Congreso Nacional de Control Automático 2006
18 - 20, Octubre, 2006, UNAM, México, DF

ISBN 970-32-4025-9

Passivity-based Control of a Hybrid Asymmetric Nine-Level Inverter

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***Abstract*— In this paper the problem of achieving a low current Total Harmonic Distortion for a low voltage single phase non-linear load, using the Passivity Based Control applied to a hybrid asymmetric nine-level inverter is shown. The controller was designed taking account of the passivity properties of the averaged state model. A simulation study was carried out in order to evaluate the performance of the control designed by the passivity approach. The simulation results show the low current THD reached in the current injected to the electrical system when the inverter is used to compensate the current harmonics of the non-linear devices. The main contribution of this paper is the calculation of two laws that allows to control two converters at different switching frequencies.**

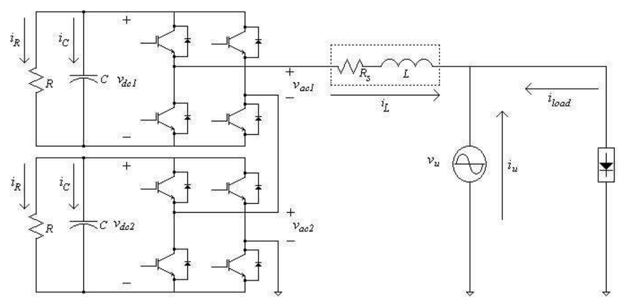

Fig. 1. Hybrid Asymmetric Multilevel Inverter

I. Introduction

The proliferation of non-linear devices in the electrical power systems has increased the amount of harmonic currents. Nowadays, equipment such as computers, printers, variable frequency drives, Uninterruptible Power Systems (UPS), Programmable Logic Controllers (PLC), among others are often found in industrial and commercial electrical systems. All these devices have in common that they draw a non-sinusoidal current. Since the electrical system is not designed to operate with this kind of current, unwanted effects as well as unwanted behavior in the equipment powered from the electrical system, are very common. Overheated conductors, transformers and motors, circuit breakers nuisance trips, capacitor bank failures, erratic behavior in electronic equipment and voltage distortion are some of the symptoms when there are harmonic currents in the power system.

Several devices are used to mitigate harmonic currents. On one side passive notch filter can be used for this purpose, however, this device can not mitigate zero sequence harmonic currents in three phase systems, because of its high zero sequence impedance. On the other hand a K-factor transformer is a good solution to block zero sequence harmonics, but its main drawback is that it can not block positive and negative sequence harmonics [1].

Another device that can be used to decrease the distortion caused by the harmonic currents is the active filter. This device has proved to be a very effective way to compensate harmonic currents in electrical systems [2]. Active filtering can be performed by hybrid asymmetrical multilevel inverters, which have been recently applied in low voltage applications [3].

A hybrid asymmetrical multilevel inverter has several arrays of power semiconductors and direct current capacitors, in order to generate a stepped voltage waveform. The addition of the output voltage of all the arrays permits to get a final high voltage, while the power semiconductors must withstand a lower voltage. Furthermore, this kind of inverter has had a wide development because it allows to reduce cost and power ratings [3].

Some of the the biggest advantages of the hybrid asymmetrical multilevel inverter are its low switching losses, low dv/dt and low voltage total harmonic distortion (because of the stepped waveform) [4]. The characteristics mentioned previously, as well as its almost sinusoidal currents allows the hybrid asymmetrical multilevel inverter to eliminate most of the harmonics [5].

The circuit of figure 1 shows a hybrid asymmetrical nine level inverter, which has two series connected converters, each one implemented with a H-bridge topology. In order to improve certain characteristics such as efficiency and compensation capacity, both converters are operated at different dc voltages and switching frequencies [2].

Since the switching frequencies are different in both converters, the converter with a lower switching frequency will compensate low order current harmonics, while the converter with a higher switching frequency will compensate high frequency current harmonics [2]. Due to the different voltage levels in the dc-link on each converter, this kind of inverter allows an improved resolution in the output voltage than the symmetrical inverters [6].

