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293

Control Design for Car-following Helly's Model [★]

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Abstract: This paper takes a well-known microscopic traffic model due to Helly. This model includes the relative velocity and the relative distance between two vehicles moving on a lane as its main variables, forming a simple and accurate expression to describe the car-following phenomena. We have conducted analysis for this model in order to design two possible control schemes, a PI Regulator and an Integral-Optimal Control, which none of them has been designed for a model like that presented here. Through suitable simulations, we have also performed comparissons between these two proposals, with some interesting results and proper observations on the features of each one of such designs suggested.

Keywords: Car-following Models; Helly's Model; PI-Controller; Optimal Control.

1. INTRODUCTION

Car-following models are representations of the system formed by two cars, one in the front and other in the back, moving in a single lane. Each vehicle has its own velocity: $v_L(t)$ for that car in front (leader) and $v_f(t)$ for the car in the back (follower), but this last one is affected by the first if it is asummed that passings are not allowed (Treiber et al., 2000).

Car-following phenomena are recognized as a base to understand traffic congestion. From a local description of pairs of vehicles it is possible to recreate cumulative effects in a lane or even in a network (Panwai and Dia, 2005), in such a way that going from relatively simple expressions it is possible to model real complex systems.

Natural progress has marked a trend in developing more accurate expressions for such phenomenon, implying more sofisticated models with an increasing number of terms and parameters to take into account since the first attempts to emulate the behavior of pairs of vehicles running on a lane (Olstam and Tapani, 2004).

However, it is always advantageous to succced in a balance between accuracy and simplicity in order to obtain precise enough calculations with low computation effort. Looking back on the first and sufficiently well established models of L.A. Pipes (1953) and W. Helly (1959), it is possible to notice that they both posses an intuitive support as well as a well-defined theoertical base.

These two models include few parameters and, once calibrated and tested, they approach to reality with very little error. A next natural step is the design of control schemes for regulation purposes. The direct variable to be controlled is the velocity $v_f(t)$, that now should be a function of the distance and the velocity of the vehicle in front, regarding safety restrictions. This regulation can be used to obtain a safety breaking system for avoiding possible collisions due to distractions, or for developing autonomous driving cars (Göhring et al., 2013).

Autonomous driving is a topic that is becoming very popular in recent years (Wang et al., 2013). Even though it needs the integration of different subsystems, the main module is that in charge with the regulation of the velocity $v_f(t)$. There are many algorithms that deal with such tasks based, for example, on multi-objective optimal schemes (Swaroop, D. 1994). Those approaches comply with the purposes implied in an autonomous driving. However, they are rather complex, and we have turn to simple models that we think they are not fully studied, like those presented here.

This document is organized as follows: in the next section Pipes' and Helly's models are described and compared, but focusing on the second one. The distinct forms to describe this model will be useful to design different control schemes as seen in Section 3 which, as far as we know, they have not been reported even though their simplicity. The performance of these schemes are presented in Section 4, where the simulated variables show interesting responses specially when they face disturbances. At the end, some pertinent conclusions are included.

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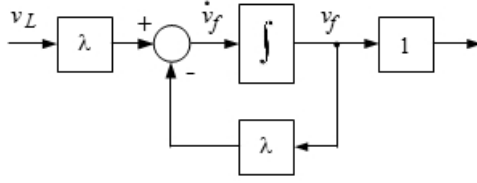Cuernavaca, Morelos, México.

294

Fig. 1. Block diagram for Pipes' model as suggested in equation (1).

2. DESCRIPTION OF MODELS

2.1 Pipes' Model

One of the first and simple approaches to the car-following phenomena is due to L. A. Pipes (1953). His proposal is very intuitive since it relates the acceleration $\dot{v}_f(t)$ of a follower driver with the difference of the velocity of the car in front $v_L(t)$ and the car behind $v_f(t)$

$$\frac{dv_f(t)}{dt} = \lambda \left[v_L(t) - v_f(t) \right] \qquad (1)$$

where λ is a sensistivity parameter, related with the reactive behavior of the driver in the follower car (Fig. 1).

