SPD-SI Control with Simple Tuning for the Global Regulation of Robot Manipulators with Bounded Inputs ⋆

Marco Mendoza * **Arturo Zavala-Río** ** **Víctor Santibáñez** *** **Fernando Reyes** ****

* *Universidad Autónoma de San Luis Potosí, Av. Salvador Nava S/N, Zona Universitaria 78290, San Luis Potosí, Mexico (e-mail: marco.mendoza@uaslp.mx)*
** *Instituto Potosino de Investigación Científica y Tecnológica, Camino a la Presa San José 2055, Lomas 4a. Sección 78216, San Luis Potosí, Mexico (e-mail: azavala@ipicyt.edu.mx)*
*** *Instituto Tecnológico de la Laguna, Apdo. Postal 49, Adm. 1, 27001 Torreón, Mexico (e-mail: vsantiba@itlalaguna.edu.mx)*
**** *Benemérita Universidad Autónoma de Puebla, Boulevard Puebla 108, Bosques de San Sebastián 72310, Puebla, Mexico (e-mail: fernando.reyes@correo.buap.mx)*

Abstract: In this paper, a Proportional-Integral-Derivative (PID) type controller for the global position stabilization of robot manipulators with bounded inputs is proposed. With respect to previous approaches of the kind, it represents a simplifying alternative through its SPD-SI structure that involves *generalized* saturation functions. Moreover, it is mainly characterized by its very simple control-gain tuning criterion, the simplest hitherto obtained while guaranteeing the global regulation objective —avoiding input saturation— in the considered analytical framework. Experimental results on a 2-degree-of-freedom direct-drive manipulator corroborate the efficiency of the proposed controller.

Keywords: PID control, simple tuning, global regulation, robot manipulators, saturation.

1. INTRODUCTION

In real applications where robot manipulators are involved, PID controllers are commonly used in practice (Rocco, 1996). Several versions of such controllers using nonlinear structures, mainly oriented to guarantee global regulation, have been developed for instance in (Kelly, 1998). Nevertheless, such modified schemes consider that actuators can furnish any required torque value, which is not realistic. Moreover, unexpected or undesirable behaviors could take place in view of the saturation nonlinearity that generally relates the controller outputs to the plant inputs in real applications (Krikelis and Barkas, 1984). Under the consideration of such a constraint, several schemes have been further presented in the literature. For instance, feedback controllers with Saturating-Proportional (SP) and Saturating-Derivative (SD) actions under exact gravity compensation (Santibáñez and Kelly, 1996) were some of the earlier proposals. Parametric dependency has been further reduced through bounded adaptive approaches with discontinuous structures (Colbaugh *et al.*, 1997) as well as continuous feedback approaches (López-Araujo *et al.*, 2013a). Nevertheless, these algorithms remain partially model dependent by involving the regression matrix implicated in the linear structural characterization of the gravity force vector with respect to its parametric coefficient set.

On the other hand, bounded PID-type controllers have been further developed alleviating the model dependence. For instance, semiglobal regulators with different saturating structures have been proposed in (Alvarez-Ramirez *et al.*, 2003) and (Alvarez-Ramirez *et al.*, 2008). The stability analyses in these studies are developed through the singular perturbation methodology which shows the existence of a suitable tuning mainly characterized by the requirement of small enough integral action gains and sufficiently high proportional and derivative ones. As far as the authors are aware, the first bounded PID-type algorithm for global position stabilization was proposed in (Gorez, 1999). Nonetheless, the structure of the controller developed therein is quite complex. Other studies have devoted efforts to solve the global PID position stabilization problem for manipulators with constrained inputs through simpler structures, giving rise to the SP-SI-SD type algorithm developed in (Meza *et al.*, 2005) *via* passivity theory and later on in (Su *et al.*, 2010) through Lyapunov stability analysis, and to the SPD-SI type scheme presented in (Santibáñez et al., 2008). However, some of these controllers were developed using a particular structure, namely involving a specific saturation function —more precisely the hyperbolic tangent—

⋆ Marco Mendoza was supported by CONACYT, Mexico. Víctor Santibáñez was supported by CONACYT (project no. 134534) and TNM (Tecnológico Nacional de México), Mexico.

