Congreso Nacional de Control
Automático, AMCA 2015,
Cuernavaca, Morelos, México.

606

Observer design for a class of Lipschitz nonlinear systems with delayed outputs: time-varying delay

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Abstract: In this paper a design of an observer for Lipschitz nonlinear systems in the presence of time-varying delay in outputs measurements is proposed. The structure of the proposed observer is based on the presence of a proportional integral term which permits the compensation of delay in the measurements outputs. The observer gain is a function of the maximum bound of the delay, the parameters of the system and it is calculated by the resolution of given LMI equation. The Lyapunov-Krasovskii functional is used to prove the asymptotical convergence to zero of the observation error. This observer can be applied to the case of systems with variable piecewise delay continuous functions, as for example, in the case of systems with piecewise delay constant functions of time and the case of time-discrete measurements with constant variable delay. The observer is validated into a *Free Piston Stirling Engine* model and the results were satisfactory, with a time-varying delay in piecewise randomly

Keywords: Time-varying delay , Lyapunov-Krasovskii, LMI and FPSE

1. INTRODUCTION

In the recent years time-delayed systems have been investigated extensively because of the delay phenomenon is often encountered in various engineering systems such as mechanical and electrical systems, communication networks, among others. The source of a time-delay may be due to the nature of the system or induced into the system due to the transmission delays associated to other components interacting with the system. A time-delay may be the origin of instability or oscillations in a system. For this reason, many researchers are devoted to investigate the different fields of automatic control for time-delayed systems, such as stability, observability, controllability and system identification, among others. For instance, in Richard (2003), different control approaches for delayed systems are presented.

The case of observer design for state estimation of linear and nonlinear undelayed systems is treated for many authors from a theoretical and practical point of view, see for instance, Gauthier et al. (1194), Gauthier and Kupka (1994), Targui et al. (2002), Targui et al. (2001), The case of observer design for state estimation of linear

and nonlinear delayed systems has been investigated Germani et al. (2002), Cacace et al. (2010), Subbaroa and Muralidhar (2008), Kazantzis and Wright (2005), Darouch (2001), Hou et al. (2002),Wang et al. (2002). The observation problem is complicated if the system output is available after a delay interval.

For instance, in Germani et al. (2002), Kazantzis and Wright (2005), Subbaroa and Muralidhar (2008) a chain of observation algorithms reconstructing the system state based on delayed measurements of the process output is proposed. In Cacace et al. (2010), a constant gain observer is proposed for a class of observable nonlinear systems with variable delay, the gain of the observer is delay-dependent and is the solution of a matrix equation.

In this paper an observer for Lipschitz nonlinear systems in the presence of time-varying delay in outputs measurements is proposed, the design of the presented observer is based on the use of a proportional integral term in the structure of the observer. The dynamical proportional integral term permits the compensation of the variable delay measurements outputs. The gain of the observer is delay-dependent and it is calculated by the resolution of a given LMI equation. The Lyapunov-

Congreso Nacional de Control
Automático, AMCA 2015,
Cuernavaca, Morelos, México.

607

Krasovskii functional is used to prove the asymptotical convergence to zero of the observation error. Lyapunov-Krasovskii functional is used to prove the asymptotical convergence to zero of the observation error with some concretes conditions.

This paper is organized as follows: in Section 2, we present some notations and preliminaries. Section 3 describes the observer synthesis. In Section 4 numerical simulations are presented in order to evaluate the observer performance and finally conclusions are discussed in Section 5.

2. PRELIMINARIES AND NOTATIONS

The following notations will be used. A^T denotes the transpose of the matrix A. For a vector x, $||x|| = \sqrt{x^T x}$ denotes its Euclidean norm. We denote by $||A|| = \sqrt{\lambda_{max}(A^T A)}$ the spectral norm of a matrix A, which is the square-root of the maximum eigenvalue of the matrix $A^T A$.

Consider the following nonlinear Lipschitz system:

$$\begin{cases} \dot{x}(t) = Ax(t) + \varphi(u(t), x(t)), & t \geq \tau_M \\ \bar{y}(t) = Cx(t - \tau(t)), & t \geq 0, \quad \tau(t) \in [0 \quad \tau_M] \\ x(-\tau_M) = \bar{x} \end{cases} \quad (1)$$

where $x \in R^n$ is the system state, $u \in R^m$ is the system known input, $\bar{y}(t) \in R^p$ denotes the measured delayed system output. A and C are constants matrices with appropriate dimensions. $\tau(t)$ represents the signal known time-varying output measurement delay, which is bounded by some $\tau_M > 0$.