Calibrating this parameter (Rosas-Jaimes et al., 2013) gives a quantitiy in the range $\lambda \in (0, 1]$, where values closer to 0 represent less reactive drivers, and values approaching 1 represent those drivers with a more reactive behavior. This model has been tested and, even though it is known that it fails for some conditions, most of the time it behaves quite accurately (Rosas-Jaimes et al., 2014).

However, one caveat about the behaviour of this model is that the only stimulus taken into account is the relative speed between cars. Some studies have shown that drivers also consider a safety distance between bumps to avoid collisions (Chung et al., 2005).

Another shortcoming of equation (1) is that the arithmetic difference between simultaneously equal but non-steady velocities sometimes result in zero values, giving zero acceleration $dv_f(t)/dt$, which is not realistic either (Ioannou et al., 2008).

2.2 Helly's Model

The car-following model due to Helly (1959) extends Pipes' model introducing a term that takes into account the distance separation between the involved cars

$$\frac{dv_f(t)}{dt} = \lambda_v \left[v_L(t) - v_f(t) \right] +$$
$$+ \lambda_x \left[x_L(t) - x_f(t) - D(t) \right] \qquad (2)$$

In equation (2) λ_v is the same sensitivity parameter as in Pipes' equation (1) that modifies the difference of velocities, meanwhile λ_x is another similar parameter for the additional term of the difference between the leader's position $x_L(t)$ and the follower's position $x_f(t)$, which is in turn affected by a desired distance $D(t)$. This last term adjusts the driver's response in such a way that it depends on the relative distance $x_R(t) = x_L(t) - x_f(t)$ and not only on the relative velocity $v_R(t) = v_L(t) - v_f(t)$, giving a more realistic approach.
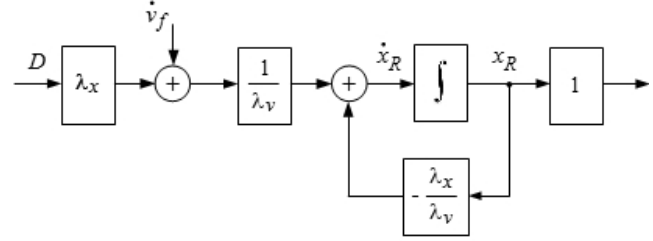

Fig. 2. Helly's model as suggested in equation (4)

Equation (2) is a well-known expression in Transportation and Traffic. With the relative quantities of v_R and x_R defined, this expression can be written in an alternative way

$$\frac{dv_f(t)}{dt} = \lambda_v v_R(t) + \lambda_x \left[x_R(t) - D(t) \right] \qquad (3)$$

From equation (3) it is possible to obtain equation (4) focusing on $x_R(t)$

$$\dot{x}_R(t) = -\frac{\lambda_x}{\lambda_v} \left[x_R(t) \right] + \frac{1}{\lambda_v} \left[\dot{v}_f(t) + \lambda_x D(t) \right] \qquad (4)$$

Fig. (2) depicts by blocks this approach to Helly's model.

From equation (4) it possible to make some interesting observations:

- The relative distance $x_R(t)$ is a state and also an output suitable to be measure.
- $\dot{x}_R(t) = v_R(t)$.
- The time derivative of the follower's velocity $\dot{v}_f$ and the desired distance $D(t)$ are inputs.
- The acceleration of the follower $\dot{v}_f$ is an estimable quantity.
- The desired distance can be viewed as a function determined beforehand or as a quantity to be estimated.

As can be seen, this model permits convenient manipulation. Another way to express Helly's model is by giving it a second order treatment. It is now evident that this model is fully represented by these two equations:

$$\dot{v}_f(t) = \lambda_v v_R(t) + \lambda_x x_R(t) - \lambda_x D(t) \qquad (5a)$$
$$\dot{x}_R(t) = v_R(t) \qquad (5b)$$

