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Discrete Prediction Based Control for Continuous Unstable Linear Time-Lag Systems

B. del Muro-Cuellar*, M. Velasco-Villa†, J.J. Álvarez-Ramírez‡, O. Jiménez-Ramírez*
*Instituto Politécnico Nacional, SEPI-ESIME-Culhuacán,
Av. Santa Ana 1000 Col San Francisco Culhuacán, CP. 04430, México D.F.
†CINVESTAV-IPN, Departamento de Ingeniería Eléctrica, Mecatrónica,
Apdo. 14-740, 07000, México DF, México.
‡Universidad Autónoma Metropolitana–Iztapalapa

***Abstract*— This work consider the regulator problem for unstable linear input-delay systems. A discrete time control strategy intended to compensate the effects of the involved time delay and to stabilize the overall closed loop system, is designed. The proposed control strategy provides a prediction of the system output, which is used to solve the considered regulation problem in the same way that it is used in a classical Smith predictor compensator.**

Keywords: Time lag systems, prediction,discretization, sampled data control.

I. Introduction

Dynamical processes with delayed inputs and states are commonly found in the modeling of electronic, mechanical, biological or chemical systems. Natural time delays can be associated to electronic components or to computational time caused by complex control algorithm. Also, time delay systems can be due to natural modeling, for example, in the case of population and chemical processes modeling (Kolmanovskii and Myshkis, 1992). When a time delay affects the input (or output) signal of the system, a common approach is to eliminate the effect of the delayed signal to deal with a system free of delay. An approximation approach uses Taylor or Pade expansions of the delay operator (Marshall, 1979; Hu and Wang, 2002). For linear systems, the most common strategy is the so-called Smith predictor compensator (SPC) (Smith, 1957; Palmor, 1996; Martinez and Camacho, 2005), which provides an estimate of future outputs to be incorporated within a feedback control function (see Figure 1). The regulation of dead time systems is analyzed in (Mirkin and Raskin, 2003) where the solution is provided by a observer prediction structure.

The main drawback of the original SPC was related to the class of systems were it could be implemented, since it was restricted to stable plants. In order to overcome this problem, a modification that allows the consideration of processes with an integrator and long time delay was reported (Astrom et al., 1994)(Matausek and Micic, 1996; Normey-Rico and Camacho, 2001). A SPC for unstable plants was proposed by (Xian et al., 2005). Further results by considering a discrete-time representation of the process was studied in (Torrico and Normey-Rico, 2005).

This paper focuses on the regulator problem for unstable time delayed linear systems. Note that the classical SPC cannot be used for this class of systems because the process instability forbids a stable cancellation of the time delay operator. To solve this situation, the proposed approach uses an observer-based scheme to obtain an estimate of the delay-free output. Subsequently, such estimate is used into a feedback function to stabilize the plant. To carry out the control strategy, based on the equivalent discrete time model of the system, a discrete time state predictor that is capable of stabilize the overall closed loop system and produce the desired output prediction, is designed.

The paper is organized as follows. Section II presents the class of systems and the classical SPC is briefly recalled. Section III develops the prediction discrete-time strategy. To show the performance of the proposed discrete time control strategy, some simulation experiments are presented in Section IV. Finally, Section V, presents some conclusions.

II. Class of systems and problem formulation

This section presents the class of systems involving time delays at the input signal (or equivalently, at the output). Consider the following class of (possibly unstable) SISO linear systems with delayed input:

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}u(t-\tau) \\ y(t) &= \bar{C}\bar{x}(t) \end{aligned} \tag{1}$$

where $\bar{x} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input, $y \in \mathbb{R}$ is the output, and $\tau \geq 0$ is the time-delay associated to the input. $\bar{A} \in \mathbb{R}^{n\times n}$, $\bar{B} \in \mathbb{R}^{n\times 1}$ and $\bar{C} \in \mathbb{R}^{1\times n}$ are matrices and vectors of system parameters and are assumed to be known. The input-output representation of system (1) can be obtained as usually by considering the Laplace transform of (1) that leads to the following expression:

$$\begin{array}{rcl} s\bar{X}(s) & = & \bar{A}\bar{X}(s) + \bar{B}e^{-\tau s}U(s) \\ Y(s) & = & \bar{C}\bar{X}(s). \end{array}$$