The following assumptions are needed for the derivation of the observer:

Assumption 1 (A1). The variable delay $\tau(t)$ is bounded, e.g. $0 \leq \tau(t) \leq \tau_M$.

Assumption 2 (A2). The function φ is globally Lipschitz in x uniformly in u i.e. for all bounded, then there exists $c > 0$ such that for all x and $z \in R^n$ one has:

$$||\varphi(u, x) - \varphi(u, z)|| \leq c||x - z||$$

Lemma 1. For $a, b \in R^n$ and $\varepsilon > 0$ we have $2a^T b \leq \varepsilon^{-1}a^T a + \varepsilon b^T b$.

Lemma 2. From Newton-Leibniz formula we have for $x(t) \in R^n$:

$$x(t - \tau(t)) = x(t) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds \quad (2)$$

3. MAIN RESULT

Let us now give our proposed observer for system (1) with delayed output $\bar{y}(t)$. Consider the following dynamical system:

$$\begin{cases} \dot{\hat{x}} = A\hat{x}(t) + \varphi(u(t), \hat{x}(t)) - K(C\hat{x}(t) - \bar{y}(t)) \\ +KC \int_{t-\tau(t)}^{t} (A\hat{x}(s) + \varphi(u(s), \hat{x}(s)))ds, \quad t \geq 0 \end{cases} \quad (3)$$

where

$$K = \frac{S^{-1}C^T}{2\tau_M ||C^T C||(||A|| + c)} \quad (4)$$

and $\hat{x}(s) = \lambda(s), \quad s \in [-\tau_M, 0]$ and $u(s) = \omega(s), \quad s \in [-\tau_M, 0]$.

The functions $\lambda(s)$ and $\omega(s)$ are known and are used to initialize system (3) in $[-\tau_M, 0]$.

Set $\mu(t) = C \int_{t-\tau(t)}^{t}(A\hat{x}(s) + \varphi(u(s), \hat{x}(s))ds$, then by the differentiation of $\mu(t)$ we can write system (3) in the simple form:

$$\begin{cases} \dot{\hat{x}} = A\hat{x}(t) + \varphi(u(t), \hat{x}(t)) - K(C\hat{x}(t) - \bar{y}(t)) + K\mu(t) \\ \dot{\mu} = CA\hat{x}(t) + C\varphi(u(t), \hat{x}(t),) \\ -(1 - \dot{\tau}(t))C(A\hat{x}(t - \tau(t)) \\ +\varphi(u(t - \tau(t)), \hat{x}(t - \tau(t)))) \end{cases} \quad (5)$$

The functions $\lambda(s)$ and $\omega(s)$ are known and are used to initialize system (5) in $[-\tau_M, 0]$ and $\mu(0) = C \int_{-\tau(0)}^{0}(A\hat{x}(s) + \varphi(u(s), \hat{x}(s)))ds$.

The following Theorem is given:

Theorem 1. For system (1) assume that **(A1)** and **(A2)** are satisfied and there exist a constant $\varepsilon > 0$ and a constant symmetric positive definite matrix S such that the following matrix inequality is satisfied.

$$SA + A^T S + \varepsilon^{-1}SS + \varepsilon c^2 I + I - \gamma C^T C \leq 0 \quad (6)$$

where $\gamma = \dfrac{1}{\tau_M ||C^T C||(||A|| + c)}$ and I is the identity matrix with appropriate dimensions, then system (3) is an asymptotic observer for system (1). The observation error $\tilde{x} = \hat{x}(t) - x(t)$ converge asymptotically to zero, then $lim_{t \to \infty}||\tilde{x}|| = 0$.

Proof.
By combining of equations (1) and (3) the dynamics of the observation error is:

Congreso Nacional de Control
Automático, AMCA 2015,
Cuernavaca, Morelos, México.

