect characterizes the lack of flatness, (J.B. Pomet, 1995) where an alternative viewpoint is developed through the notions of dynamic equivalence and linearization and using the language of jet bundles, and the excellent volume (H. Sira-Ramírez and S.K. Agrawal, 2004) where a fine recollection of examples is presented .

A differential system

$$\dot{x} = f(x,u), \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m \quad m \leq n$$

is said to be differentially flat if there is a vector $y \in \mathbb{R}^m$, which is called the flat output, such that

1) $y, \dot{y}, \ddot{y}, \ldots$ are linearly independent
2) y is a function of x and a finite number of derivatives of u
3) There are two functions ϕ and ψ such that

$$x = \phi(y, \dot{y}, \ldots, y^{(\alpha)}), \quad u = \psi(y, \dot{y}, \ldots, y^{(\alpha+1)}),$$

for certain multi-index $\alpha = (\alpha_1, \ldots, \alpha_m)$ and

$$y^{(\alpha)} = \left(\frac{d^{\alpha_1} y_1}{dt^{\alpha_1}}, \ldots, \frac{d^{\alpha_m} y_m}{dt^{\alpha_m}}\right)$$

As was mentioned before, there is no a general rule for determining wether or not a given system is flat. Although many examples of engineering interest are flat, and in some cases an educated guessing allows to find the flat output y having a relevant physical interpretation.

IV. EQUIVALENCE OF SYSTEMS

Some of the results about flatness are better understood, by means of the equivalence of systems technique. For then let $\mathcal{M}$ be a differential manifold and let $F \in C^\infty(T\mathcal{M}, \mathbb{R}^{n-m})$, an implicit system is written as follows

$$F(x, \dot{x}) = 0, \quad \text{rank}\left(\frac{\partial F}{\partial \dot{x}}\right) = n - m$$

Any system $\dot{x} = f(x, u)$ can be taken into this form: rank$[f_u] = m$ implies $u = \mu(x, \dot{x}_{n-m+1}, \ldots, x_n)$, for then

$$F_i(x, \dot{x}) = \dot{x}_i - f_i(x, \mu(x, \dot{x}_{n-m+1}, \ldots, x_n))$$

Two systems $(\mathcal{M}, F), (\mathcal{N}, G)$ with rank$\,(F_{\dot{x}}) = n - m$ and rank$\,(G_{\dot{y}}) = p - q$ are equivalent in $x_0 \in \mathcal{M}$ and $y_0 \in \mathcal{N}$ if:

1) There is $\Phi = (\varphi_1, \varphi_2, \ldots) \in C^\infty(\mathcal{N}, \mathcal{M})$ such that

$$\Phi(y_0) = x_0, \quad \frac{d\varphi_i}{dt} = \varphi_{i+1}$$

and any solution $t \mapsto y(t)$ of $G(y, \dot{y}) = 0$ satisfies $F(\varphi_1(y(t)), \varphi_2(y(t))) = 0$

2) There is $\Psi = (\psi_1, \psi_2, \ldots) \in C^\infty(\mathcal{M}, \mathcal{N})$ such that

$$\Psi(x_0) = y_0, \quad \frac{d\psi_i}{dt} = \psi_{i+1}$$

and any solution $t \mapsto y(t)$ of $F(x, \dot{x}) = 0$ satisfies $G(\psi_1(x(t)), \psi_2(x(t))) = 0$

If two systems are equivalent then they have the same co-ranks $m = q$.

Given a trajectory $t \mapsto x(t)$ of system $F(x, \dot{x}) = 0, x \in \mathcal{M}$, the implicit system

$$\left(\frac{\partial F}{\partial x}(x, \dot{x})\right)\xi(t) + \left(\frac{\partial F}{\partial \dot{x}}(x, \dot{x})\right)\dot{\xi}(t) = 0, \quad \xi \in T\mathcal{M}$$

is called *the linear approximation* around x

Proposition 4.1: If two systems are equivalent then the corresponding linear approximations are also equivalent.