This expression can be rewritten as

$$\begin{array}{rcl} \frac{Y(s)}{U(s)} & = & \bar{C}\left[sI - \bar{A}\right]^{-1}\bar{B}e^{-\tau s} \\ & = & \frac{N(s)}{D(s)}e^{-\tau s} = G(s)e^{-\tau s} \end{array} \quad (2)$$

where $N(s)$ and $D(s)$ are polynomials in the variable s. Note that a traditional output feedback control strategy as

$$U(s) = [R(s) - Y(s)]Q(s)$$

leads to a closed loop system of the form

$$\frac{Y(s)}{R(s)} = \frac{Q(s)G(s)e^{-\tau s}}{1 + Q(s)G(s)e^{-\tau s}}$$

where the term $e^{-\tau s}$ in the denominator complicates the stability analysis of the feedback system.

II-A. *Smith predictor compensator*

A classical SPC for a system of the class (2) is shown in Figure 1. The main idea behind a SPC strategy is based on the modeling of the system as

$$W(s) = G(s)U(s) \quad (3a)$$

$$Y(s) = e^{-\tau s}W(s), \quad (3b)$$

and to design an estimator (predictor) for the intermediate signal $W(s)$ (not available for measurement). The smith predictor control scheme use this signal in the controller $Q(s)$ depicted also in Figure 1, in order to compensate the effects of the time delay $e^{-\tau s}$ on the overall closed loop system. It is easy to see that the closed loop system is for the compensation strategy of Figure 1 it is given by,

$$\frac{Y(s)}{R(s)} = \frac{G(s)Q(s)}{1 + G(s)Q(s)}e^{-\tau s}$$

Under ideal conditions ($i.e.$, exact knowledge), the SPC allows to keep out of the closed loop the time delay term. Unfortunately, the classical Smith predictor scheme is able to deal only with stable plants (Palmor, 1996; Smith, 1957), and several modifications of the same strategy can only manage with some special class of unstable systems (Astrom et al., 1994; Majhi and Atherton, 1998; Matausek and Micic, 1996; Torrico and Normey-Rico, 2005; Xian et al., 2005).

Based on the original Smith predictor idea (estimation of signal $W(s)$), regardless of the stability of the system, in the following a methodology based on the discrete-time model of system (1) (equivalently (2)) that allows the design of an alternative discrete-time predictor for signal $W(s)$, is presented. The convergence of the error prediction is assured by considering the decomposition of the discrete time delay and the use of an appropriate output injection strategy.

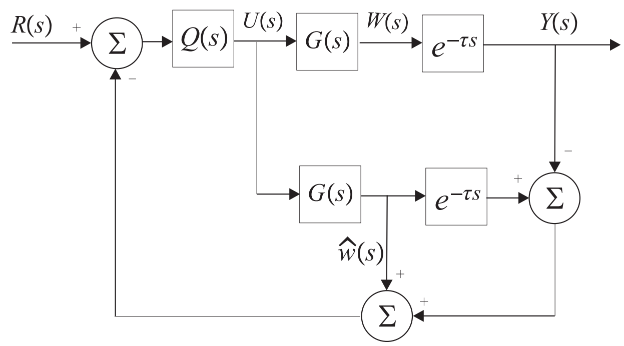

Figura 1. The Smith Predictor Scheme.

III. Prediction strategy

In order to described the alternative prediction strategy proposed in this work, consider the discretization (Astrom and Wittenmark, 1997) of system (1) (equivalently, 2) under the assumption that the input time delay satisfies $\tau = \alpha T$ and subject to a sampling period T. For doing this, consider now $G(z)$, the z-transform of $G(s)$, under the action of a sampling and hold device (a zero order hold for instance). A discrete state representation (observable and controllable) can easily be obtained for $G(z)$ (or equivalently for system (1) with $\tau = 0$ (system without delay) as,

$$\begin{array}{rcl} v(k+1) & = & A_p v(k) + B_p u(k) \\ w(k) & = & C_p v(k) + D_p u(k) \end{array} \quad (4)$$

Note that in the case of a zero order hold, the matrices A_P, B_P, C_P can be directly obtained from system (1) with,

$$A_p = e^{\bar{A}T} = \mathcal{L}^{-1}(sI - \bar{A})^{-1}\mid_{t=T}$$

$$B_p = \int_0^T e^{\bar{A}(T-\tau)}\bar{B}d\tau$$

$$C_p = \bar{C},\ D_p = \bar{D}$$

where $\mathcal{L}^{-1}$ is the inverse of the Laplace operator. In order to simplify the developments of the paper it is assumed that the discretization process produces a system for which $D_p = 0$.