But, $v_R(t) = v_L(t) - v_f(t)$ as previously mentioned, and then equations (5) can be written as

$$\dot{v}_f(t) = -\lambda_v v_f(t) + \lambda_x x_R(t) + \left[\lambda_v v_L(t) - \lambda_x D(t) \right] \qquad (6a)$$
$$\dot{x}_R(t) = -v_f(t) + v_L(t) \qquad (6b)$$

which can be express as a matrix form

$$\begin{bmatrix} \dot{v}_f \\ \dot{x}_R \end{bmatrix} = \begin{bmatrix} -\lambda_v & \lambda_x \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_f \\ x_R \end{bmatrix} + \begin{bmatrix} \lambda_v & -\lambda_x \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_L \\ D \end{bmatrix} \qquad (7)$$

where the state is formed by the velocity of the follower v_f and the distance between the vehicles x_R. Besides, leader's velocity v_L and the desired distance D are properly indicated as inputs. It is convenient to keep the output as $x_R(t)$ in this case too, and then the output is

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} v_f \\ x_R \end{bmatrix} \qquad (8)$$

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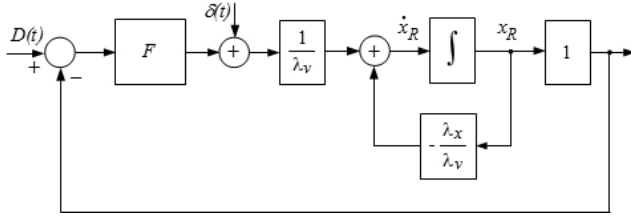Automático, AMCA 2015,
Cuernavaca, Morelos, México.

295

Fig. 3. Block diagram for output feedback applied to Helly's model based on (4)

3. CONTROL SCHEMES

3.1 Output Feedback

By a proper device [1], it is possible to obtain the distance $x_R(t)$ between a leader vehicle and a follower one.

This distance must not be less than a desired distance $D(t)$ for safety purposes. This quantity can be treated as a reference for a control to be developed.

Fig. 3 shows the plant that represents Helly's model by equation (4), included in a feedback control scheme, where the function F stands for a controller designed to make $x_R(t) \to D(t)$. A clasical feedback is one achieved by pole placement like a proportional plus integral (PI), a proportional plus derivative (PD) or a proportional plus integral plus derivative (PID) controller.

It possibly comes to mind to tune up the gains for such controllers by a method like Ziegler–Nichols, but in this case the plant, Helly's model, is a first order system represented by the transfer function (9), it does not has a delay and it cannot oscillate by moving its only pole.

$$\frac{x_R}{u} = \frac{\frac{1}{\lambda_v}}{s + \frac{\lambda_x}{\lambda_v}} \tag{9}$$

A PD controller tends to accelerate the response, and a PI controller tends to improve the steady state error. The first of them could generate an uncomfortable or even a dangerous situation if the acceleration achieved is too high. On the other hand, disturbances and variations on a distance $D(t)$ that change with time could be better track if the difference with $x_R(t)$ is minimized.

Following this reasoning, a function F as in equation (10) is proposed

$$F = k\frac{s+a}{s} = k + \frac{ka}{s} \tag{10}$$

In this way, a pure integrator is compensated by a very near zero, increasing the order of the control system and improving the response because the diference $D(t) - x_R(t)$ tends to zero.

It has been determined that it is not convenient for this case to use a PID controller. In a real situation, it requieres more effort to construct and to operate such a regulator, and its performance is not completly justified when compared with the simpler PI regulator presented.

[1] There are a number of devices used to measure the separation between a car that follows another in front of it, using radar, or paterrn recognition from the image of a camera, to say some of the most common.

3.2 Optimal Control

In order to design an optimal control scheme by output feedback, let the desired distance $D(t)$ be the reference signal and let the leader vehicle velocity $v_L(t)$ a disturbance.

The input vector $u(t)$ to affect the system is then

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} v_L \\ k_2(D - x_R) + k_1 v_f \end{bmatrix} \tag{11}$$

in such a way that u_2 is equivalent to

$$u_2 = \begin{bmatrix} k_1 & k_2' \end{bmatrix} \begin{bmatrix} v_f \\ x_R \end{bmatrix} - k_2' D \tag{12}$$

where $k_2' = -k_2$.

