Consensus in Delayed Robot Networks using only Position Measurements

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Abstract: This paper provides a solution to the leaderless consensus problem in networks of robots with interconnecting time-delays for which velocity measurements are not available, under the assumption that the interconnection graph is undirected and connected. The controller has the structure of the Proportional plus damping scheme with gravity cancellation and the estimated velocities are obtained using the Immersion and Invariance observer. Simulations, with ten robots, show the performance of the proposed approach.

Keywords: Euler-Lagrange Systems, Consensus, Time-Delays.

1. INTRODUCTION

For networks of multiple agents, the consensus control objective is to reach an agreement between certain coordinates of interest using a distributed controller. There are mainly two consensus problems: the leader-follower, where a network of follower agents has to be synchronized with a given leader, and the leaderless, where all agents agree at a certain coordinates value. The solutions to these problems has recently attracted the attention of the research community in different fields, such as biology, physics, control theory and robotics (refer to (Olfati-Saber et al., 2007; Scardovi and Sepulchre, 2009; Ren, 2008), for solutions with linear agents, and to (Yu et al., 2011; Scardovi et al., 2009; Stan and Sepulchre, 2007; Zhao et al., 2009), for solutions with some classes of nonlinear agents).

The practical applications of the solutions to the leaderless consensus problem are diverse and range from formation control of multiple unmanned aerial vehicles to the synchronization of swarms of mobile robots. A particular example is a robot teleoperator, where two mechanical manipulators are coupled by a communication channel that, in general, induces time-delays (Anderson and Spong, 1989). The control objective in these systems is that when the human operator moves the *local* manipulator, the *remote* manipulator has to track its position, and the force interaction of this last with the environment has to be reflected back to the operator (Nuño et al., 2011). The results reported in the present paper are a generalization of the stability condition reported in (Nuño et al., 2009) to the case of networks of robots without velocity measurements. A direct application of the consensus controllers reported here is the teleoperation of multiple-remote devices, the collaboration of multiple users via a multiple-local multiple-remote system, among others (Malysz and Sirouspour, 2011; Rodriguez-Seda et al., 2010).

Recently, a full-order globally exponentially convergent velocity observer has been proposed for a general class of mechanical systems with or without constraints (Astolfi et al., 2010, 2009). The adopted approach is based on the notions of Immersion and Invariance (I&I), where the objective consists in finding a certain manifold $\mathcal{M}$, in the extended state–space of the plant and the observer, that should be rendered attractive and invariant. Finally, in order to provide a Lyapunov–based stability analysis, a dynamic scaling as well as some high–gain terms are introduced, see also (Karagiannis and Astolfi, 2008; Astolfi et al., 2007). Recently, the I&I observer has been ported to the bilateral teleoperators control with interconnecting delays (Sarras et al., 2015).

The main contributions of this paper are: i) a sufficient condition for the solution of the leaderless consensus problem in networks, modeled as undirected weighted static graphs, of fully–actuated robots controlled by simple Proportional plus damping injection (P+d) schemes (Nuño et al., 2013b) when velocity measurements are not available; ii) the interconnection graph can exhibit asymmetric variable time-delays, with the only assumption that such delays are bounded and that the bounds are known; iii) the controller only depends on position measurements and the full-order I&I observer (Astolfi et al., 2010) is used to find a velocity estimation.

To streamline the presentation, throughout the paper the following notation is introduced. Lower case letters denote scalar functions, e.g. t, bold lower case letters denote vectors, e.g. $\mathbf{x}$, and bold upper case letters denote matrices, e.g. $\mathbf{A}$. $\mathbf{I}_k$ represents the identity matrix of size $k \times k$. $\mathbf{1}_k$ and $\mathbf{0}_k$ represent column vectors of size k with all entries equal to one and to zero, respectively. Additionally, we define $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$. $\lambda_m\{\mathbf{A}\}$ and $\lambda_M\{\mathbf{A}\}$ represent the minimum and maximum eigenvalues of matrix $\mathbf{A}$, respectively while $||\mathbf{A}||$ denotes the matrix–induced 2-norm. $|\mathbf{x}|$ stands for the standard Euclidean norm of vector $\mathbf{x}$. For any function $\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\mathbf{f}\|_\infty := \sup_{t\geq 0} |\mathbf{f}(t)|$, and the $\mathcal{L}_2$-norm as $\|\mathbf{f}\|_2 :=$

$(\int_0^\infty |\mathbf{f}(t)|^2 dt)^{\frac{1}{2}}$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_\infty < \infty\}$ and $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_2 < \infty\}$, respectively.

The following lemma, borrowed from (Nuño et al., 2009), will be used in the proof of the main result of the paper.

Lemma 1. (Nuño et al., 2009). For any vector signals $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, any variable time-delay $0 \leq T(t) \leq {}^*T < \infty$ and any constant $\alpha > 0$, the following inequality holds

$$-\int_0^t \mathbf{x}^\top(\sigma) \int_{-T(\sigma)}^0 \mathbf{y}(\sigma+\theta) d\theta d\sigma \leq \frac{\alpha}{2}\|\mathbf{x}\|_2^2 + \frac{{}^*T^2}{2\alpha}\|\mathbf{y}\|_2^2,$$

where $\|\cdot\|_2$ stands for the $\mathcal{L}_2$-norm.

2. NETWORK DYNAMICS

The dynamical behavior of the network accounts for a twofold: i) the dynamics of nodes and ii) the interconnection topology.

2.1 Node Description

Every ith-node contains a fully actuated n-DoF robot manipulator with revolute joints and its dynamic behavior satisfies the following EL-equation of motion

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) = \boldsymbol{\tau}_i \quad (1)$$

where $\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i \in \mathbb{R}^n$, are the joint positions, velocities and accelerations, respectively; $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix; $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal effects matrix, defined via the Christoffel symbols of the first kind; $\mathbf{g}_i(\mathbf{q}_i) \in \mathbb{R}^n$ is the gravitational torques vector and $\boldsymbol{\tau}_i \in \mathbb{R}^n$ is the torque exerted by the actuators.

The EL-system (1) enjoys the following properties (Kelly et al., 2005; Spong et al., 2005):

P1. For all $\mathbf{q}_i \in \mathbb{R}^n$, $m_{mi}\mathbf{I}_n \leq \mathbf{M}_i(\mathbf{q}_i) \leq m_{Mi}\mathbf{I}_n$, where $m_{mi} := \lambda_m\{\mathbf{M}_i(\mathbf{q}_i)\}$ and $m_{Mi} := \lambda_M\{\mathbf{M}_i(\mathbf{q}_i)\}$.
P2. Matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew-symmetric.
P3. For all $\mathbf{q}_i, \mathbf{a}, \mathbf{b} \in \mathbb{R}^n$, $\exists k_{ci} \in \mathbb{R}_{>0}$ such that $|\mathbf{C}_i(\mathbf{q}_i, \mathbf{a})\mathbf{b}| \leq k_{ci}|\mathbf{a}||\mathbf{b}|$.
P4. If $\dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$ then $\frac{d}{dt}\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is a bounded operator.