Definition 4.1: $(\mathcal{M}, F)$ is flat in x_0 if it is equivalent to $(\mathbb{R}^m, 0)$, that is, if trajectories $t \mapsto x(t)$ are the image of a trivialization Φ, such that, $\Phi(y_0) = x_0$. Equivalently, for each curve $t \mapsto y(t)$

$$x(t) = (x, \dot{x}, \ddot{x}, \ldots) = \Phi(\varphi_1(y(t)), \varphi_2(y(t)), \ldots)$$

Proposition 4.2: If a system is flat then it is equivalent to its linear approximation.

Proposition 4.3: If $(\mathcal{M}, F)$ is flat in x_0, then

1) Its linear approximation is controllable.
2) If x_0 is an equilibrium point, the system is locally controllable around x_0.

V. THE FLATNESS OF THE ELASTO-ROBOT

We now present a particular case inspired in a robotic mechanism consisting of a prismatic pair coupled with a revolute and having a vibratory element at the end-effector, see figure 3.

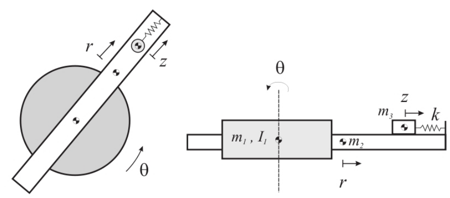

Fig. 3. Robot with oscillatory end-effector

The parameters involved are the following

$$\begin{aligned} a &= \text{Disk radius} \\ \theta &= \text{Angular displacement} \\ r &= \text{Parallel displacement} \\ m_2 &= \text{Prismatic-pair mass} \\ z &= \text{Vibration} \\ m_3 &= \text{Terminal-effector mass} \end{aligned}$$

In order to write the Euler-Lagrange equations we consider the kinetic (T_i) and potential (V_i) energies for each of the elements, obtained by means of the superposition principle, κ denotes the constant associated to the vibration.

Revolute.

$$\begin{aligned} T_1 &= \frac{1}{2} I \dot{\theta}^2 \\ V_1 &= 0 \end{aligned}$$

Prismatic pair.

$$\begin{aligned} T_2 &= \frac{1}{2} m_2 (\dot{r}^2 + r^2 \dot{\theta}^2) \\ V_2 &= 0 \end{aligned}$$

Terminal-effector.

$$\begin{aligned} T_3 &= \frac{1}{2}m_3((\dot{r}-\dot{z})^2+(r-z)^2\dot{\theta}^2) \\ V_3 &= \frac{1}{2}(z-r^2\kappa \end{aligned}$$

The Lagrangian is the following

$$\begin{aligned} \mathcal{L} &= 2\left[T_1+T_2+T_3-(V_1+V_2+V_3)\right] \\ &= I\dot{\theta}^2+(m_2+m_3)\dot{r}^2+(m_2+m_3)r^2\dot{\theta}^2+m_3\dot{z}^2 \\ &\quad -2m_3\dot{r}\dot{z}-m_3z\dot{\theta}^2-r^2\kappa-z^2\kappa+2rz\kappa, \end{aligned}$$

From which we get the Euler-Lagrange equations

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right)-\frac{\partial\mathcal{L}}{\partial\theta} &= \tau_1, \\ \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{r}}\right)-\frac{\partial\mathcal{L}}{\partial r} &= \tau_2, \\ \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{z}}\right)-\frac{\partial\mathcal{L}}{\partial z} &= 0, \end{aligned}$$

for then

$$\begin{aligned} \left(I+m_2r^2+m_3\left(r-z\right)^2\right)\ddot{\theta} &+ \\ 2m_3\left(\dot{r}r-z\dot{r}-\dot{z}r+\dot{z}z\right)\dot{\theta} &+ \\ 2m_2r\dot{r}\dot{\theta} &= \tau_1, \\ \left(m_2+m_3\right)\ddot{r}-m_3\ddot{z}-m_2r\dot{\theta}^2 &- \\ m_3r\dot{\theta}^2+m_3z\dot{\theta}^2+\kappa\ r-\kappa\ z &= \tau_2, \\ m_3\ddot{z}-m_3\ddot{r}-m_3z\dot{\theta}^2+m_3r\dot{\theta}^2 &- \\ \kappa\ r+\kappa\ z &= 0, \end{aligned}$$

The torque forces $(u,v)=(\tau_1,\tau_2)$, are control parameters. We define the state variables

$$\begin{aligned} x_1 &= \theta, & x_4 &= \dot{x}_1, \\ x_2 &= r, & x_5 &= \dot{x}_2, \\ x_3 &= z, & x_6 &= \dot{x}_3, \end{aligned}$$

For then $M(x)\dot{x}+V(x,\dot{x})+G(x)=\tau$, coordinates $x=(x_1,x_2,x_3,x_4,x_5,x_6)$ in the manifold

$$\mathcal{M}=(0,2\pi)\times(0,R)\times(0,Q)\times(0,2\pi)\times(0,R)\times(0,Q)$$

for certain fixed values for R and Q.