In order to develop an optimal control, it is possible to find out a solution for Riccati Equation (13)

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{13}$$

where the matrices A, B, R, and Q are defined as

$$A = \begin{bmatrix} -\lambda_v & \lambda_x \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} \lambda_v & -\lambda_x \\ 1 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} \lambda_x & 0 \\ 0 & \lambda_v \end{bmatrix} \qquad R = \begin{bmatrix} \lambda_v/2 & 0 \\ 0 & \lambda_x \end{bmatrix}$$

where A is the matrix of coefficients of the state and B is the matrix of coefficients of the inputs, for the system described by equation (7). Matrices Q and R are symmetric and positive definite, as can be noticed by the elements with values λ_v and λ_x included in them.

This equation has values in such a way that minimizes the performance index expressed by equation (14)

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt \tag{14}$$

To satisfy equation (13) it is sufficient that P has values as in (15)

$$P = \begin{bmatrix} \dfrac{-\lambda_v}{\lambda_v + \lambda_x} + \dfrac{\sqrt{\lambda_v^2 + \lambda_v\lambda_x + \lambda_x^2}}{\lambda_v + \lambda_x} & 0 \\ 0 & \dfrac{\sqrt{2}}{2}\lambda_v \end{bmatrix} \tag{15}$$

With the defined and calculated values, then it is possible to obtain a gain matrix K to achieve an optimal state feedback

$$K = R^{-1}B^T P =$$

$$= \begin{bmatrix} \dfrac{-2\lambda_x}{\lambda_v + \lambda_x} + \dfrac{2\sqrt{\lambda_v^2 + \lambda_v\lambda_x + \lambda_x^2}}{\lambda_v + \lambda_x} & \dfrac{\sqrt{2}}{2}\lambda_x \\ \dfrac{\lambda_v}{\lambda_v + \lambda_x} - \dfrac{\sqrt{\lambda_v^2 + \lambda_v\lambda_x + \lambda_x^2}}{\lambda_v + \lambda_x} & 0 \end{bmatrix} \tag{16}$$

Notice that, in this way, the matrix gain K is in close relation with the sensitivity parameters λ_v and λ_x.

To improve the response and to vanish the error signal in a more efficient way, an integral block with a gain $G_I = 0.25$ is added to the control scheme, which is depicted complete in Fig. 4.

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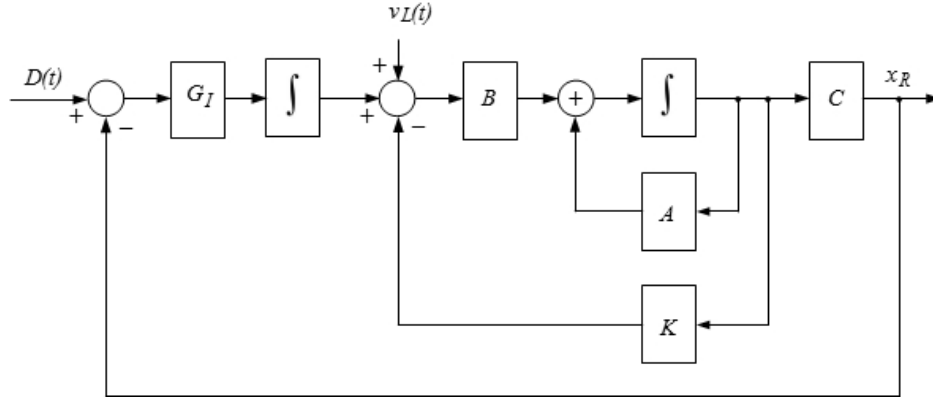Automático, AMCA 2015,
Cuernavaca, Morelos, México.

296

Fig. 4. Block diagram for optimal-integral control for Helly's model

Notice that this is a state feedback scheme and it should be necessary to know the state $[v_f \quad x_R]^T$. Fortunately, by the proposed second order model, these two quantities are measurable by proper devices. Besides, as $u(t) = -Kx$ then the performance index can be written as in (17)

$$J = \int_0^\infty \left(x^T Q x + x^T K^T R K x \right) dt =$$

$$= \int_0^\infty x^T \left(Q + K^T R K \right) x \, dt \qquad (17)$$

The argument in (17), $M = x^T \left(Q + K^T R K \right) x$, is expressed by λ_v, λ_x and the state, being J a quadratic expression in which

$$J = \int_0^\infty \left[M_{11} v_f^2 + (M_{12} + M_{21}) v_f x_R + M_{22} x_R^2 \right] dt \quad (18)$$

This expression has quadratic and cross terms of v_f and x_R, in close analogy with kinetic and potential energy.