Let be the following factorization of the inertia matrix, borrowed from (Astolfi et al., 2010):

$$\mathbf{M}_i(\mathbf{q}_i) = \mathbf{T}_i^\top(\mathbf{q}_i)\mathbf{T}_i(\mathbf{q}_i),$$

Since $\mathbf{M}_i(\mathbf{q}_i)$ is symmetric and it satisfies **P1**, $\mathbf{T}_i(\mathbf{q}_i)$ always exists. Further, let the mappings $\mathbf{L}_i : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ and $\mathbf{F}_i : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ be defined as

$$\begin{aligned} \mathbf{L}_i(\mathbf{q}_i) &:= \mathbf{T}_i^{-1}(\mathbf{q}_i) \\ \mathbf{F}_i(\mathbf{q}_i, \boldsymbol{\tau}_i) &:= \mathbf{L}_i^\top(\mathbf{q}_i)(\boldsymbol{\tau}_i - \mathbf{g}_i(\mathbf{q}_i)), \end{aligned} \quad (2)$$

and consider the following coordinate transformation

$$\mathbf{x}_i := \mathbf{T}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i. \quad (3)$$

Then, using (2) and (3), dynamics (1) can be transformed into the new system

$$\begin{aligned} \dot{\mathbf{q}}_i &= \mathbf{L}_i(\mathbf{q}_i)\mathbf{x}_i \\ \dot{\mathbf{x}}_i &= \mathbf{S}_i(\mathbf{q}_i, \mathbf{x}_i)\mathbf{x}_i + \mathbf{F}_i(\mathbf{q}_i, \boldsymbol{\tau}_i), \end{aligned} \quad (4)$$

with the mapping $\mathbf{S}_i : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ given by

$$\mathbf{S}_i(\mathbf{q}_i, \mathbf{x}_i) := \left[\dot{\mathbf{T}}(\mathbf{q}_i) - \mathbf{L}_i^\top(\mathbf{q}_i)\mathbf{C}_i(\mathbf{q}_i, \mathbf{x}_i)\right]\mathbf{L}_i(\mathbf{q}_i). \quad (5)$$

The mapping $\mathbf{S}_i$ has the following properties, related to the properties of the Coriolis matrix $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ (Astolfi et al., 2010, 2009):

P5. $\mathbf{S}_i(\mathbf{q}_i, \mathbf{x}_i)$ is skew–symmetric.
P6. $\mathbf{S}_i(\mathbf{q}_i, \mathbf{x}_i)$ is linear in the second argument, i.e., $\mathbf{S}_i(\mathbf{q}_i, \alpha_1\mathbf{a} + \alpha_2\mathbf{b})\mathbf{c} = \alpha_1\mathbf{S}_i(\mathbf{q}_i, \mathbf{a})\mathbf{c} + \alpha_2\mathbf{S}_i(\mathbf{q}_i, \mathbf{b})\mathbf{c}$, for all $\mathbf{q}_i, \mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^n$ and any scalars α_1, α_2.
P7. There exists a mapping $\bar{\mathbf{S}}_i : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ such that $\mathbf{S}_i(\mathbf{q}_i, \mathbf{x}_i)\mathbf{z}_i = \bar{\mathbf{S}}_i(\mathbf{q}_i, \mathbf{z}_i)\mathbf{x}_i$, for all $\mathbf{q}_i, \mathbf{x}_i, \mathbf{z}_i \in \mathbb{R}^n$.

2.2 Interconnection Topology

The interconnection of the N robots is modeled using graph theory via the Laplacian matrix $\mathbf{W} := [w_{ij}] \in \mathbb{R}^{N\times N}$, whose elements are defined as

$$w_{ij} = \begin{cases} \sum_{j\in\mathcal{N}_i} a_{ij} & i = j \\ -a_{ij} & i \neq j \end{cases} \quad (6)$$

where $\mathcal{N}_i$ is the set of agents transmitting information to the ith robot, $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$ otherwise.

Similar to passivity-based (energy-shaping) synchronization (Nuño et al., 2013a; Arcak, 2007) and in order to ensure that the interconnection forces are generated by the gradient of a potential function, the following assumption is used in this paper:

A1. The network graph is *undirected and connected.*

By construction, $\mathbf{W}$ has a zero row sum. Moreover, Assumption **A1**, ensures that $\mathbf{W}$ is symmetric, has a single zero-eigenvalue and the rest of its spectrum has positive real parts. Thus, rank$(\mathbf{W}) = N - 1$. Therefore, null$(\mathbf{W}) = \alpha\mathbf{1}_N$, for any $\alpha \in \mathbb{R}$.

Regarding the interconnecting delays, it is assumed that:

A2. The information exchange, from the j-th robot to the i-th robot, is subject to a variable time-delay $T_{ji}(t)$ with a known upper-bound ${}^*T_{ji}$. Hence, it holds that $0 \leq T_{ji}(t) \leq {}^*T_{ji} < \infty$. Moreover, $\dot{T}_{ji}(t)$ is bounded.

3. I&I VELOCITY OBSERVER

For completeness, let us review the design of the I&I velocity observer reported in (Astolfi et al., 2010; Karagiannis and Astolfi, 2008). First, for the EL-system (4), it is proposed the manifold

$$\mathcal{M}_i = \{(\mathbf{q}_i, \mathbf{x}_i, \boldsymbol{\xi}_i, \hat{\mathbf{x}}_i, \hat{\mathbf{q}}_i \;:\; \boldsymbol{\xi}_i + \boldsymbol{\beta}_i(\mathbf{q}_i, \hat{\mathbf{q}}_i, \hat{\mathbf{x}}_i) - \mathbf{x}_i = \mathbf{0}\} \quad (7)$$

where $\boldsymbol{\xi}_i, \hat{\mathbf{q}}_i, \hat{\mathbf{x}}_i \in \mathbb{R}^n$ are (part of) the observer state, whose dynamics, as well as the mapping $\boldsymbol{\beta}_i \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, are defined below. To prove that the manifold $\mathcal{M}_i$ is attractive and invariant, it is shown that the off–the–manifold coordinates

$$\mathbf{z}_i = \boldsymbol{\xi}_i + \boldsymbol{\beta}_i(\mathbf{q}_i, \hat{\mathbf{q}}_i, \hat{\mathbf{x}}_i) - \mathbf{x}_i, \quad (8)$$

whose norm determines the distance of the state from the manifold $\mathcal{M}_i$, is such that:

C1. (*Invariance*) $\mathbf{z}(0) = \mathbf{0} \Rightarrow \mathbf{z}(t) = \mathbf{0}$, for all $t > 0$
C2. (*Attractivity*) $\mathbf{z}(t)$ asymptotically (exponentially) converges to zero.