608

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) - KC(\hat{x}(t) - x(t - \tau(t)))$$
$$+ \varphi(u(t), \hat{x}(t)) - \varphi(u(t), x(t))$$
$$+ KC \int_{t-\tau(t)}^{t} (A\hat{x}(s) + \varphi(u(s), \hat{x}(s)))ds \quad (7)$$

Then by the use of Lemma 2,

$$\dot{\tilde{x}}(t) = (A - KC)\tilde{x}(t) + \varphi(u(t), \hat{x}(t))$$
$$- \varphi(u(t), x(t)) - KC \int_{t-\tau(t)}^{t} (Ax(s)$$
$$+ \varphi(u(s), x(s)))ds$$
$$+ KC \int_{t-\tau(t)}^{t} (A\hat{x}(s) + \varphi(u(s), \hat{x}(s)))ds \quad (8)$$

Then by the use of the equation of $x(t) = \hat{x}(t) - \tilde{x}(t)$, we obtain:

$$\dot{\tilde{x}}(t) = (A - KC)\tilde{x}(t) + \varphi(u(t), \hat{x}(t)) - \varphi(u(t), x(t))$$
$$+ KC \int_{t-\tau(t)}^{t} (A\tilde{x}(s) + \varphi(u(s), \hat{x}(s))$$
$$- \varphi(u(s), x(s)))ds \quad (9)$$

Consider now the following Lyapunov-Krasovskii functional:

$$V(\tilde{x}) = \tilde{x}^T(t)S\tilde{x}(t) + ||SKC||(||A|| + c)$$
$$\int_{-\tau_M}^{0} \int_{\nu}^{0} ||\tilde{x}(t + s)||^2 ds d\nu \quad (10)$$

where τ_M is the constant introduced in Assumption **(A1)**.

The time derivative of the functional V is:

$$\dot{V} = 2\tilde{x}^T(t)S(A - KC)\tilde{x}(t) + 2\tilde{x}^T(t)S(\varphi(u(s), \hat{x}(s))$$
$$- \varphi(u(s), x(s)))$$
$$+ 2\tilde{x}^T(t)SKC \int_{t-\tau(t)}^{t} (A\tilde{x}(s) + \varphi(u(s), \hat{x}(s))$$
$$- \varphi(u(s), x(s)))ds$$
$$+ \tau_M ||SKC||(||A|| + c)||\tilde{x}(t)||^2$$
$$- ||SKC||(||A|| + c) \int_{t-\tau_M}^{t} ||\tilde{x}(s)||^2 ds$$

$$\dot{V} \leq \tilde{x}^T(t)(S(A - KC) + (A - KC)^T S)\tilde{x}(t)$$
$$+ 2\tilde{x}^T(t)S(\varphi(u(s), \hat{x}(s)) - \varphi(u(s), x(s)))$$
$$+ \int_{t-\tau(t)}^{t} 2\tilde{x}^T(t)SKC(A\tilde{x}(s) + \varphi(u(s), \hat{x}(s))$$
$$- \varphi(u(s), x(s)))ds$$
$$+ \tau_M ||SKC||(||A|| + c)||\tilde{x}(t)||^2$$
$$- ||SKC||(||A|| + c) \int_{t-\tau(t)}^{t} ||\tilde{x}(s)||^2 ds \quad (11)$$

From the Lipschitz Assumption (A2) and by the use of Lemma 1 we obtain,

$$2\tilde{x}^T(t)SKC(A\tilde{x}(s) + \varphi(u(s), \hat{x}(s)) - \varphi(u(s), x(s)))$$
$$\leq ||SKC||(||A|| + c)||\tilde{x}(t)||^2$$
$$+ ||SKC||(||A|| + c)||\tilde{x}(s)||^2 \quad (12)$$

then it follows that,

$$\int_{t-\tau(t)}^{t} 2\tilde{x}^T(t)SKC(A\tilde{x}(s) + \varphi(u(s), \hat{x}(s))$$
$$- \varphi(u(s), x(s)))ds$$
$$\leq \tau(t)||SKC||(||A|| + c)||\tilde{x}(t)||^2$$
$$+ ||SKC||(||A|| + c) \int_{t-\tau(t)}^{t} ||\tilde{x}(s)||^2 ds$$
$$\leq \tau_M ||SKC||(||A|| + c)||\tilde{x}(t)||^2$$
$$+ ||SKC||(||A|| + c) \int_{t-\tau(t)}^{t} ||\tilde{x}(s)||^2 ds \quad (13)$$