This is related with the nature of J and the physical meaning of the terms in it, where the minimization of this performance index is directly related with the optimization of the energy terms in which the velocity $v_f(t)$ of the follower and the separation between cars $x_R(t)$ are involved.

Finally, we catch the attention in the fact, based on what we have seek and found, that none of this controllers have been designed and implemented taking Helly's model as a base, instead of the simplicity of the model and the direct processes of reaching such regulation schemes.

4. SIMULATIONS

In this section some convenient simulations are carried out for both control schemes presented in the last section. All these simulations were perfomed with the values $\lambda_v = 0.5$ and $\lambda_x = 0.7$, which are common in practice (Lee, 1966), (Burnham et al., 1974).

For the PI control (10) it is proposed that

$a = 0.1 \frac{\lambda_x}{\lambda_v}$
$k = 5$

For a first simulation, a desired distance $D(t) = 6$ m has been chosen, with an initial separation of $D(0) = 10$ m.

Fig. 5 shows the behavior of the relative distance $x_R(t)$ between two vehicles in a car-following scenario affected by this control. In Fig. 5 it is also possible to notice that the transient has a very high change at the beginning, which represents a high acceleration achieved by the follower, but then this signal tends to smooth until it reaches the desired value in a soft manner.
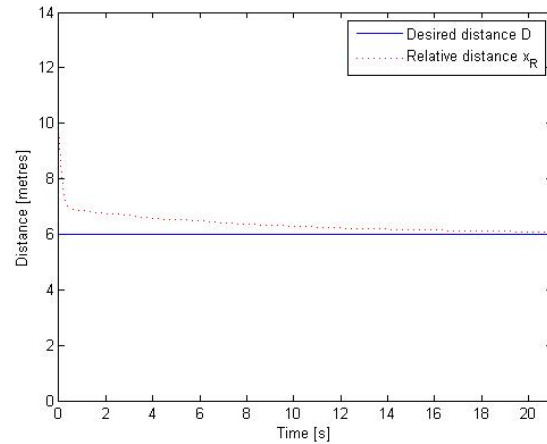

Fig. 5. Response of the relative distance $x_R(t)$ in relation to the desired distance $D(t) = 6$ m, using a PI controller.

If a PD controller were used, it is possible that this accceleration could be higher, which represents a condition that could be uncomfortable or even dangerous for any possible travellers in that car.

The desired velocity $D(t)$ is not necessarily constant and it could be convenient that it varies.

Leaving all the values used the same, but making $D(t)$ a senoid, with a frequency of $\omega = 1$ rad/s and an amplitude of ± 2 m gives the behaviour depicted in Fig. 6.

As in the previous case, the acceleration at the beginning is high for the follower and similar remarks can be done for this situation. On the other hand, it can be seen that the controller tries to track the senoid $D(t)$. However, due to the design values, it never succeeds in achieving a zero error, and it can be seen that the follower has a slow response whenever the leader is changing velocity $v_L(t)$.
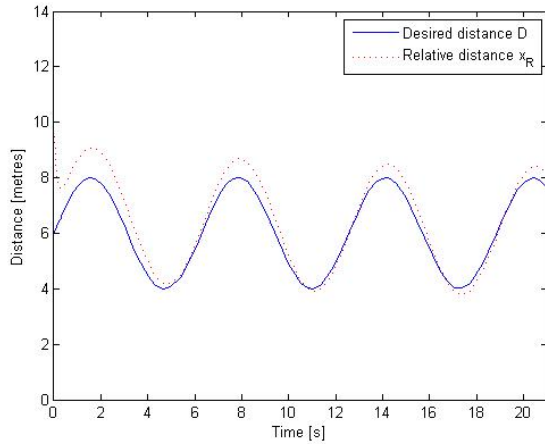
Congreso Nacional de Control
Automático, AMCA 2015,
Cuernavaca, Morelos, México.