Then, if $\lim_{t\to\infty} |\mathbf{z}(t)| = \mathbf{0}$, an asymptotic estimate of $\mathbf{x}_i$ is given by $\boldsymbol{\xi}_i + \boldsymbol{\beta}_i$. Convergence to the manifold $\mathcal{M}_i$ can be proved by examining the $\mathbf{z}_i$–dynamical behavior that is given by

$$\dot{\mathbf{z}}_i = \dot{\boldsymbol{\xi}}_i + \nabla_{\mathbf{q}_i}\boldsymbol{\beta}_i \mathbf{L}_i(\mathbf{q}_i)\mathbf{x}_i + \nabla_{\hat{\mathbf{q}}_i}\boldsymbol{\beta}_i\dot{\hat{\mathbf{q}}}_i + \nabla_{\hat{\mathbf{x}}_i}\boldsymbol{\beta}_i\dot{\hat{\mathbf{x}}}_i - \\ -\mathbf{S}_i(\mathbf{q}_i,\mathbf{x}_i)\mathbf{x}_i - \mathbf{F}(\mathbf{q}_i,\boldsymbol{\tau}_i).$$

Defining

$$\dot{\boldsymbol{\xi}}_i := \mathbf{S}_i(\mathbf{q}_i,\boldsymbol{\xi}_i+\boldsymbol{\beta}_i)(\boldsymbol{\xi}_i+\boldsymbol{\beta}_i) - \nabla_{\hat{\mathbf{q}}_i}\boldsymbol{\beta}_i\dot{\hat{\mathbf{q}}}_i - \nabla_{\hat{\mathbf{x}}_i}\boldsymbol{\beta}_i\dot{\hat{\mathbf{x}}}_i - \\ - \nabla_{\mathbf{q}_i}\boldsymbol{\beta}_i\mathbf{L}_i(\mathbf{q}_i)(\boldsymbol{\xi}_i+\boldsymbol{\beta}_i) + \mathbf{F}_i(\mathbf{q}_i,\boldsymbol{\tau}_i) \tag{9}$$

together with **P6** and **P7**, yields

$$\dot{\mathbf{z}}_i = [\mathbf{S}_i(\mathbf{q}_i,\mathbf{x}_i) + \bar{\mathbf{S}}_i(\mathbf{q}_i,\boldsymbol{\xi}_i+\boldsymbol{\beta}_i) - \nabla_{\mathbf{q}_i}\boldsymbol{\beta}_i\mathbf{L}_i(\mathbf{q}_i)]\mathbf{z}_i. \tag{10}$$

The desired objective consists in finding a certain mapping $\boldsymbol{\beta}_i$ such that the $\mathbf{z}_i$–dynamics reduces to $\dot{\mathbf{z}}_i = [\mathbf{S}_i(\mathbf{q}_i,\mathbf{x}_i) - k_i\mathbf{I}]\,\mathbf{z}_i$, where $k_i \in \mathbb{R}_{>0}$, which ensures $\mathbf{z}_i = \mathbf{0}$ to be Globally Exponentially Stable (GES). That is, a mapping $\boldsymbol{\beta}_i$ that solves the partial differential equation

$$\nabla_{\mathbf{q}_i}\boldsymbol{\beta}_i = [k_i\mathbf{I} + \bar{\mathbf{S}}_i(\mathbf{q}_i,\boldsymbol{\xi}_i+\boldsymbol{\beta}_i)]\mathbf{T}_i(\mathbf{q}_i).$$

However, in general, such a $\boldsymbol{\beta}_i$ may not exist. Hence, an approximate solution has been proposed by defining an *ideal* $\nabla_{\mathbf{q}_i}\boldsymbol{\beta}_i$ as

$$\mathbf{H}_i(\mathbf{q}_i,\boldsymbol{\xi}_i+\boldsymbol{\beta}_i) := [k_i\mathbf{I} + \bar{\mathbf{S}}_i(\mathbf{q}_i,\boldsymbol{\xi}_i+\boldsymbol{\beta}_i)]\mathbf{T}_i(\mathbf{q}_i), \tag{11}$$

and

$$\boldsymbol{\beta}_i(\mathbf{q}_i,\hat{\mathbf{q}}_i,\hat{\mathbf{x}}_i) := \mathbf{H}_i(\hat{\mathbf{q}}_i,\hat{\mathbf{x}}_i)\mathbf{q}_i. \tag{12}$$

The above choices yield

$$\nabla_{\mathbf{q}_i}\boldsymbol{\beta}_i = \mathbf{H}_i(\mathbf{q}_i,\boldsymbol{\xi}_i+\boldsymbol{\beta}_i) - \mathbf{H}_i(\mathbf{q}_i,\boldsymbol{\xi}_i+\boldsymbol{\beta}_i) + \mathbf{H}_i(\hat{\mathbf{q}}_i,\hat{\mathbf{x}}_i) \\ = \mathbf{H}_i(\mathbf{q}_i,\boldsymbol{\xi}_i+\boldsymbol{\beta}_i) - \boldsymbol{\Delta}_{\mathbf{q}_i}(\mathbf{q}_i,\hat{\mathbf{x}}_i,\mathbf{e}_{\mathbf{q}_i}) - \\ - \boldsymbol{\Delta}_{\mathbf{x}_i}(\mathbf{q}_i,\hat{\mathbf{x}}_i,\mathbf{e}_{\mathbf{x}_i}), \tag{13}$$

where we used, for the second line, the definitions

$$\mathbf{e}_{\mathbf{q}_i} := \hat{\mathbf{q}}_i - \mathbf{q}_i, \quad \mathbf{e}_{\mathbf{x}_i} := \hat{\mathbf{x}}_i - (\boldsymbol{\xi}_i + \boldsymbol{\beta}_i), \tag{14}$$

and the fact that mappings $\boldsymbol{\Delta}_{\mathbf{q}_i}, \boldsymbol{\Delta}_{\mathbf{x}_i} : \mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n \to \mathbb{R}^{n\times n}$ always exist and are such that, for all $\mathbf{q}_i, \mathbf{x}_i, \hat{\mathbf{q}}_i, \hat{\mathbf{x}}_i$,

$$\boldsymbol{\Delta}_{\mathbf{q}_i}(\mathbf{q}_i,\hat{\mathbf{x}}_i,\mathbf{0}) = \mathbf{0}, \quad \boldsymbol{\Delta}_{\mathbf{x}_\mathbf{i}}(\mathbf{q}_\mathbf{i},\hat{\mathbf{x}}_\mathbf{i},\mathbf{0}) = \mathbf{0}. \tag{15}$$

Substituting (11) and (13) in (10), yields

$$\dot{\mathbf{z}}_i = [\mathbf{S}_i(\mathbf{q}_i,\mathbf{x}_i) - k_i\mathbf{I}]\,\mathbf{z}_i + \boldsymbol{\Delta}_{\mathbf{q}_i}(\mathbf{q}_i,\hat{\mathbf{x}}_i,\mathbf{e}_{\mathbf{q}_i})\mathbf{L}_i(\mathbf{q}_i)\mathbf{z}_i + \\ + \boldsymbol{\Delta}_{\mathbf{x}_i}(\mathbf{q}_i,\hat{\mathbf{x}}_i,\mathbf{e}_{\mathbf{x}_i})\mathbf{L}_i(\mathbf{q}_i)\mathbf{z}_i.$$