consequently, we obtain,

$$\dot{V} \leq \tilde{x}^T(t)(S(A - KC) + (A - KC)^T S)\tilde{x}(t)$$
$$+ 2\tilde{x}^T(t)S(\varphi(u(s), \hat{x}(s)) - \varphi(u(s), x(s)))$$
$$+ \tau_M ||SKC||(||A|| + c)||\tilde{x}(t)||^2$$
$$+ ||SKC||(||A|| + c) \int_{t-\tau(t)}^{t} ||\tilde{x}(s)||^2 ds$$
$$+ \tau_M ||SKC||(||A|| + c)||\tilde{x}(t)||^2$$
$$- ||SKC||(||A|| + c) \int_{t-\tau(t)}^{t} ||\tilde{x}(s)||^2 ds \quad (14)$$

now by the use of the Lipschitz assumption 2 and Lemma 1 we have

$$2\tilde{x}^T(t)S(\varphi(u(s), \hat{x}(s)) - \varphi(u(s), x(s)))$$
$$\leq 2||S\tilde{x}(t)||||\varphi(u(s), \hat{x}(s)) - \varphi(u(s), x(s))||$$
$$\leq \varepsilon c^2 \tilde{x}^T(t)\tilde{x}(t) + \varepsilon^{-1}\tilde{x}^T(t)SS\tilde{x}(t) \quad (15)$$

then,

$$\dot{V} \leq \tilde{x}^T(t)(S(A - KC) + (A - KC)^T S)\tilde{x}(t)$$
$$+ \varepsilon c^2 \tilde{x}^T(t)\tilde{x}(t) + \varepsilon^{-1}\tilde{x}^T(t)SS\tilde{x}(t)$$
$$+ 2\tau_M ||SKC||(||A|| + c)||\tilde{x}(t)||^2$$
$$\leq \tilde{x}^T(t)(S(A - KC) + (A - KC)^T S + \varepsilon c^2 I$$
$$+ \varepsilon^{-1}SS + 2\tau_M ||SKC||(||A|| + c))\tilde{x}(t) \quad (16)$$

now replacing K in (16) we obtain:

$$\dot{V} \leq \tilde{x}^T(t)(SA + A^T S + \varepsilon^{-1}SS + \varepsilon c^2 I + I$$
$$- \gamma C^T C)\tilde{x}(t) \quad (17)$$

where γ, then if there exists $\varepsilon > 0$ and $S > 0$ such that:

$$SA + A^T S + \varepsilon^{-1}SS + \varepsilon c^2 I + I - \gamma C^T C \leq 0 \quad (18)$$

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Congreso Nacional de Control
Automático, AMCA 2015,
Cuernavaca, Morelos, México.

609

consequently, $\dot{V} \leq 0$ which means that system (9) is asymptotically stable and it results that $\lim_{t \to +\infty} ||\tilde{x}(t)|| \to 0$. Then, system (3) is an asymptotic observer for system (1). This completes the proof of Theorem 1.

3.1 Application to the case of systems with piecewise delay continuous function

In some practical applications the variable delay is given in a form of piecewise continuous function:

$$\tau(t) = w_k(t), \quad t \in [t_k \quad t_{k+1}[, \quad k \in N$$

where $w_k(t)$ is a continuous function for $t \in [t_k \quad t_{k+1}[$ with $t_{k+1} - t_k > 0$, then system (1) can be written as:

$$\begin{cases} \dot{x}(t) = Ax(t) + \varphi(u(t), x(t)) \\ \bar{y}(t) = Cx(t - w_k(t)), \quad t \in [t_k \quad t_{k+1}[\end{cases} \quad (19)$$

then for $t \in [t_k \quad t_{k+1}[$ it is easy to see that system (19) is of the form (1) then Theorem can be applied to system (19).

Remark 1. The case $w_k(t) = \tau_k, \quad t \in [t_k \quad t_{k+1}[$ where $\tau_k \geq 0$ correspond the case $\bar{y}(t) = Cx(t - \tau_k)$ for $t \in [t_k \quad t_{k+1}[$ constance piecewise variable delay then observer () can be applied with $\dot{\tau}(t) = 0$ for $t \in [t_k \quad t_{k+1}[$ and $\tau_M = max\{\tau_k\}$.

Remark 2. The case $w_k(t) = t - t_k + \tau_k, \quad t \in [t_k - \tau_k \quad t_{k+1} - \tau_{k+1}[$ corspond to the case $\bar{y}(t) = Cx(t_k - \tau_k)$ for $t \in [t_k - \tau_k \quad t_{k+1} - \tau_{k+1}[$ which is the case of discreet time measurements with constant variable delay. then observer (5) can be applied with $\dot{\tau}(t) = 1$ for $t \in [t_k - \tau_k \quad t_{k+1} - \tau_{k+1}[$ and $\tau_M = max\{t_{k+1} - \tau_{k+1} - (t_k - \tau_k)\}$.