Defining,

$$\hat{x}(k) = \left[\begin{array}{cccc} \hat{x}_1(k) & \hat{x}_2(k) & \cdots & \hat{x}_\alpha(k) \end{array}\right]^T,$$

the prediction $\hat{w}(k)$ of the signal $w(k)$ is obtained by a predictor of the form,

$$\begin{array}{l} \hat{x}(k+1) = \left[\begin{array}{ccccc} 0 & 1 & & & \\ 0 & 0 & 1 & & \\ & & & \ddots & \\ & & & & 1 \\ 0 & & & & \end{array}\right]\hat{x}(k) \\ \quad + \left[\begin{array}{c} 0 \\ \\ 0 \\ 1 \end{array}\right]\hat{w}(k) - G_1 e_y(k) \end{array} \quad (5)$$

$$
\begin{aligned}
\hat{v}(k+1) &= A_p\hat{v}(k) + B_p u(k) - G_2 e_y(k) \\
\hat{w}(k) &= C_p\hat{v}(k)
\end{aligned} \tag{6}
$$

where

$$e_y(k) = y(k) - \hat{y}(k),\ \hat{y}(k) = \hat{x}_1(k)$$

and

$$
\begin{aligned}
G_1 &= \begin{bmatrix} g_1 & g_2 & .. & g_\alpha \end{bmatrix}^T \\
G_2 &= \begin{bmatrix} g_\alpha & g_{\alpha+1} & .. & g_{\alpha+n} \end{bmatrix}^T.
\end{aligned}
$$

The structure of the proposed predictor it is shown in Figure 2.

Considering the preceding developing it is now possible to formally state the prediction result of our work. This will be done in the following lemma,

Lemma 1: Consider system (1) and the compensator (5)-(6). It is always possible to find a real valued vector $G = [\ G_1^T \ \mid \ G_2^T\ ]^T$, $G_1 \in \mathbb{R}^{\alpha\times 1}$ $G_2 \in \mathbb{R}^{n\times 1}$ such that the output $\hat{w}(k)$ of the compensator (5)-(6) provides the estimation of the signal $w(k)$ for the original system (1) (equivalently, system (3a)). This is, $\lim\limits_{K\to\infty}[w(k)-\hat{w}(k)] = 0$.

Demostración: The discrete-time representation for the delay term e^{-hs} can be easily found as z^{-k}. A discrete state space representation for this model produces,

$$
\begin{aligned}
x_d(k+1) &= A_d x_d(k) + B_d u_d(k) \\
y(k) &= C_d x_d(k)
\end{aligned} \tag{7}
$$

with

$$x_d(k) = \begin{bmatrix} x_{d1}(k) & x_{d2}(k) & \cdots & x_{d\alpha}(k) \end{bmatrix}^T$$

$$
A_d = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{\alpha\times\alpha}
$$

$$
\begin{aligned}
B_d &= \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^T \in \mathbb{R}^{\alpha\times 1} \\
C_d &= \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{1\times\alpha}.
\end{aligned}
$$

From the basic properties of the z-transform it is well known that the two cascade systems $G(z)$ and $z^{-\alpha}$ (or (4) and (7)) are equivalent to system $G(z)z^{-\alpha}$. Then, considering $u_d(k) = w(k)$ the complete discrete-time model (observable) for system (1) takes the form,