The mappings $\boldsymbol{\Delta}_{\mathbf{y}_i}$, $\boldsymbol{\Delta}_{\mathbf{x}_i}$ play the role of disturbances that are dominated with a dynamic scaling of the form $\boldsymbol{\eta}_i = \frac{1}{r_i}\mathbf{z}_i$ and a proper choice of the observer dynamics. From the dynamic scaling, the dynamical behavior of $\boldsymbol{\eta}_i$ is given by $\dot{\boldsymbol{\eta}}_i = \frac{1}{r_i}\dot{\mathbf{z}}_i - \frac{\dot{r}_i}{r_i}\boldsymbol{\eta}_i$, that is

$$\dot{\boldsymbol{\eta}}_i = [\mathbf{S}_i(\mathbf{q}_i,\mathbf{x}_i) - k_i\mathbf{I}]\,\boldsymbol{\eta}_i + [\boldsymbol{\Delta}_{\mathbf{q}_i} + \boldsymbol{\Delta}_{\mathbf{x}_i}]\,\mathbf{L}_i\boldsymbol{\eta}_i - \frac{\dot{r}_i}{r_i}\boldsymbol{\eta}_i. \tag{16}$$

Using $U_i = \frac{1}{2}|\boldsymbol{\eta}_i|^2$, setting the following r_i–dynamics

$$\dot{r}_i = -\frac{k_i}{4}(r_i - 1) + \frac{r_i}{k_i}(||\boldsymbol{\Delta}_{\mathbf{q}_i}\mathbf{L}_i||^2 + ||\boldsymbol{\Delta}_{\mathbf{x}_i}\mathbf{L}_i||^2), \tag{17}$$

with $r_i(0) \geq 1$ and the fact that, for $r_i > 0$, $\frac{r_i - 1}{r_i} \leq 1$, yields $\dot{U}_i \leq -\frac{k_i}{4}|\boldsymbol{\eta}_i|^2$. Thus $\boldsymbol{\eta}_i = \mathbf{0}$ is GES. Note that $\mathbf{z}_i = \mathbf{0}$ will be GES if it can be proved that $r_i \in \mathcal{L}_\infty$. Before going through this proof, let us show that $\mathbf{e}_{\mathbf{q}_i} = \mathbf{0}$ and $\mathbf{e}_{\mathbf{x}_i} = \mathbf{0}$ are also GES. First, let us start by setting

$$\dot{\hat{\mathbf{q}}}_i := \mathbf{L}_i(\mathbf{q}_i)(\boldsymbol{\xi}_i + \boldsymbol{\beta}_i) - \psi_{2_i}(\mathbf{q}_i, r_i)\mathbf{e}_{\mathbf{q}_i} \tag{18}$$

$$\dot{\hat{\mathbf{x}}}_i := \mathbf{F}_i(\mathbf{q}_i,\boldsymbol{\tau}_i) + \mathbf{S}_i(\mathbf{q}_i,\boldsymbol{\xi}_i+\boldsymbol{\beta}_i)(\boldsymbol{\xi}_i+\boldsymbol{\beta}_i) - \psi_{1_i}(\mathbf{q}_i, r_i)\mathbf{e}_{\mathbf{x}_i},$$

where $\psi_{1_i}, \psi_{2_i} : \mathbb{R}^n\times\mathbb{R} \to \mathbb{R}_{>0}$ will be defined later. Using the previous expressions, (14) can be written as

$$\dot{\mathbf{e}}_{\mathbf{x}_i} = r_i\nabla_{\mathbf{q}_i}\boldsymbol{\beta}_i\mathbf{L}_i(\mathbf{q}_i)\boldsymbol{\eta}_i - \psi_{1_i}(\mathbf{q}_i, r_i)\mathbf{e}_{\mathbf{x}_i} \\ \dot{\mathbf{e}}_{\mathbf{q}_i} = r_i\mathbf{L}(\mathbf{q}_i)\boldsymbol{\eta}_i - \psi_{2_i}(\mathbf{q}_i, r_i)\mathbf{e}_{\mathbf{q}_i}. \tag{19}$$

Now, let us define the proper Lyapunov candidate function $V_i = U_i + \frac{1}{2}(|\mathbf{e}_{\mathbf{x}_i}|^2 + |\mathbf{e}_{\mathbf{q}_i}|^2)$. After applying Young's inequality, $\dot{V}_i$ evaluated along (19), yields

$$\dot{V}_i \leq -\left(\frac{k_i}{4} - 1\right)|\boldsymbol{\eta}_i|^2 - \left(\psi_{2_i} - \frac{1}{2}r_i^2||\mathbf{L}_i||^2\right)|\mathbf{e}_{\mathbf{q}_i}|^2 - \\ - \left(\psi_{1_i} - \frac{1}{2}r_i^2||\nabla_{\mathbf{q}_i}\boldsymbol{\beta}_i||^2||\mathbf{L}_i||^2\right)|\mathbf{e}_{\mathbf{x}_i}|^2.$$

Setting $\psi_{1_i} := \frac{1}{2}r_i^2||\nabla_{\mathbf{q}_i}\boldsymbol{\beta}_i||^2||\mathbf{L}_i||^2 + \psi_{4_i}$, $\psi_{2_i} := \frac{1}{2}r_i^2||\mathbf{L}_i||^2 + \psi_{5_i}$ and $k_i := 4(1+\psi_{3_i})$, where $\psi_{3_i}, \psi_{4_i}, \psi_{5_i} \in \mathbb{R}_{>0}$ will be explicitly defined later, then $\dot{V}_i \leq -\psi_{3_i}|\boldsymbol{\eta}_i|^2 - \psi_{4_i}|\mathbf{e}_{\mathbf{x}_i}|^2 - \psi_{5_i}|\mathbf{e}_{\mathbf{q}_i}|^2$. Hence, $\mathbf{e}_{\mathbf{x}_i} = \mathbf{0}$ and $\mathbf{e}_{\mathbf{q}_i} = \mathbf{0}$ are also GES.

Finally, in order to prove that $r \in \mathcal{L}_\infty$, define $W_i := V_i + \frac{1}{2}r_i^2$ and evaluate its time-derivative along (17), such that

$$\dot{W}_i \leq -\dot{V}_i + \frac{r_i^2}{4(1+\psi_{3_i})}(||\boldsymbol{\Delta}_{\mathbf{q}_i}\mathbf{L}_i||^2 + ||\boldsymbol{\Delta}_{\mathbf{x}_i}\mathbf{L}_i||^2),$$

where the fact that $\frac{(r-1)}{r} \leq 1$, for $r > 0$, has been used.

Now, (15) ensures the existence of mappings $\bar{\boldsymbol{\Delta}}_{\mathbf{y}_i}$, $\bar{\boldsymbol{\Delta}}_{\mathbf{x}_i} : \mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n \to \mathbb{R}^{n\times n}$ such that

$$||\boldsymbol{\Delta}_{\mathbf{q}_i}(\mathbf{q}_i,\hat{\mathbf{x}}_i,\mathbf{e}_{\mathbf{q}_i})|| \leq ||\bar{\boldsymbol{\Delta}}_{\mathbf{y}_i}(\mathbf{q}_i,\hat{\mathbf{x}}_i,\mathbf{e}_{\mathbf{q}_i})||\,|\mathbf{e}_{\mathbf{q}_i}| \\ ||\boldsymbol{\Delta}_{\mathbf{x}_i}(\mathbf{q}_i,\hat{\mathbf{x}}_i,\mathbf{e}_{\mathbf{x}_i})|| \leq ||\bar{\boldsymbol{\Delta}}_{\mathbf{x}_i}(\mathbf{q}_i,\hat{\mathbf{x}}_i,\mathbf{e}_{\mathbf{x}_i})||\,|\mathbf{e}_{\mathbf{x}_i}|.$$