Despite all the advantages mentioned above, the main drawback of the hybrid asymmetrical multilevel inverters is the necessity of several separated dc sources [7], connected in series. This fact can limit the applications for this kind of inverters [8].

In this paper, a hybrid asymmetrical nine-level inverter with a capacitor voltage source in each converter is presented and its control laws are calculated using a passivity-based approach. This inverter was intended as a current harmonics filter for a low voltage system.

II. Problem formulation

The hybrid asymmetrical nine-level inverter shown in figure 1 has two converters. Each converter voltage has been identified as v_{dc1} and v_{dc2}. Both converters have a capacitor whose value is C and it is used as the voltage source. The inverter has a line reactance whose inductance value is L and a wiring resistance, R_s. The losses in each converter are represented by a resistor whose value is R. Although the capacitor and losses resistor values are the same in both converters they can have different values.

A non-linear device is connected to Point of Common Coupling (PCC), as well as the hybrid asymmetrical nine-level inverter. The non-linear device draws a current i_{load} while the current drawn by the inverter is i_L. The voltage delivered by the utility is represented by a sinusoidal voltage source v_u and i_u is the current drawn into the utility.

A. Inverter model

A brief introduction to the asymmetrical hybrid multilevel inverter was given in the last section, however other aspects must be considered as a part of the problem formulation:

- The voltage source for each converter is a capacitor and the voltage ratio between both converters is 1:3, i.e. $3v_{dc1} = v_{dc2}$.
- The switching frequency ratio between both converters is 3:1, i.e. $f_{c1} = 3f_{c2}$.

There are some remarks that must be taken into account. On one hand, the voltage ratio between both converters can be different, however, a ratio of 1:3 allows to get nine different levels using only two converters. On the other hand, the switching frequency ratio makes possible avoid repeating voltage levels during the switching of all converters; for example, the voltage level $-4v_{dc}$ is

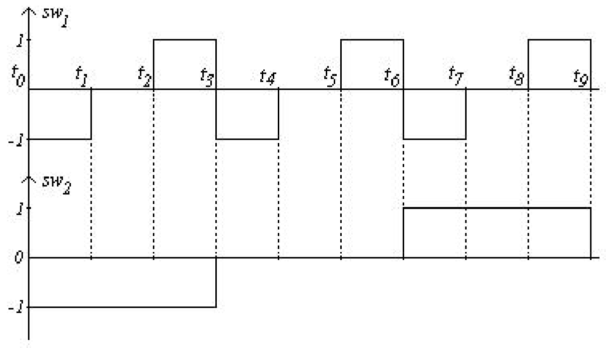

Fig. 2. Relation between switching frequencies

sw_1	sw_2	Level sw_1	Level sw_2	Output voltage
-1	-1	$-v_{dc1}$	$-v_{dc2}$	$-4v_{dc}$
0	-1	0	$-v_{dc2}$	$-3v_{dc}$
1	-1	v_{dc1}	$-v_{dc2}$	$-2v_{dc}$
-1	0	$-v_{dc1}$	0	$-v_{dc}$
0	0	0	0	0
1	0	v_{dc1}	0	v_{dc}
-1	1	$-v_{dc1}$	v_{dc2}	$2v_{dc}$
0	1	0	v_{dc2}	$3v_{dc}$
1	1	v_{dc1}	v_{dc2}	$4v_{dc}$

TABLE I
SWITCHING COMBINATIONS AND OUTPUT VOLTAGE LEVELS

generated only once during the switching as well as the other levels (as shown in table I). However, different voltage and frequency switching ratios can be used. Since that the combined frequency of the inverter is 420 Hz, the highest current harmonic that can be compensated is the seventh.

There are two control inputs, one for the high frequency converter, sw_1, and the other for the low frequency converter sw_2. These inputs take discrete values in the set $\{-1, 0, 1\}$. Since the switching frequency of sw_1 is three times the switching frequency of sw_2, there are nine possible combinations of the discrete values. If v_{dc2} is considered the reference voltage, i.e. $v_{dc} = v_{dc2}$, then nine different voltage levels can be generated by the hybrid asymmetrical nine-level inverter, because $3v_{dc1} = v_{dc2}$ as shown in table I.