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) \\
y(k) &= Cx(k)
\end{aligned} \tag{8}
$$

where $x(k) = [\ x_d(k)^T \ \ v(k)^T\ ]^T$, and the α-row of A is $[\ 0 \ \ .. \ \ 0 \ \ C_p\ ]$ and $x_\alpha(k) = w(k)$. Then, an output predictor for the original system can be design as a state observer for (8), as is given by equations (5)-(6). From the proposed prediction strategy it can be easily proved that $\lim\limits_{K\to\infty}[x(k) - \hat{x}(k)] = 0$ if the vector G is computed in such a way that the eigenvalues of $(A - GC)$ form a stable set. From the observability properties of the pair (A, C) it is assured the existence of the require design vector parameters G. Finally, from the fact that $w(k) = C_p v(k) = [\ \mathbf{0} \ \mid \ C_p\ ]x(k)$, it is obtained the desired result, this is, $\lim\limits_{K\to\infty}[w(k) - \hat{w}(k)] = 0$. ∎

IV. SIMULATION RESULTS

The aim of this section is to present some academic examples, all of them containing a dead-time in the forward path, to illustrate the goodness of the proposed prediction strategy. The first case study consists of an unstable first order system taken from (Xian et al., 2005). The performance of the system with the proposed predictor are compared with the results in (Xian et al., 2005) using the same compensator used in the referred paper.

The second case consists of a more complicate plant: a second order unstable system. It is exposed to parametric perturbations and to different initial conditions.

Example 1. Consider the unstable delayed system(Xian et al., 2005),

$$\frac{Y(s)}{U(s)} = \frac{4e^{-5s}}{10s-1} = G_1(s)e^{-5s}. \tag{9}$$

A discrete-time version of this system, considering a ZOH is given by,

$$G(z) = \frac{0{,}4207z^{-5}}{(z-1{,}105)} = G_1(z)z^{-5}.$$

Note that the term $G_1(z)$ corresponds to the subsystem without delay $G_1(s)$ and z^{-5} correspond to the discretization of the time delay term e^{-5s}. Now, an observable representation in state variables can be obtained by considering,

$$
\begin{aligned}
A &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0{,}4207 \\ 0 & 0 & 0 & 0 & 0 & 1{,}105 \end{bmatrix}, \\
B &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T, \\
C &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
D &= [0].
\end{aligned}
$$

In order to stabilize $(A - GC)$, the vector G is computed as,

$$G = [1{,}005{,}1{,}1105{,}1{,}2271{,}1{,}356{,}1{,}4984{,}3{,}9355]$$

that locates the poles of the system at $[0{,}1, 0, 0, 0, 0, 0]$.

As

$$
\begin{aligned}
\hat{w}(k) &= C_p v(k) = \begin{bmatrix} \mathbf{0} & \mid & C_p \end{bmatrix} \begin{bmatrix} x_d(k) \\ v(k) \end{bmatrix} \\
&= 0{,}4207x_6.
\end{aligned}
$$

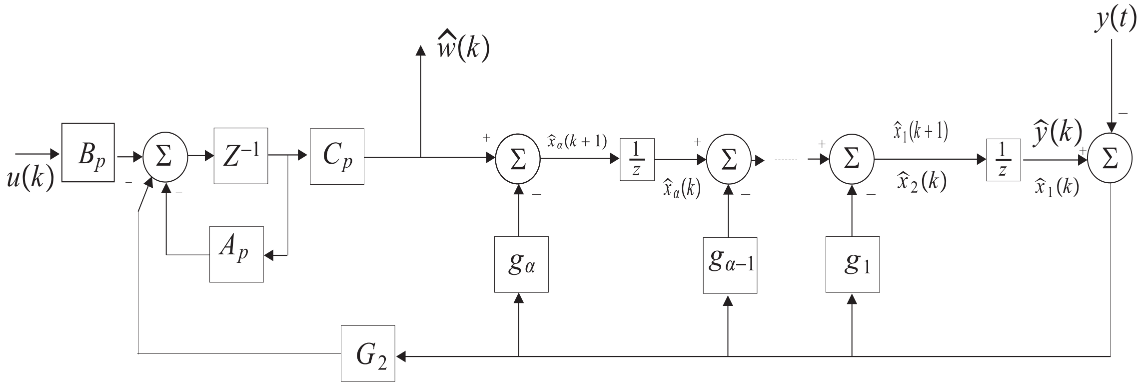

Figura 2. The proposed predictor.

The controller used in (Xian et al., 2005) can be implemented now by considering

$$u(k) = (1{,}25)r(k) - (1{,}5)\hat{w}(k) \tag{10}$$

with $r(k)$ an unitary step input.