and applying the control gains to the saturating actions externally. This constrains the control gains to be small enough to guarantee the avoidance of the input saturation phenomenon, which severely restricts their choice resulting in important limitations on the closed-loop performance. Furthermore, the stability analyses of the previous PID-type approaches give rise to several complex conditions on the control gains that hinder the tuning task. Such limitations have motivated this work, where a PID-type global regulator for robot manipulators with constrained inputs is proposed. It keeps an SPD-SI structure that involves *generalized* saturation functions and releases the control gains from saturation avoidance inequalities. More importantly, the developed algorithm is mainly characterized by its very simple control-gain tuning criterion; simplification of the tuning conditions for PID-type controllers has been a research subject for several years (Kelly, 1995) and had never been achieved to be as simple as it is shown in this paper. Experimental tests on a 2-degree-of-freedom (DOF) direct-drive manipulator corroborate the contributed result. It is worth further pointing out that the SPD-SI approach designed in this work extends the previous result in (Mendoza *et al.*, 2014), where the proposed control scheme kept an SP-SI-SD structure.

2. PRELIMINARIES

Let $X \in \mathbb{R}^{m\times n}$ and $y \in \mathbb{R}^n$. Throughout this paper, X_{ij} represents the element of X at its i^{th} row and j^{th} column, and y_i denotes the i^{th} element of y. 0_n stands for the origin of $\mathbb{R}^n$ and I_n represents the $n\times n$ identity matrix. $\|\cdot\|$ stands for the standard Euclidean norm for vectors, *i.e.* $\|y\| = \sqrt{\sum_{i=1}^n y_i^2}$, and induced norm for matrices, *i.e.* $\|X\| = \sqrt{\lambda_{\max}\{X^TX\}}$ where $\lambda_{\max}\{X^TX\}$ represents the maximum eigenvalue of X^TX. For a continuous scalar function $\psi : \mathbb{R} \mapsto \mathbb{R}$, ψ'denotes its derivative, when differentiable, $D^+\psi$ its upper right-hand (Dini) derivative, *i.e.* $D^+\psi(\varsigma) = \limsup_{h\to 0^+} \frac{\psi(\varsigma+h)-\psi(\varsigma)}{h}$, with $D^+\psi = \psi'$ at points of differentiability (Khalil, 2002, Appendix C.2), and ψ^{-1} its inverse, when invertible.

Consider the n-DOF serial rigid manipulator dynamics with viscous friction (Kelly *et al.*, 2005)

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau \tag{1}$$

where q, $\dot{q}$, $\ddot{q} \in \mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity, and acceleration vectors, $H(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, and $C(q,\dot{q})\dot{q}$, $F\dot{q}$, $g(q)$, $\tau \in \mathbb{R}^n$ are respectively the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with $F \in \mathbb{R}^{n\times n}$ being a positive definite constant diagonal matrix whose entries $f_i > 0$, $i = 1,\dots,n$, are the viscous friction coefficients, and $g(q) = \nabla\mathcal{U}(q)$, with $\mathcal{U}(q)$ being the gravitational potential energy, or equivalently

$$\mathcal{U}(q) = \mathcal{U}(q_0) + \int_{q_0}^{q} g^T(r)dr \tag{2a}$$

for any q, $q_0 \in \mathbb{R}^n$, with [1]

$$\begin{aligned}\int_{q_0}^{q} g^T(r)dr = &\int_{q_{01}}^{q_1} g_1(r_1, q_{02},\dots,q_{0n})dr_1 \\ &+ \int_{q_{02}}^{q_2} g_2(q_1, r_2, q_{03},\dots,q_{0n})dr_2 \\ &+ \cdots + \int_{q_{0n}}^{q_n} g_n(q_1,\dots,q_{n-1},r_n)dr_n\end{aligned} \tag{2b}$$

Some well-known properties characterizing such a dynamical model are recalled here (Kelly *et al.*, 2005, Chap. 4). Subsequently, we denote $\dot{H}$ the rate of change of H, *i.e.*, $\dot{H} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n} : (q,\dot{q}) \mapsto \left[\frac{\partial H_{ij}}{\partial q}(q)\dot{q}\right]$.