Once the two inputs, sw_1 and sw_2, of the circuit of the figure 1 have been considered, the dynamic behavior of the complete inverter can be represented by the following set of differential equations

$$L\dot{x_1} + R_s x_1 - sw_1 x_2 - sw_2 x_3 = -V_u \quad (1)$$

$$C\dot{x_2} + sw_1 x_1 + R^{-1} x_2 = 0 \quad (2)$$

$$C\dot{x_3} + sw_2 x_1 + R^{-1} x_3 = 0 \quad (3)$$

where $x_1 = i_L$, $x_2 = v_{dc1}$ and $x_3 = v_{dc2}$

Due to the discrete nature of the inputs sw_1 and sw_2, the usual mathematical methods for analysis and design can not be used to solve this problem and find the control laws. To cope with this situation, an average technique was used to get a continuous approximation of both inputs, namely μ_1 and μ_2.

In order to get the averaged model for the circuit of figure 1, sw_1 and sw_2 were defined as piecewise functions

$$sw_1 = \begin{cases} -1 & t_0 < t \leq t_1 \quad t_3 < t \leq t_4 \quad t_6 < t \leq t_7 \\ 0 & t_1 < t \leq t_2 \quad t_4 < t \leq t_5 \quad t_7 < t \leq t_8 \\ 1 & t_2 < t \leq t_3 \quad t_5 < t \leq t_6 \quad t_8 < t \leq t_9 \end{cases}$$

$$sw_2 = \begin{cases} -1 & t_0 < t \leq t_3 \\ 0 & t_3 < t \leq t_6 \\ 1 & t_6 < t \leq t_9 \end{cases}$$

This functions allowed to get the following equivalent compact form for the hybrid asymmetric nine-level inverter

$$D_B\dot{x} + R_Bx + sw_1M_1x + sw_2M_2x = \varepsilon_B \tag{4}$$

where D_B=diag {L,C,C}, R_B=diag {R_s,R^{-1},R^{-1}}, $\varepsilon_B = [v_u, 0, 0]^T$ and

$$M_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; M_2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

In order to find the state, a differential equation such as $\dot{x} = A(u)x + B$ can be written like the following integral equation [9]

$$x(\tau_1) - x(\tau_0) = \int_{\tau_0}^{\tau 1} (A(u)x + B)d\tau$$

If a time interval T is defined as a sampling period for a PWM modulation scheme, with τ_0 and τ_1 denoting the principle and end of the period respectively, then considering the above equation and the time intervals specified for the switching positions sw_1 and sw_2, the average state model can be calculated dividing the state difference into T and taking limits as $T \to 0$ [9]. The averaged state model is

$$D_B\dot{x} + R_Bx + 3\mu_1M_1x + \mu_2M_2x = \varepsilon_B \tag{5}$$

where μ_1 and μ_2 belong to the continuous set {-1,1}.

and (1), (2) and (3) have the following equivalent averaged state model

$$L\dot{x_1} + R_sx_1 - 3\mu_1x_2 - \mu_2x_3 = -V_u \tag{6}$$

$$C\dot{x_2} + 3\mu_1x_1 + R^{-1}x_2 = 0 \tag{7}$$

$$C\dot{x_3} + \mu_2x_1 + R^{-1}x_3 = 0 \tag{8}$$

In the technique presented to calculate the averaged state model for the hybrid asymmetric nine-level inverter, the discrete values of the switching functions sw_1 and sw_2 were used to obtain the equivalent equations of the converter for every switching combination and finally get the averaged state model. In [10] a different approach was used to determine the averaged state model for a three-level converter used in a synchronous rectifier application because quadratic functions were obtained to relate the voltage and currents in the ac side with the voltage and currents in the dc side.

B. Control problem

Once the averaged state model has been determined, the main problem was to design a control law that allows to mitigate the harmonic currents of i_{load} while the voltage in the direct current capacitors reaches the desired values on each converter. This control objective implies that in order to mitigate the harmonic current of i_{load}, the desired current that must be reached is the current drawn by the load, but without the fundamental component of the current, thus cancelling some of the harmonic currents and i_{net} should be a almost sinusoidal current.