4. ILLUSTRATIVE EXAMPLE

The performance of the proposed observer shall be illustrated through an observer design involving a Free-Piston Stirling Engine (**FPSE**). A **FPSE** is a heat engine that operates by cyclic compression and expansion of air or the gas, the working fluid, at different temperature levels such that is net conversion of heat energy to mechanical work. In **FPSE** the power generated by the engine is related to the length of the piston x_p and displacer strokes x_d. The **FPSE** used in this paper is a split type, and the isothermal model Ulusoy (1994) and Zheng et al. (2012) is adopted. The equations of motion for the displacer and piston can be written as (20), and defining $x_1 = x_p$, $x_2 = \dot{x}_p$, $x_3 = x_d$ y $x_4 = \dot{x}_d$:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \bar{f}_L & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ \bar{f}_L x_2 - \bar{f}_C x_1^2 x_2 + \bar{k}_C x_1^3 + k_{pp} p_m(t) x_1 + k_{pd} p_m(t) x_3 \\ 0 \\ k_{dd} p_m(t) x_1 + c_{dd} p_m(t) x_2 + k_{dp} p_m(t) x_3 + c_{dp} p_m(t) x_4 \end{bmatrix}$$

$$\quad (20)$$

with:

$$\bar{y}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t - \tau(t))$$

and with constant terms:

$$k_{dd} = -\frac{\bar{A}_r}{\bar{m}_d}(\gamma \bar{a}_s - \bar{a}_d) \quad k_{dp} = -\frac{\bar{A}_r}{\bar{m}_d}\bar{a}_p \quad k_{pd} = \frac{\bar{a}_d}{\bar{m}_p}$$

$$k_{pp} = -\frac{\bar{a}_p}{\bar{m}_p} \quad c_{dd} = k_1 \frac{A_{dp}}{\bar{m}_d} \quad c_{dp} = k_2 \frac{A_{dp}}{\bar{m}_d}$$

The numerical values used in this work were obtained using the parameters of **FPSE** presented in Ulusoy (1994), we can see in Table (1). The input is the mean pressure $p_m(t) = 0.7MPa$ for $t \geq 0$. The initial conditions for the system are $x(0) = [\,0.5 \ 0 \ 0.5 \ 0\,]^T$. This systems can be represented in the form (1).

Table 1. Parameter of the **FPSE**

Parameter	Value	Parameter	Value
$\bar{A}_r$	2.17×10^{-4}	$\bar{a}_p$	0.2593
$\bar{m}_d$	1.2378×10^5	$\bar{m}_p$	1.8015×10^6
γ	1.635	A_{dp}	0.9833
$\bar{a}_s$	0.1434	k_1	-0.028
$\bar{a}_d$	0.1339	k_2	0.021
$\bar{k}_C$	0.0080	$\bar{f}_C$	7.1709×10^{-6}
$\bar{f}_L$	-0.0770		

The value of the Lipschitz constant for this system is $c = 3.726$. An observer for the nonlinear **FPSE** model can be represented in the form (5), the gain is calculated using to Theorem (1) on LMI based solution with $\varepsilon = 0.02$ and $\tau_M = 1$ (sec), the observer gain founded to be:

$$K = \begin{bmatrix} 181.9553 & 1.1358 \\ 65.2878 & 0.4769 \\ 1.1358 & 4.6732 \\ 2.1225 & 5.2571 \end{bmatrix}$$

The initial conditions for the observer is $\hat{x}(\tau(0)) = [\,0 \ 0 \ 0 \ 0\,]^T \ \forall t \in [\,0 \ \tau(0)\,]$. The chain observer is simulated for a time-delayed output, with the delay $\tau(t)$ time-varying. The value of $\tau(t)$ varies randomly in the ranges $\tau \in (0.5, 1)$ and seconds $\Delta t = 10$ sec, ver Fig. (2).

The observations plots are as expected, in Fig. (1), as we can see clearly the convergence of the observations states.