The latter sustains the fact that

$$||\boldsymbol{\Delta}_{\mathbf{q}_i}\mathbf{L}_i||^2 \leq ||\mathbf{L}_i||^2||\bar{\boldsymbol{\Delta}}_{\mathbf{q}_i}||^2|\mathbf{e}_{\mathbf{q}_i}|^2 \\ ||\boldsymbol{\Delta}_{\mathbf{x}_i}\mathbf{L}_i||^2 \leq ||\mathbf{L}_i||^2||\bar{\boldsymbol{\Delta}}_{\mathbf{x}_i}||^2|\mathbf{e}_{\mathbf{x}_i}|^2. \tag{20}$$

Therefore, setting

$$\psi_{4_i} = \frac{r_i^2}{4(1+\psi_{3_i})}||\mathbf{L}_i||^2||\bar{\boldsymbol{\Delta}}_{\mathbf{x}_i}||^2 + \psi_{6_i}$$

$$\psi_{5_i} = \frac{r_i^2}{4(1+\psi_{3_i})}||\mathbf{L}_i||^2||\bar{\boldsymbol{\Delta}}_{\mathbf{q}_i}||^2 + c_{1_i},$$

and using (20) returns

$$\dot{W}_i \leq -\psi_{3_i}|\boldsymbol{\eta}_i|^2 - \psi_{6_i}|\mathbf{e}_{\mathbf{x}_i}|^2 - c_{1_i}|\mathbf{e}_{\mathbf{q}_i}|^2 \leq 0, \tag{21}$$

where c_{1_i} is any positive constant and $\psi_{6_i} \in \mathbb{R}_{>0}$ will be defined in the next section.

W_i is positive definite and radially unbounded, w.r.t. r_i, and $\dot{W}_i \leq 0$, this implies that $r_i \in \mathcal{L}_\infty$ as required. Hence $\mathbf{z}_i = \mathbf{0}$ is GES.

Remark 1. The complete observer dynamics is given by (9), (16), (17) and (18) and the observed velocity $\dot{\hat{\mathbf{q}}}_i$ is defined as $\dot{\hat{\mathbf{q}}}_i := \mathbf{L}_i(\mathbf{q}_i)\hat{\mathbf{x}}_i$, where $\hat{\mathbf{x}}_i = \boldsymbol{\xi}_i + \boldsymbol{\beta}_i + \mathbf{e}_{\mathbf{x}_i}$.

4. CONSENSUS IN NETWORKS OF ROBOTS

The problem that this paper solves is the following:

Consensus Problem. Consider a network with N (different) robots, modeled as in (1). Assume that the interconnection graph satisfies **A1** and **A2**. Furthermore, suppose that velocity measurements $\dot{\mathbf{q}}_i$ are not available. Under this scenario, find a controller to ensure that all robot positions reach a consensus with zero velocity, i.e., for all $i \in \bar{N}$, there exists $\mathbf{q}_c \in \mathbb{R}^n$ such that $\lim_{t\to\infty}\mathbf{q}_i(t) = \mathbf{q}_c$ and $\lim_{t\to\infty}\dot{\mathbf{q}}_i(t) = \mathbf{0}$.

4.1 Proposed Controller

When velocities are available, the P+d controller (Nuño et al., 2013b) is capable of solving the consensus problem described above. In this paper we borrow the same structure as the P+d scheme but without using velocity measurements. The proposed scheme is given by

$$\boldsymbol{\tau}_i = \mathbf{g}_i(\mathbf{q}_i) - p_i \sum_{j\in\mathcal{N}_i} a_{ij}\left(\mathbf{q}_i - \mathbf{q}_j(t - T_{ji}(t))\right) - d_i\hat{\dot{\mathbf{q}}}_i, \quad (22)$$

where $p_i, d_i \in \mathbb{R}_{>0}$ are the proportional and the damping (like) gains, respectively.

The closed-loop system (1) and (22) is

$$\mathbf{M}_i\ddot{\mathbf{q}}_i + \mathbf{C}_i\dot{\mathbf{q}}_i + p_i \sum_{j\in\mathcal{N}_i} a_{ij}\left(\mathbf{q}_i - \mathbf{q}_j(t - T_{ji}(t))\right) + d_i\hat{\dot{\mathbf{q}}}_i = \mathbf{0}.$$

Defining $\mathcal{E}_i = \frac{1}{2p_i}\dot{\mathbf{q}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i + \frac{1}{4}\sum_{j\in\mathcal{N}_i} a_{ij}|\mathbf{q}_i - \mathbf{q}_j|^2$, and evaluating $\dot{\mathcal{E}}_i$ along (1) and (22), yields

$$\dot{\mathcal{E}}_i = -\frac{d_i}{p_i}\dot{\mathbf{q}}_i^\top\hat{\dot{\mathbf{q}}}_i - \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\mathbf{q}}_i^\top\left(\mathbf{q}_j - \mathbf{q}_j(t - T_{ji}(t))\right), \quad (23)$$

where Property **P2**, Assumption **A1**, the properties of the Laplacian matrix **W** and mimicking part of the proof of Proposition 2 in (Aldana et al., 2015) have been employed to compute (23).

On one hand, the fact that, from (8), (14) and the dynamic scaling $\mathbf{z}_i = r_i\boldsymbol{\eta}_i$, $\hat{\mathbf{x}}_i - \mathbf{x}_i = r_i\boldsymbol{\eta}_i + \mathbf{e}_{\mathbf{x}i}$. Hence,

$$\begin{aligned}\hat{\dot{\mathbf{q}}}_i &= \dot{\mathbf{q}}_i + \mathbf{L}_i(\mathbf{q}_i)(\hat{\mathbf{x}}_i - \mathbf{x}_i)\\ &= \dot{\mathbf{q}}_i + \mathbf{L}_i(\mathbf{q}_i)(r_i\boldsymbol{\eta}_i + \mathbf{e}_{\mathbf{x}i}).\end{aligned} \quad (24)$$

On the other hand, with

$$\mathbf{q}_j - \mathbf{q}_j(t - T_{ji}(t)) = \int_{t-T_{ji}(t)}^{t}\dot{\mathbf{q}}_j(\theta)d\theta,$$

$\dot{\mathcal{E}}_i$ can be written as

$$\dot{\mathcal{E}}_i = -\frac{d_i}{p_i}|\dot{\mathbf{q}}_i|^2 - \frac{d_i}{p_i}\dot{\mathbf{q}}_i^\top\mathbf{L}_i(r_i\boldsymbol{\eta}_i + \mathbf{e}_{\mathbf{x}i}) - \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\mathbf{q}}_i^\top\int_{t-T_{ji}(t)}^{t}\dot{\mathbf{q}}_j(\theta)d\theta.$$

The term $-\dot{\mathbf{q}}_i^\top\mathbf{L}_i(r_i\boldsymbol{\eta}_i + \mathbf{e}_{\mathbf{x}i})$ can be bounded, using Young's inequality, as

$$-\dot{\mathbf{q}}_i^\top\mathbf{L}_i(r_i\boldsymbol{\eta}_i + \mathbf{e}_{\mathbf{x}i}) \le \frac{1}{2}|\dot{\mathbf{q}}_i|^2 + r_i^2||\mathbf{L}_i||^2|\boldsymbol{\eta}_i|^2 + ||\mathbf{L}_i||^2|\mathbf{e}_{\mathbf{x}i}|^2.$$