Based on the last statements, the formulation of the control problem is:

Given the hybrid asymmetric nine-level inverter model (6)-(8) design two control laws, $\mu_1 = \mu(x_1, x_2, x_3)$ and $\mu_2 = \mu(x_1, x_2, x_3)$ such that

$$\lim_{t\to\infty} x_1 - x_{1d} = 0; \quad \lim_{t\to\infty} x_2 - X_{2d} = 0; \quad \lim_{t\to\infty} x_3 - X_{3d} = 0$$

guarantee internal stability and with x_{1d}, X_{2d} and X_{3d} as the behaviors of the averaged state model

The following fact must be taken into account: the controller was designed for an averaged state model and even though simulations can be performed, an experimental verification should be done, in order to verify that the inherently discontinuous system can be successfully controlled by the two continuous control laws. In order to confirm the results obtained by the simulations, the experimental stage is being carried out at this moment.

III. CONTROLLER DESIGN

Considering the average state model (5), the total energy stored in the circuit is

$$V = \frac{1}{2}x^T D x$$

and because of the skew-symmetric structure of M_1 and M_2, the time derivative of V along the system trajectories is

$$\dot{V} = -x^T R_B x + x^T \varepsilon$$

This time derivative shows that the system is passive from the input ε to the output x.

Now the passivity-based controller design methodology can be applied. The design process of the controller is divided into two parts: *energy shaping* and *damping injection*.

If the error between the system behavior and the desired behavior is defined as $\tilde{x} = x - x_d$, the following dynamics are obtained

$$D_B\dot{\tilde{x}} + (R_B + 3\mu_1 M_1 + \mu_2 M_2)\tilde{x} = \Phi \quad (9)$$

$$\Phi = \varepsilon_B - \{D_B\dot{x}_d + (R_B + 3\mu_1 M_1 + \mu_2 M_2)x_d\} \quad (10)$$

where (9) is the error dynamic and (10) is the auxiliary dynamic.

In order to accomplish the energy shaping step, the next desired energy-like function is considered

$$V_d = \frac{1}{2}\tilde{x}^T D_B \tilde{x}$$

This function has global minimum at $\tilde{x} = 0$ because $D = D^T > 0$. If now V_d is the function that describes the energy storage in the system, it can be proved that (9) has also some passivity properties. The time derivative of V_d along the trajectories of the error dynamics is

$$\dot{V}_d = -\tilde{x}^T R_B \tilde{x} + \tilde{x}^T \Phi \quad (11)$$

which shows that the error dynamic (9) is passive from the input Φ to the output $\tilde{x}$, because $\dot{V}_d < \tilde{x}^T\Phi$. It is a fact well known that if $\Phi = 0$ then the minimum energy point $\tilde{x} = 0$ is stable [11]. In order to improve the stability properties of the system, damping is injected into the system considering that

$$\Phi = -K\tilde{x} \quad (12)$$

where the matrix K is

$$K = \begin{bmatrix} k_1 & k_2 & k_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

If (12) can be achieved then the time derivative of V_d would be

$$\dot{V}_d = -\tilde{x}^T(R_B + K)\tilde{x} \quad (13)$$

and the condition $R_B + K = (R_B + K)^T > 0$ implies that $\lim_{t\to\infty} \tilde{x} = 0$

But since K is not a symmetrical matrix the above condition can not be accomplished. The decomposition of the K matrix into a symmetrical matrix and a skew-symmetrical matrix allows to overcome this problem, because if $K = K_{sym} + K_{skew}$ then

$$\dot{V}_d = -\tilde{x}^T(R_B + K_{sym})\tilde{x} \quad (14)$$

and since $R_B + K_{sym} = (R_B + K_{sym})^T > 0$ then $\lim_{t\to\infty} \tilde{x} = 0$.