5. CONCLUSION

In this paper, a new approach to the Lipschitz nonlinear observer design problem in the presence of time-varying delayed output measurements was proposed. Lyapunov-Krasovskii functional is used to prove the asymptotical convergence to zero of the observation error. The proposed observer is applied into an **FPSE** model and satisfying simulation results are obtained, the time-varying delay is piecewise randomly. An advantage of the observer is that the delay is dynamically compansed in the

Congreso Nacional de Control
Automático, AMCA 2015,
Cuernavaca, Morelos, México.

610

(a) Estimation position x_1

(b) Estimation velocity x_2

(c) Estimation position x_3
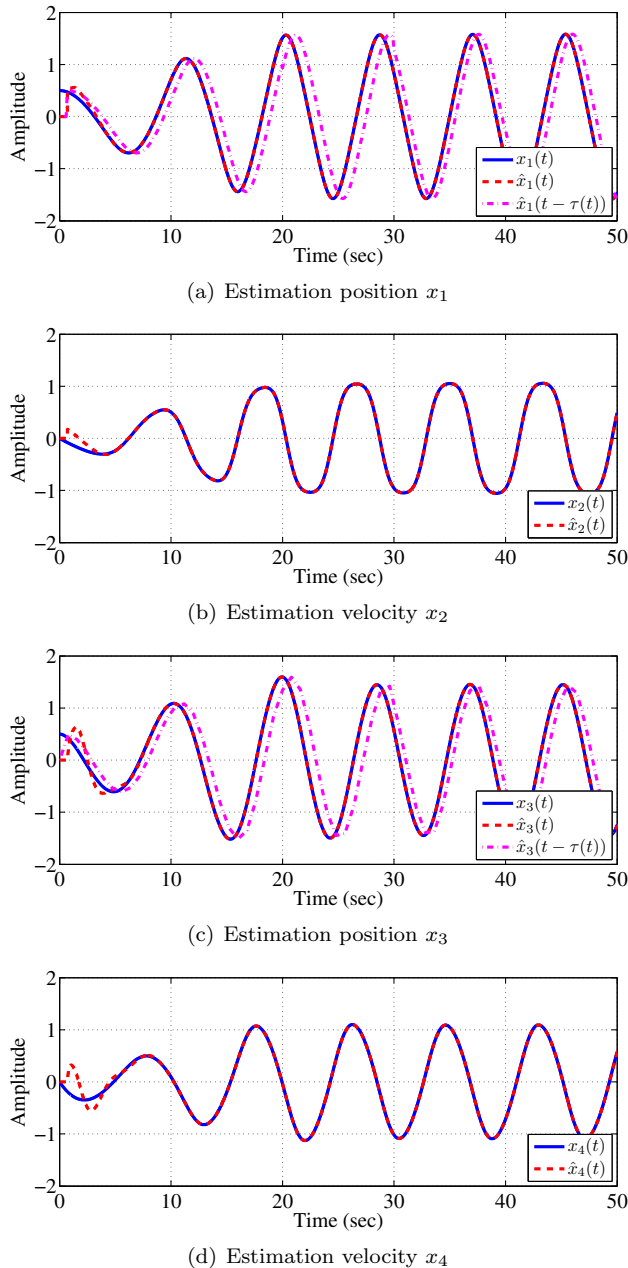

(d) Estimation velocity x_4

Fig. 1. Estimation of the delayed state **FPSE** with $y_1 = x_1(t - \tau(t))$ and $y_2 = x_3(t - \tau(t))$

observer gain. This observer can be applied to the case of systems with piecewise delay continuous functions. Furthermore, in the case of time-discreet measurements with constant variable delay.

Fig. 2. Evolution of the delay $\tau(t)$ (**FPSE**)

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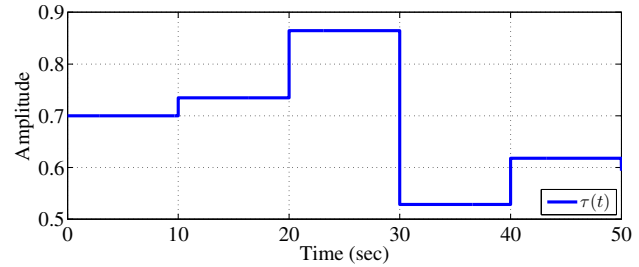M. Darouach, Linear Functional Observers for Systems with Delays in State Variables, *IEEE Trans. Automatic Control*, vol. 46(3), pp. 491-496, 2001.