Hence, we get

$$\begin{aligned}\dot{\mathcal{E}}_i = &-\frac{d_i}{2p_i}|\dot{\mathbf{q}}_i|^2 + \frac{d_i}{p_i}\left(r_i^2||\mathbf{L}_i||^2|\boldsymbol{\eta}_i|^2 + ||\mathbf{L}_i||^2|\mathbf{e}_{\mathbf{x}i}|^2\right) -\\ &-\sum_{j\in\mathcal{N}_i} a_{ij}\dot{\mathbf{q}}_i^\top\int_{t-T_{ji}(t)}^{t}\dot{\mathbf{q}}_j(\theta)d\theta.\end{aligned} \quad (25)$$

Now, using (21), defining $\mathcal{T}_i := \mathcal{E}_i + W_i$ and setting $\psi_{3_i} = \frac{d_i}{p_i}r_i^2||\mathbf{L}_i||^2 + c_{2_i}$ and $\psi_{6_i} = \frac{d_i}{p_i}||\mathbf{L}_i||^2 + c_{3_i}$, for any $c_{2_i}, c_{3_i} \in \mathbb{R}_{>0}$, yields

$$\begin{aligned}\dot{\mathcal{T}}_i \le &-\frac{d_i}{2p_i}|\dot{\mathbf{q}}_i|^2 - \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\mathbf{q}}_i^\top\int_{t-T_{ji}(t)}^{t}\dot{\mathbf{q}}_j(\theta)d\theta -\\ &- c_{1_i}|\mathbf{e}_{\mathbf{q}i}|^2 - c_{2_i}|\boldsymbol{\eta}_i|^2 - c_{3_i}|\mathbf{e}_{\mathbf{x}i}|^2.\end{aligned}$$

Defining $\boldsymbol{\chi}_i := \mathrm{col}(\boldsymbol{\eta}_i, \mathbf{e}_{\mathbf{x}i}, \mathbf{e}_{\mathbf{q}_i})$ and $c_{m_i} := \min\{c_{1_i}, c_{2_i}, c_{3_i}\}$, the previous inequality can be further written as

$$\dot{\mathcal{T}}_i \le -\frac{d_i}{2p_i}|\dot{\mathbf{q}}_i|^2 - \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\mathbf{q}}_i^\top\int_{t-T_{ji}(t)}^{t}\dot{\mathbf{q}}_j(\theta)d\theta - c_{m_i}|\boldsymbol{\chi}_i|^2. \quad (26)$$

Proposition 1. Controller (22) and the *I&I* observer, solve the Consensus Problem, provided that the controller gains satisfy the following condition, for any $\alpha_i > 0$:

$$\frac{d_i}{p_i} > w_{ii}\alpha_i + \sum_{j\in\mathcal{N}_i}\frac{{}^*T_{ij}^2 a_{ji}}{\alpha_j}, \qquad \forall i \in \bar{N}, j \in \mathcal{N}_i. \quad (27)$$

Proof. Defining $\mathcal{T} := \sum_{i\in\bar{N}}\mathcal{T}_i$, using (26) and integrating $\dot{\mathcal{T}}$, from 0 to t, yields

$$\begin{aligned}\mathcal{T}(t) - \mathcal{T}(0) \le -\sum_{i\in\bar{N}}\Bigg[&c_{m_i}\|\boldsymbol{\chi}_i\|_2^2 + \frac{d_i}{2p_i}\|\dot{\mathbf{q}}_i\|_2^2 +\\ &+\sum_{j\in\mathcal{N}_i} w_{ij}\int_0^t\dot{\mathbf{q}}_i^\top(\sigma)\int_{\sigma-T_{ji}(t)}^{\sigma}\dot{\mathbf{q}}_j(\theta)d\theta d\sigma\Bigg]\end{aligned}$$

Invoking Lemma 1 on the last term, with $\alpha_i \in \mathbb{R}_{>0}$, and using the fact that $w_{ii} = \sum_{j\in\mathcal{N}_i} a_{ij}$, returns

$$\begin{aligned}\mathcal{T}(0) \ge \sum_{i\in\bar{N}}\Bigg(c_{m_i}\|\boldsymbol{\chi}_i\|_2^2 + \sum_{j\in\mathcal{N}_i}\frac{a_{ij}}{2}\Bigg[&\left(\frac{d_i}{p_i w_{ii}} - \alpha_i\right)\|\dot{\mathbf{q}}_i\|_2^2 -\\ &-\frac{{}^*T_{ji}^2}{\alpha_i}\|\dot{\mathbf{q}}_j\|_2^2\Bigg]\Bigg) + \mathcal{T}(t).\end{aligned}$$

Using $\mathbf{Q} := \mathrm{col}(\|\dot{\mathbf{q}}_1\|_2^2, ..., \|\dot{\mathbf{q}}_N\|_2^2) \in \mathbb{R}^N$ and

$$\boldsymbol{\Psi} = \begin{bmatrix} \frac{d_1}{p_1} - w_{11}\alpha_1 & -\frac{{}^*T_{21}^2 a_{12}}{\alpha_1} & \dots & -\frac{{}^*T_{N1}^2 a_{1N}}{\alpha_1} \\ -\frac{{}^*T_{12}^2 a_{21}}{\alpha_2} & \frac{d_2}{p_2} - w_{22}\alpha_2 & \dots & -\frac{{}^*T_{N2}^2 a_{2N}}{\alpha_2} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{{}^*T_{1N}^2 a_{N1}}{\alpha_N} & -\frac{{}^*T_{2N}^2 a_{N2}}{\alpha_N} & \dots & \frac{d_N}{p_N} - w_{NN}\alpha_N \end{bmatrix},$$

we can write $\mathcal{T}(0) \ge \mathbf{1}_N^\top\boldsymbol{\Psi}\mathbf{Q} + \sum_{i\in\bar{N}} c_{m_i}\|\boldsymbol{\chi}_i\|_2^2 + \mathcal{T}(t)$.

It can be clearly seen that if the sum of the elements of every column of $\boldsymbol{\Psi}$ is strictly positive then there exist $\lambda_i > 0$ such that $\mathcal{T}(0) \ge \sum_{i\in\bar{N}}\left(\lambda_i\|\dot{\mathbf{q}}_i\|_2^2 + c_{m_i}\|\boldsymbol{\chi}_i\|_2^2\right) + \mathcal{T}(t)$. Hence $\dot{\mathbf{q}}_i \in \mathcal{L}_2$, for all $i \in \bar{N}$. A sufficient condition for the existence of $\lambda_i > 0$ is to choose d_i, for all $i \in \bar{N}$, to fulfill (27). Thus, $\dot{\mathbf{q}}_i \in \mathcal{L}_2$ and $\mathcal{T} \in \mathcal{L}_\infty$. This last ensures that $|\mathbf{q}_i - \mathbf{q}_j|$, $\dot{\mathbf{q}}_i \in \mathcal{L}_\infty$, for all $i \in \bar{N}$ and $j \in \mathcal{N}_i$. Furthermore, from (24), $\dot{\mathbf{q}}_i, r_i, \boldsymbol{\eta}_i, \mathbf{e}_{\mathbf{x}i} \in \mathcal{L}_\infty$ and **P1** imply that $\hat{\dot{\mathbf{q}}}_i \in \mathcal{L}_\infty$. From the closed-loop system (1) and (22), the fact that $\dot{\mathbf{q}}_i, \hat{\dot{\mathbf{q}}}_i \in \mathcal{L}_\infty$ and $\dot{\mathbf{q}}_i \in \mathcal{L}_2$, it is shown that $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$ and hence, using Barbalat's Lemma, $\lim_{t\to\infty}\dot{\mathbf{q}}_i(t) = \mathbf{0}$. This last and since $\mathbf{z}_i = \mathbf{0}$ and $\mathbf{e}_{\mathbf{x}i} = \mathbf{0}$ are GES, ensure that $\lim_{t\to\infty}\hat{\dot{\mathbf{q}}}_i(t) = \mathbf{0}$. Further, $|\mathbf{q}_i - \mathbf{q}_j|, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$ and **P4** imply that $\frac{d}{dt}\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$, meaning that $\ddot{\mathbf{q}}_i$ is uniformly continuous, and since