In Figure 3 it is compared the output $y(t)$ of system (9)-(10) with the output $y1(t)$ obtained in (Xian et al., 2005). Although the performance of both systems are very similar, it is possible to see that when the initial conditions are such that $y(0) = 0{,}5$. The controller proposed in this work presents a better performance, getting a faster error convergence as can be observed in Figure 4. The robustness of the predictor can also be tested by considering a 20 % variation on its value; this is, $\tau = 0{,}52$ sec producing the results presented in Figure 5 where the disturbance is easily compensated. Finally, in figure 6, the performance of the prediction error $e_w(t) = w(t) - \hat{w}(t)$ it is shown for the signal $e(t)$ when the initial conditions are $y(0) = y1(0) = 0{,}5$ and $\tau = 5$. Signals $e1(t)$ and $e2(t)$ show the case when the initial conditions are set to zero and $\tau = 5{,}2$ and $\tau = 4{,}8$ respectively.

Example 2. Consider now the unstable second order time-delay system,

$$\frac{Y(s)}{U(s)} = \frac{(s+2)e^{-0{,}4s}}{(s+1)(s-2)}.$$

A discrete-time version of this system, considering a ZOH and $T = 0{,}1\,\text{sec}$. is,

$$G(z) = \frac{0{,}11588(z - 0{,}8182)z^{-4}}{(z - 0{,}9048)(z - 1{,}221)}.$$

In this case the observable representation in state space

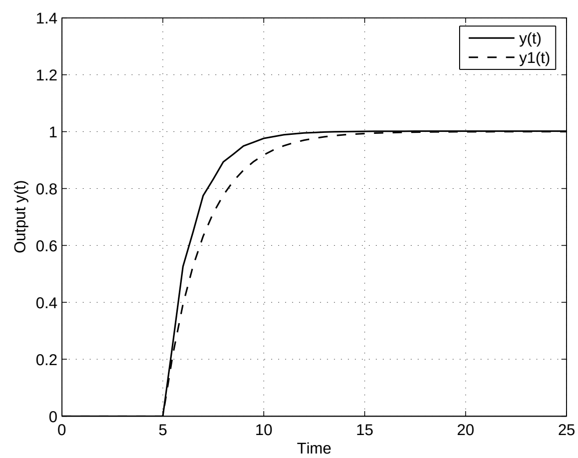

Figura 3. Output of both systems: ideal conditions.

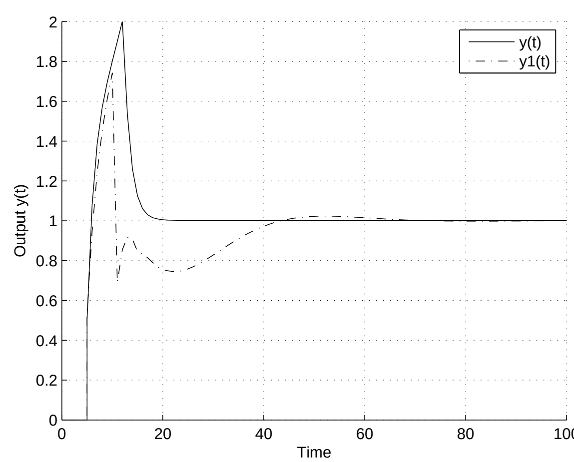

Figura 4. Output of both systems $y(t)$ and $y_1(t)$

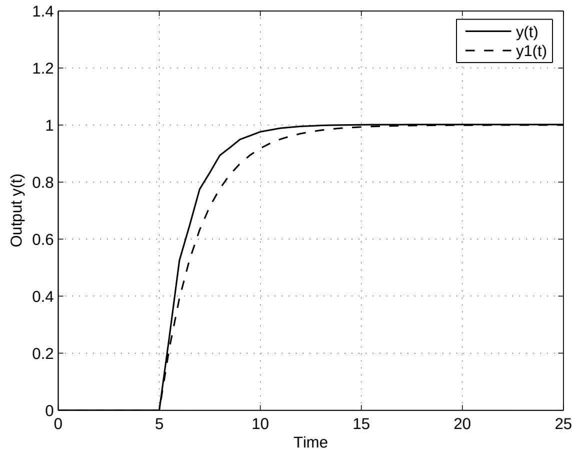

Figura 5. Output when the time delay $\tau = 0{,}5\,\mathrm{sec}$.