$M(x)$ denotes the inerty matrix, $V(x,\dot{x})$ the Coriolis vector and $G(x)$ the vector potential. By writing $I+m_2x_2^2+m_3(x_2-x_3)^2=J$, with $J>I$, and assuming $J=1$, $m_2=1$, $a^2-x^2>0$, with $a=\sqrt{J-I}$, we get

$$x_2-x_3=\frac{1}{\sqrt{m_3}}\sqrt{a^2-x_2^2}, \tag{3}$$

in conclusion we obtain the following non-linear control system

$$\begin{aligned} \dot{x}_1 &= x_4 \qquad (4) \\ \dot{x}_2 &= x_5 \qquad (5) \\ \dot{x}_3 &= x_6 \qquad (6) \\ \dot{x}_4 &= -2m_2x_2x_4x_5-2\sqrt{m_3}x_4(x_5-x_6)\sqrt{a^2-x_2^2} \\ &\quad +u \qquad (7) \\ \dot{x}_5 &= x_2x_4^2+v \qquad (8) \\ \dot{x}_6 &= \frac{\kappa}{\sqrt{m_3^3}}\sqrt{a^2-x_2^2}+x_3x_4^2+v \qquad (9) \end{aligned}$$

VI. Flatness-based control

We now show that the above control system is flat, for that we consider the output

$$y=(L_1,L_2)=(x_1,x_2). \tag{10}$$

Equation 8 yields

$$v=\ddot{L}_2-L_2L_1^2, \tag{11}$$

for then equation 7 implies

$$\ddot{L}_1=-2L_2\dot{L}_1\dot{L}_2-2\sqrt{m_3}\dot{L}_1(\dot{L}_2-x_6)\sqrt{a^2-L_2^2}+u,$$

Now by using (11) and (3), we get

$$\begin{aligned} x_6 &= \frac{\ddot{L}_1-\ddot{L}_2+2L_2\dot{L}_1\dot{L}_2+L_2L_1^2}{2\sqrt{m_3}\dot{L}_1\sqrt{a^2-L_2^2}}+\dot{L}_2, \qquad (12) \\ x_3 &= L_2-\frac{1}{m_3}\sqrt{a^2-L_2^2}, \end{aligned}$$

and together with

$$\begin{aligned} x_1 &= L_1, & x_4 &= \dot{L}_1, \\ x_2 &= L_2, & x_5 &= \dot{L}_2, \end{aligned}$$

$$\begin{aligned} u &= \frac{\dot{L}_1(L_1^{(3)}-L_2^{(3)}+2L_2\ddot{L}_1\ddot{L}_2+2L_2\ddot{L}_1\dot{L}_2)}{2\sqrt{m_3}\dot{L}_1^2\sqrt{a^2-L_2^2}} \\ &+ \frac{\dot{L}_1(2\dot{L}_1\dot{L}_2^2+2L_1L_2\dot{L}_1+\dot{L}_2L_1^2)}{2\sqrt{m_3}\dot{L}_1^2\sqrt{a^2-L_2^2}} \\ &+ \frac{(\ddot{L}_1-\ddot{L}_2+2L_2\dot{L}_1\dot{L}_2+L_1L_2^2)(\dot{L}_1\dot{L}_2L_2)}{2\sqrt{m_3}\dot{L}_1^2(a^2-L_2^2)}, \qquad (13) \end{aligned}$$

completes the description of the system in terms of the flat output and a finite number of derivatives. In conclusion we can write

$$\begin{aligned} x &= \Theta(y,\dot{y},\ddot{y}) \\ u &= \Phi(y,\dot{y},\ddot{y}) \\ v &= \Psi(y,\dot{y},\ddot{y},y^{(3)}) \end{aligned} \tag{14}$$

Once x_1 and x_2 are controlled, so are x_3, x_4, x_5 and x_6 and open-loop controls are given by expressions (11) and (13).