$$\lim_{t\to\infty}\int_0^t\ddot{\mathbf{q}}_i(\sigma)d\sigma = \lim_{t\to\infty}\dot{\mathbf{q}}_i(t) - \dot{\mathbf{q}}_i(0) = -\dot{\mathbf{q}}_i(0),$$

then $\lim_{t\to\infty}\ddot{\mathbf{q}}_i(t) = \mathbf{0}$.

Finally, convergence to zero of accelerations, velocities and estimated velocities ensure that $(\mathbf{W} \otimes \mathbf{I}_n)\mathbf{q} = \mathbf{0}$ is part of the Globally Asymptotically Stable (GAS) equilibrium, where $\mathbf{q} = \text{col}(\mathbf{q}_1^\top, \ldots, \mathbf{q}_N^\top)$. Thus, from the properties of the Laplacian matrix $\mathbf{W}$, there exists $\mathbf{q}_c \in \mathbb{R}^n$ such that $\lim_{t\to\infty} \mathbf{q}(t) = (\mathbf{1}_N \otimes \mathbf{q}_c)$. This completes the proof.

$\square$

5. SIMULATIONS

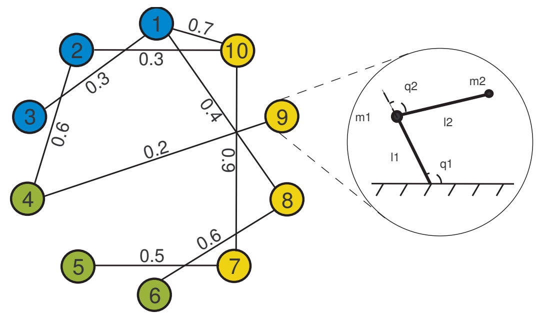

Fig. 1. Weighted network composed of ten 2-DOF nonlinear manipulators with revolute joints. The network is composed by three different groups of manipulators, and the members of each group are equal.

This section presents some numerical simulations that illustrate the consensus result reported here. The simulations employ a network of ten 2-DoF nonlinear manipulators with revolute joints (Fig. 1). The corresponding nonlinear dynamics are modeled using (1) with the inertia and Coriolis matrices given by

$$\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix} \alpha_i + 2\beta_i \mathrm{c}_{2_i} & \delta_i + \beta_i \mathrm{c}_{2_i} \\ \delta_i + \beta_i \mathrm{c}_{2_i} & \delta_i \end{bmatrix},$$

$$\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \begin{bmatrix} -\beta_i \mathrm{s}_{2_i} \dot{q}_{2_i} & -\beta_i \mathrm{s}_{2_i}(\dot{q}_{1_i} + \dot{q}_{2_i}) \\ \beta_i \mathrm{s}_{2_i} \dot{q}_{1_i} & 0 \end{bmatrix}$$

and the elements of the gravity vector given by $g_{1_i} = \frac{1}{l_{2_i}} g\delta_i \mathrm{c}_{12_i} + \frac{1}{l_{1_i}}(\alpha_i - \delta_i)\mathrm{c}_{1_i}$ and $g_{2_i} = \frac{1}{l_{2_i}} g\delta_i \mathrm{c}_{12_i}$, where $\alpha_i := l_{2_i}^2 m_{2_i} + l_{1_i}^2(m_{1_i} + m_{2_i})$, $\beta_i := l_{1_i} l_{2_i} m_{2_i}$ and $\delta_i := l_{2_i}^2 m_{2_i}$. c_{2_i}, s_{2_i} and c_{12_i} stand for the short notation of $\cos(q_{2_i})$, $\sin(q_{2_i})$ and $\cos(q_{1_i} + q_{2_i})$, respectively. q_{k_i} and $\dot{q}_{k_i}$ are the joint position and velocity, respectively, of link k of manipulator i, with $k \in \{1, 2\}$. l_{k_i} and m_{k_i} are the respective lengths and masses of each link.

The mapping $\mathbf{T}_i(\mathbf{q}_i)$, found using the Cholesky factorization, and its inverse $\mathbf{L}_i(\mathbf{q}_i)$ are given by

$$\mathbf{T}_i(\mathbf{q}_i) = \begin{bmatrix} \sqrt{m_{11_i} - \dfrac{m_{12_i}^2}{m_{22_i}}} & 0 \\ \dfrac{m_{12_i}}{\sqrt{m_{22_i}}} & \sqrt{m_{22_i}} \end{bmatrix},$$

and

$$\mathbf{L}_i(\mathbf{q}_i) = \begin{bmatrix} \dfrac{1}{T_{11_i}} & 0 \\ -\dfrac{T_{21_i}}{T_{11_i} T_{22_i}} & \dfrac{1}{T_{11_i}} \end{bmatrix},$$

respectively, where m_{jk_i} and T_{jk_i} are the jk-element of the matrices $\mathbf{M}_i(\mathbf{q}_i)$ and $\mathbf{T}_i(\mathbf{q}_i)$, respectively. Note that the mapping $\mathbf{T}_i(\mathbf{q}_i)$ satisfies $\mathbf{M}_i(\mathbf{q}_i) = \mathbf{T}_i^\top(\mathbf{q}_i)\mathbf{T}_i(\mathbf{q}_i)$, as required. The jk-elements of the skew-symmetric matrix $\mathbf{S}_i(\mathbf{q}_i, \mathbf{x}_i)$ are: $S_{11_i} = S_{22_i} = 0$; $S_{12_i} = -S_{21_i} = p_{1_i}(q_{2_i})x_{1_i} + p_{2_i}(q_{2_i})x_{2_i}$, where $p_{1_i}(q_{2_i}) = \frac{\beta_i \mathrm{s}_{2_i}}{T_{11_i}^2 T_{22_i}}\left(1 - \frac{T_{21_i}}{T_{22_i}}\right)$ and $p_{2_i}(q_{2_i}) = \frac{\beta_i \mathrm{s}_{2_i}}{T_{11_i} T_{22_i}^2}$.