To guarantee that $R_B + K_{sym} = (R_B + K_{sym})^T > 0$ the following condition must be satisfied

$$R_s + k_1 - \frac{R}{4}(k_2^2 + k_3^2) > 0 \quad (15)$$

The auxiliary dynamic equation is

$$D_B\dot{x}_d + R_B x_d + 3\mu_1 M_1 x_d + \mu_2 M_1 x_d - k^T\tilde{x} = \varepsilon_B \quad (16)$$

where

$$k^T\tilde{x} = k_1\tilde{x}_1 + k_2\tilde{x}_2 + k_3\tilde{x}_3$$

And the next constraint between the voltage converters must also be satisfied in order to determine μ_1 and μ_2

$$n x_{2d} = x_{3d} \quad (17)$$

where n is the voltage ratio between the high frequency converter and the low frequency converter.

From (17) and (16) the following relation between μ_1 and μ_2 is obtained

$$\mu_2 = 3n\mu_1 \tag{18}$$

Finally, considering that the voltage ratio between both converters is $n = 3$, the control laws for the hybrid asymmetrical nine-level inverter are

$$\dot{\mu}_1 = \frac{v + 30(RC)^{-1}x_{2d}\mu_1 + 90C^{-1}x_{1d}\mu_1^2 - k^T\tilde{x}}{30x_{2d}} \tag{19}$$

$$\dot{\mu}_2 = \frac{v + \frac{10}{3}(RC)^{-1}x_{2d}\mu_2 + \frac{10}{9}C^{-1}x_{1d}\mu_2^2 - k^T\dot{\tilde{x}}}{\frac{10}{3}x_{2d}} \tag{20}$$

where

$$v = L\ddot{x}_{1d} + R_s\dot{x}_{1d} + \dot{V}_{red}$$

IV. Simulation Results

The first step for the simulation of the system with the control laws calculated in the previous section was to measure the current of a non-linear device. The current drawn by a desktop and a laptop computer, shown in brown color in figure 3, was measured with a power quality monitor and its spectrum was calculated 4.

The current THD of the two non-linear devices is 0.89 p.u. (89.16 %), most of it caused by the third harmonic whose root mean square value is almost 0.80 p.u. (80 %) of the rms value of the current fundamental component. This spectrum was considered as the desired current.

The desired voltages for both converters are 50 V, for the high frequency converter, and 150 V, for the low frequency converter. The inverter and the ac side (utility voltage source and line reactance) parameters are: $L = 5mH$, $R_s = 0.8\Omega$, $v_u = 127\sqrt{2}\sin 377t$, $R = 4M\Omega$ and $C = 22000\mu F$.

In order to find the best values for k_1, k_2 and k_3, several simulations were performed in Simulink using the averaged state model (5) and the control laws calculated (19) and (20). The values found for the damping injection are:

$$k_1 = 50,\ k_2 = 0,\ k_3 = 0$$

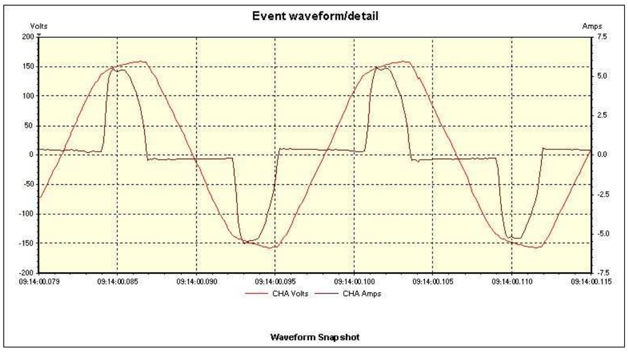

Fig. 3. Non-sinusoidal current

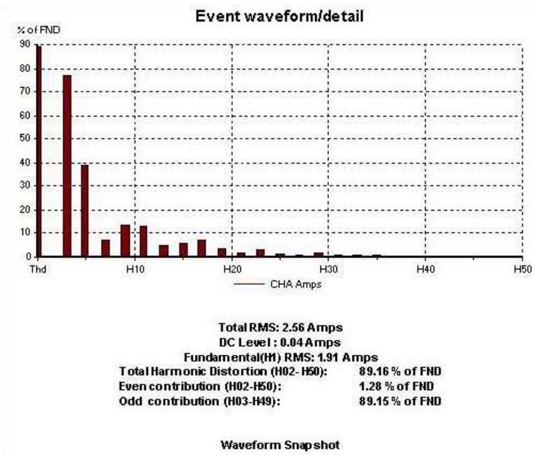

Fig. 4. Spectrum analysis of current

A very good approach to the desired current was obtained with $k_1 = 50$ and due to (15) the values for k_2 and k_3 are very small (approximately 5×10^{-5}), so both values were neglected and finally the values used for both were $k_2 = k_3 = 0$. Since a dc component was present in x_1, several test were performed to find the best initial conditions for the control laws. The best results were obtained with the following values

$$\mu_1(0) = -0.135,\ \mu_2(0) = 0$$

which allowed to reach the desired values for the voltage in both converters.