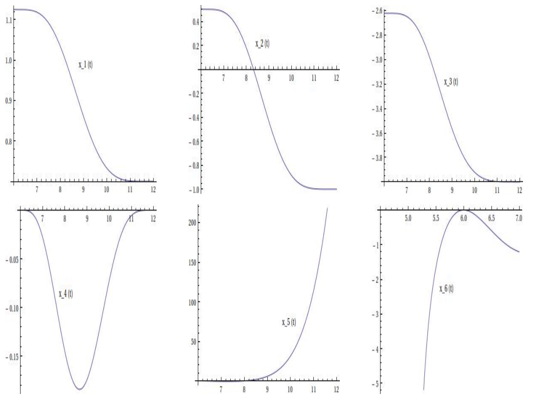

Fig. 4. Estimated state variables of the elast-robot

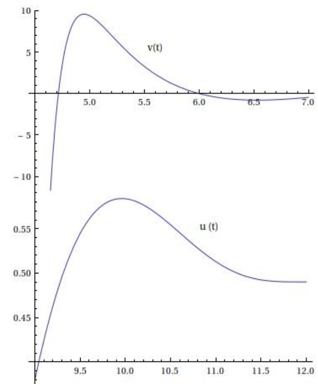

Fig. 5. Action of control parameters

VII. Simulation and numerical experiments

Given a desired reference trajectory for an angle $\theta = F(t)$ and a displacement $r = G(t)$, represented by $F^*(t)$ and $G^*(t)$, respectively, the desired law control can be obtained by

$$v = \ddot{L}_2 - L_2 L_1^2, \tag{15}$$

and

$$\begin{aligned} u &= \frac{\dot{L}_1(\tilde{u} - \tilde{v} + 2L_2\ddot{L}_1\ddot{L}_2 + 2L_2\ddot{L}_1\dot{L}_2)}{2\sqrt{m_3}\dot{L}_1^2\sqrt{a^2 - L_2^2}} \\ &+ \frac{\dot{L}_1(2\dot{L}_1\dot{L}_2^2 + 2L_1L_2\dot{L}_1 + \dot{L}_2L_1^2)}{2\sqrt{m_3}\dot{L}_1^2\sqrt{a^2 - L_2^2}} \\ &+ \frac{(\ddot{L}_1 - \ddot{L}_2 + 2L_2\dot{L}_1\dot{L}_2 + L_1L_2^2)(\dot{L}_1\dot{L}_2L_2)}{2\sqrt{m_3}\dot{L}_1^2(a^2 - L_2^2)}, \end{aligned} \tag{16}$$

where

$$\tilde{v} = \ddot{F}^*(t) - \lambda_2(\dot{F}(t) - \dot{F}^*(t)) - \lambda_1(F(t) - F^*(t)), \tag{17}$$

$$\tilde{u} = \ddot{G}^*(t) - \gamma_2(\dot{G}(t) - \dot{G}^*(t)) - \gamma_1(G(t) - G^*(t)), \tag{18}$$

for certain parameters λ_1, λ_2, γ_1 and γ_2. In practice, this control law is difficult to synthesize, however, through the use of Laplace transform, one can write the system like system string, resulting in the trivial integration.

Figure 4 presents the evolution of the state vector for the flat-output, whereas figure 5 does it for the control parameters.

VIII. Conclusions and perspectives

In this paper we have presented a brief introduction to vibratory systems together with a general FBC strategy. A definition of differentially flat system in the framework of differential geometric control theory was presented, together with results about controllability and linearization.

For a particular case of study, consisting of a mechanism of a revolute coupled with a prismatic pair and a oscillatory end-effector we have proposed a flat output that provides the synthesis along with some basic simulation of the state variables.

A future research should include a more general setting for vibratory systems as closed loop systems together with theoretical results linking the vibratory nature of the system with flatness and defect (lack of flatness). In particular causal reasons for defect, should be investigated for general vibratory systems. Further research could pursue a completion of a catalog of flat vibratory systems. Comparative numerical simulation with various extreme cases should be implemented as well as with some other vibration control paradigms.

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