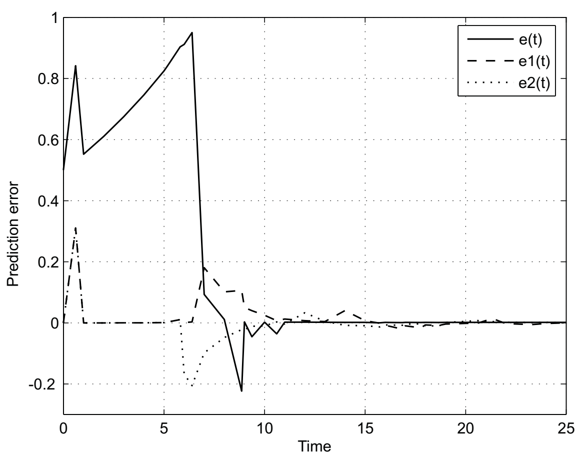

Figura 6. $e_w(t) = w(t) - \hat{w}(t)$ when the system is perturbed.

variables can be obtained as,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0{,}0948 & 0{,}1159 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1{,}1050 & 2{,}1260 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ D = [0]\,.$$

The closed loop poles are located at $[0{,}1, 0, 0, 0, 0, 0]$ by considering a vector,

$$G = [2{,}0260,\ 3{,}2023,\ 4{,}5693,\ 6{,}1758,\ 52{,}7997,\ 112{,}9090].$$

Considering,

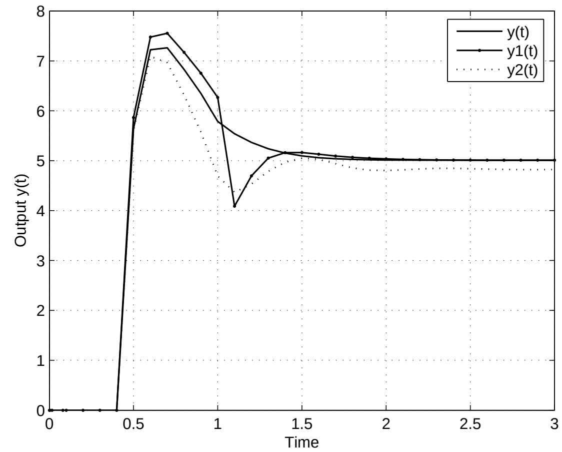

Figura 7. The output on ideal conditions and with disturbances.

$$\begin{aligned} \hat{w}(k) = C_p v(k) &= \left[\begin{array}{c|c} \mathbf{0} & C_p \end{array}\right] \begin{bmatrix} x_d(k) \\ v(k) \end{bmatrix} \\ &= \begin{bmatrix} -0{,}0948 & 0{,}1159 \end{bmatrix} \begin{bmatrix} x_5 & x_6 \end{bmatrix}^T, \end{aligned}$$

it is implemented the controller,

$$u(k) = H(z)[r(k) - \hat{w}(k)]$$

where $H(z)$ provides a digital PI action and it is given as,

$$H(z) = \frac{9{,}7(z - 0{,}9048)}{(z-1)}.$$

In Figure 7 it is shown performance of the regulation strategy: Signal $y(t)$ shows the case of ideal conditions $y(0) = 0$. Signal $y1(t)$ consider the case $y(0) = 0{,}2$ and signal $y2(t)$ shows the response of the system when the plant parameters are perturbed in the following way:

$$G_p(s) = \frac{(s+2{,}2)e^{-0{,}4s}}{(s+1{,}1)(s-1{,}8)}.$$

It is possible to appreciate the adequate performance of the system in all the situations.

V. CONCLUSIONS

In this work it is consider the prediction problem associated with an unstable linear system with dead time. The Smith predictor compensator is analyzed and an alternative discrete-time prediction strategy is proposed. The future value of the system is obtained by the consideration of a discrete-time observer that allows to obtain the desired future value at least at the sampling period. The proof of the error convergence is formally stated and the performance of the proposed methodology is shown by some simulations results.

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