297

Fig. 6. Response of the relative distance $x_R(t)$ in relation to the desired distance $D(t) = 2\sin(t) + 6$ m, using a PI controller.

If now the interest is on a variable value for the speed and acceleration of the leader car, while the desired distance is keep in a constant value, then Fig. 7 plots an oscillatory behavior due to the acceleration and desceleration due to the leader vehicle, but that the controller tries to compensates making that the fluctuation of the relative distance x_R surrounds the desired value of D.

All this trajectories can be viewed as a fair balance, in which the tracking is suffcently well behaved, taken into account that a more rapid response could originate higher accelerations as mentioned earlier.
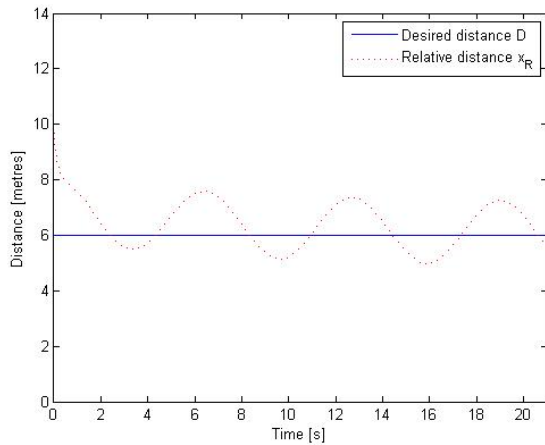

Fig. 7. Response of the relative distance $x_R(t)$ in relation to the velocity of the leader $v_L(t) = 5\sin(t) + 30$ m, using a PI controller.

In the case of the integral-optimal control, Fig.8 shows the response of the relative distance between vehicles in comparison with a constant value for $D(t)$ of 6 m. If the velocity of the leader is $v_L(t) = 30$ m/s (a little more than 100 km/hr), it can be seen that $x_R(t)$ increases at the beginning, implying a slow motion of the follower while the leader moves faster when starting, and then an acceleration of such amount that the desired value is reached in a soft manner during a process of about 20 seconds.
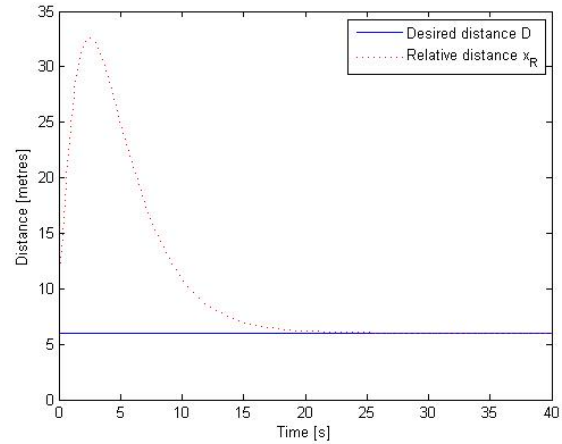

Fig. 8. Response of the relative distance $x_R(t)$ in relation to the desired distance $D(t) = 6$ m, for an integral-optimal control.

To establish proper comparissons, the distance $D(t)$ is varied as was the case for the PI regulator, in such a way that $D(t) = 2\sin(t) + 6$ m and $v_L(t)$ is kept at 30 m/s.