Matrix $\bar{\mathbf{S}}_i(\mathbf{q}_i, \boldsymbol{\xi}_i + \boldsymbol{\beta}_i)$ is

$$\bar{\mathbf{S}}_i = \begin{bmatrix} p_{1_i}(q_{2_i})(\xi_{2_i} + \beta_{2_i}) & p_{2_i}(q_{2_i})(\xi_{2_i} + \beta_{2_i}) \\ -p_{1_i}(q_{2_i})(\xi_{1_i} + \beta_{1_i}) & -p_{2_i}(q_{2_i})(\xi_{1_i} + \beta_{1_i}) \end{bmatrix}$$

and the mappings $\mathbf{H}_i(\hat{\mathbf{q}}_i, \hat{\mathbf{x}}_i)$, $\boldsymbol{\Delta}_{\mathbf{y}_i}$, $\boldsymbol{\Delta}_{\mathbf{x}_i}$ are given by

$$\mathbf{H}_i = \begin{bmatrix} k_i T_{11_i}(\hat{q}_{2_i}) + p_{3_i}(\hat{q}_{2_i})\hat{x}_{2_i} & T_{22_i} p_{2_i}(\hat{q}_{2_i})\hat{x}_{2_i} \\ k_i T_{21_i}(\hat{q}_{2_i}) - p_{3_i}(\hat{q}_{2_i})\hat{x}_{1_i} & k_i T_{22_i} - T_{22_i} p_{2_i}(\hat{q}_{2_i})\hat{x}_{1_i} \end{bmatrix},$$

$$\boldsymbol{\Delta}_{\mathbf{q}_i} = \begin{bmatrix} k_i\left[T_{11_i}(q_{2_i}) - T_{11_i}(\hat{q}_{2_i})\right] + p_{4_i}\hat{x}_{2_i}, & T_{22_i} ep_{2_i}\hat{x}_{2_i} \\ k_i\left[T_{21_i}(q_{2_i}) - T_{21_i}(\hat{q}_{2_i})\right] - p_{4_i}\hat{x}_{1_i}, & T_{22_i} ep_{2_i}\hat{x}_{1_i} \end{bmatrix}$$

and

$$\boldsymbol{\Delta}_{\mathbf{x}_i} = \begin{bmatrix} -p_{3_i}(q_{2_i})e_{x_{2_i}} & -T_{22_i} p_{2_i}(q_{2_i})e_{x_{2_i}} \\ p_{3_i}(q_{2_i})e_{x_{1_i}} & T_{22_i} p_{2_i}(q_{2_i})e_{x_{1_i}} \end{bmatrix},$$

where $p_{3_i}(q_{2_i}) = T_{11_i}(q_{2_i})p_{1_i}(q_{2_i}) + T_{21_i}(q_{2_i})p_{2_i}(q_{2_i})$, $ep_{4_i} := p_{3_i}(q_{2_i}) - p_{3_i}(\hat{q}_{2_i})$ and $ep_{2_i} := p_{2_i}(q_{2_i}) - p_{2_i}(\hat{q}_{2_i})$. Clearly, $\mathbf{e}_{\mathbf{x}_i} = \mathbf{e}_{\mathbf{x}_i} = \mathbf{0}$ implies that $\boldsymbol{\Delta}_{\mathbf{q}_i} = \boldsymbol{\Delta}_{\mathbf{x}_i} = \mathbf{0}$.

For simplicity, the interconnection variable time–delay for all agents is the same and it emulates an ordinary UDP/IP Internet delay with a normal Gaussian distribution with mean, variance and seed equal to 0.45, 0.005 and 0.35, respectively Salvo-Rossi et al. (2006). Such delays are shown in Fig. 2 and, clearly, ${}^*T_{ij} = 0.7$s. It should be underscored that compared to the real Internet delays in Nuño et al. (2009), these delays are larger.

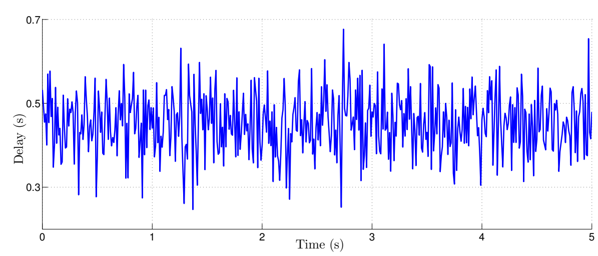

Fig. 2. Emulated UDP/IP Internet delay.

The network is composed of three different groups of robots, with equal members at each group. The parameters are: $m_1 = 4$kg, $m_2 = 2$kg and $l_1 = l_2 = 0.4$m, for Agents 1, 2 and 3; $m_1 = 3$kg, $m_2 = 2.5$kg, $l_1 = 0.6$m and $l_2 = 0.5$m for Agents 4, 5 and 6; $m_1 = 3.5$kg, $m_2 = 2.5$kg, $l_1 = 0.3$m and $l_2 = 0.35$m for Agents 7, 8, 9 and 10.

The proportional gains p_i for the controllers (22) are all 10Nm. Setting $\alpha_i = 1$, using ${}^*T_{ij} = 0.7$ and $p_i = 10$Nm, condition (27) transforms to $d_i > 17w_{ii}$, where w_{ii} corresponds to the ith–diagonal element of the Laplacian of the graph in Fig. 1. The damping gains are set to: $d_1 = 24$, $d_2 = 15.4$, $d_3 = 5.2$, $d_4 = 19$, $d_5 = 8.6$, $d_6 = 10.4$, $d_7 = d_9 = 14$, $d_8 = 18$ and $d_{10} = 28$.

The initial positions are
$\mathbf{q}(0) = [4, 3, -2, 5, 3, 6, 1, 3, 2, 4, 5, 3, 1, -2, 0, 2, 4, -1, 2, 3]^\top$,
and all the initial velocities are set to zero.

Fig. 3 shows the position behavior of the robot network, from which it can be concluded that all the robots position asymptotically converge to a common position.

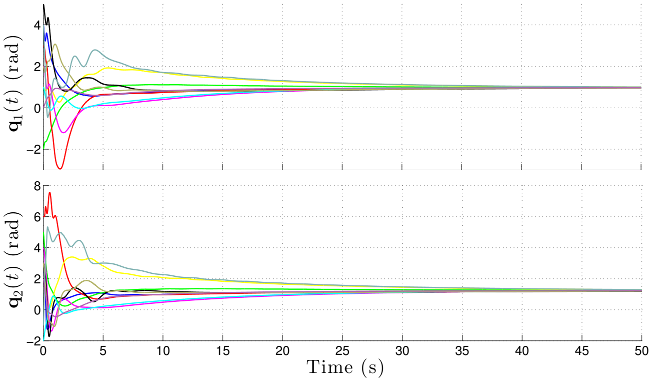

Fig. 3. Positions of the ten agents.

6. CONCLUSIONS

This paper reports a partial state feedback controller that solves the leaderless consensus problem in networks of robots. Using the I&I velocity observer, it is shown that the network can find a consensus provided that a sufficient condition on the controller's gains is satisfied. The robot interconnection graph is assumed to be undirected and connected and it might exhibit variable time-delays. Simulations, using a ten robot network, show that consensus is asymptotically achieved. Since the final value theorem cannot be applied to the closed-loop system, the final consensus point has not been analytically determined. A future research avenue is the inclusion of time-varying interconnection topologies.

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