The simulations performed with the values mentioned above were intended to evaluate the steady state operation of the hybrid asymmetrical nine-level inverter. The voltage in the high and low frequency converters are shown in figures 5 and 6 respectively.

The THD with the active filter now is slightly below 0.05 pu, shown in fig 8, value that is below the maximum limit of 5% (0.05 pu) of typical susceptibility for critical power sources [12].

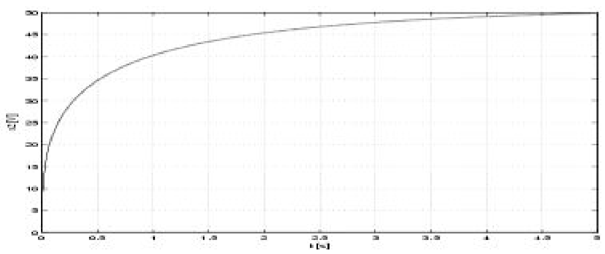

Fig. 5. Voltage of the high freq. converter, x_2

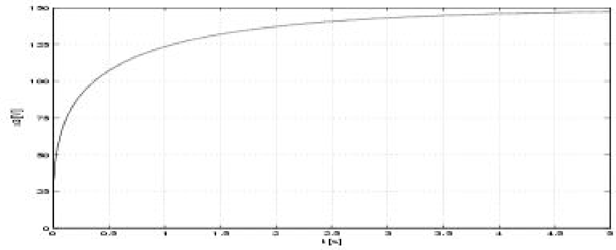

Fig. 6. Voltage of the low freq. converter, x_3

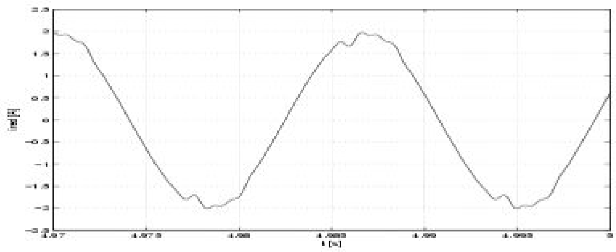

Fig. 7. Utility current, i_u

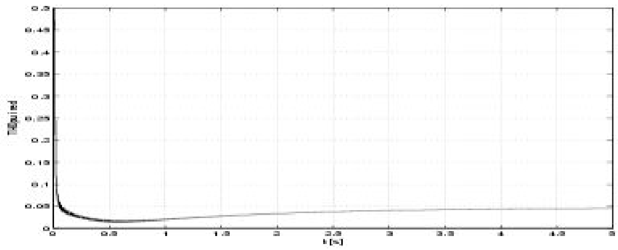

Fig. 8. THD utility current, pu

Because of the value of k_1, the start up time for x_2 and x_3 is approximately 5 seconds. When the value of k_1 was decreased the start up times improved, however the total harmonic distortion in x_1 was increased. Although a precharge method similar to the proposed in [13] based on the used or precharge resistors could have been used, in this first stage of the design of the control laws the possibility of precharging the capacitors was not considered.

V. CONCLUSIONS

A Passivity-Based Control was designed for a hybrid asymmetrical nine-level inverter achieving a good performance, because a low current THD was reached in the utility current i_u (less that 0.05 pu) in steady state. The Passivity-Based Control was possible since an averaged stated model was calculated. Two dynamic control laws were needed because it was required to control each converter at different switching frequencies, in order to reach a low utility current THD.

VI. ACKNOWLEDGEMENTS

The authors would like to thank the support given by Direccion General de Estudios de Posgrado, UNAM.

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