Fig. 9 shows the evolution of the relative distance between cars as this variable is tried to be adjusted by the control. As can be seen, the tracking is not perfect, mainly by the same reason explained in the case of the PI control: the acceleration must not be high for the follower. The transition at the very beginning denotes drastic changes in order to achieve tracking.
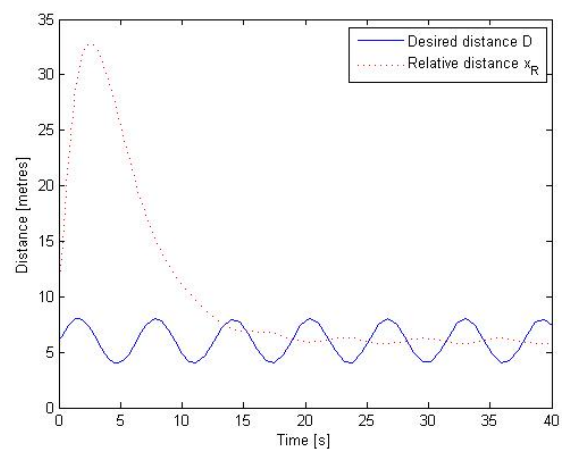

Fig. 9. Response of the relative distance $x_R(t)$ in relation to the desired distance $D(t) = 2\sin(\omega t) + 6$ m, while $V_L(t)$ is 30 m/s, for an integral-optimal control.

In order to show another point of view for this scheme, now a fluctuation in the leader's velocity in such a way that $v_L(t) = 5\sin(t) + 30$ m/s is applied, leaving the desired distance $D(t)$ constant at 6 m. It is noticeable that the control scheme tryies to adjust the response in a way that is depicted in Fig. 10.

In this case, the leader car is accelerating and breaking in an oscillating way, and the control is trying to adjust the trajectory of the follower in order to track it.
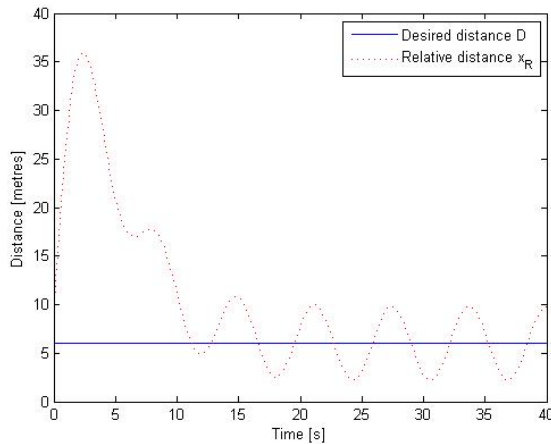
Congreso Nacional de Control
Automático, AMCA 2015,
Cuernavaca, Morelos, México.

298

Fig. 10. Response of the relative distance $x_R(t)$ in relation to the desired constant distance $D(t) = 6$ m, while $v_L(t) = 5\sin(t) + 30$, for an integral-optimal control.

As can be seen, the relative distance $x_R(t)$ oscillates around the desired distance $D(t) = 6$ m.

5. CONCLUSIONS

We have shown the well-known Helly's model as an extension of the Pipes' model. The former adds a relative distance term as a way to improve the accuracy of what is seen in reality about the car-following phenomena.

Helly's model is presented here not only in its most famous form as a differential equation, but also as a block diagram and as a state representation, a treatment that is not reported in other media as far as we know.

These representations lead directly to the design of control schemes in order to regulate the velocity developed by the follower vehicle. In this way, the desired distance $D(t)$ and the leader's velocity $v_L(t)$ are seen as inputs or as disturbances.

The controllers here presented behave well enough. One of them is a PI regulator. The behavior achieved minimize the difference $D(t) - x_R(t)$ in a reasonable period of time, with a reasonable amount of acceleration in the cases where the desirable distance $D(t)$ stayed constant and when varied as a senoid signal.

For the integral-optimal regulator, similar results have obtained. However, this scheme asures that the energy cost is the minimum to use for regulating the movement of the follower, due to the optimal part, and the error between $x_R(t)$ and $D(t)$ tends to zero, due to the integral part.

The integral-optimal regulator can be seen as more precise arrengement to control the system, but the PI regulator is easier to implement. It will depend on the especifications wanted, which of these control is used in a real application.

It is important to remark that thess control schemes have not been found reported for a model like Helly's, even though its simplicity is appealing and the desing process is very direct. This is an important observation for those possible applications in which could be easy to implement some of these regulators.

Finally, it is also desirable to show stability analysis to those schemes presented here, as well as to establish their relationship with phenomena that imply disipative effects, for example. However, because the limited space, those studies must be further presented in future documents.

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