Property 1. $H(q)$ is a continuously differentiable matrix function being positive definite, symmetric and bounded on $\mathbb{R}^n$, *i.e.* such that $\mu_m I_n \le H(q) \le \mu_M I_n$, $\forall q \in \mathbb{R}^n$, for some constants $\mu_M \ge \mu_m > 0$.

Property 2. The Coriolis matrix $C(q,\dot{q})$ satisfies:

2.1. $\|C(q,\dot{q})\| \le k_C\|\dot{q}\|$, $\forall (q,\dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, for some constant $k_C \ge 0$;
2.2. for all $(q,\dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, $\dot{q}^T\left[\frac{1}{2}\dot{H}(q,\dot{q}) - C(q,\dot{q})\right]\dot{q} = 0$ and actually $\dot{H}(q,\dot{q}) = C(q,\dot{q}) + C^T(q,\dot{q})$.

Property 3. The viscous friction coefficient matrix satisfies $f_m\|\dot{q}\|^2 \le \dot{q}^TF\dot{q} \le f_M\|\dot{q}\|^2$, $\forall \dot{q} \in \mathbb{R}^n$, where $0 < f_m \triangleq \min_i\{f_i\} \le \max_i\{f_i\} \triangleq f_M$.

Property 4. The gravity force term $g(q)$ is a continuously differentiable bounded vector function with bounded Jacobian matrix [2] $\frac{\partial g}{\partial q}$. Equivalently, every element of the gravity force vector, $g_i(q)$, $i = 1,\dots,n$, satisfies:

4.1. $|g_i(q)| \le B_{gi}$, $\forall q \in \mathbb{R}^n$, for some constant $B_{gi} > 0$;
4.2. $\frac{\partial g_i}{\partial q_j}$, $j = 1,\dots,n$, exist and are continuous and such that $\left|\frac{\partial g_i}{\partial q_j}(q)\right| \le \left\|\frac{\partial g}{\partial q}(q)\right\| \le k_g$, $\forall q \in \mathbb{R}^n$, for some positive constant k_g, and consequently $|g_i(x) - g_i(y)| \le \|g(x) - g(y)\| \le k_g\|x - y\|$, $\forall x, y \in \mathbb{R}^n$.

Let us suppose that the absolute value of each input τ_i is constrained to be smaller than a given saturation bound $T_i > 0$, *i.e.*, $|\tau_i| \le T_i$, $i = 1,\dots,n$. More precisely, letting u_i represent the control variable (controller output) relative to the i^{th} degree of freedom, we have that

$$\tau_i = T_i \operatorname{sat}(u_i/T_i) \tag{3}$$

where $\operatorname{sat}(\cdot)$ is the standard saturation function, *i.e.* $\operatorname{sat}(\varsigma) = \operatorname{sign}(\varsigma)\min\{|\varsigma|, 1\}$.

From Eqs. (1),(3), one sees that $T_i \ge B_{gi}$ (see Property 4.1), $\forall i \in \{1,\dots,n\}$, is a necessary condition for the robot to be stabilizable at any desired equilibrium configuration $q_d \in \mathbb{R}^n$. Thus, the following assumption turns out to be important within the analytical setting considered here.

Assumption 1. $T_i > \alpha B_{gi}$, $i = 1,\dots,n$, for some $\alpha \ge 1$.

Functions fitting the following definition will be involved.

Definition 1. Given a positive constant M, a nondecreasing Lipschitz-continuous function $\sigma : \mathbb{R} \to \mathbb{R}$ is said to be a *generalized saturation* with bound M if

[1] Since $g(q)$ is the gradient of the gravitational potential energy $\mathcal{U}(q)$, a scalar function, then, for any q, $q_0 \in \mathbb{R}^n$, the inverse relation $\mathcal{U}(q) = \mathcal{U}(q_0) + \int_{q_0}^{q} g^T(r)dr$ is independent of the integration path (Khalil, 2002, p. 120). Eq. (2b) considers integration along the axes. This way, on every axis (*i.e.* at every integral in the right-hand side of (2b)), the corresponding coordinate varies (according to the specified integral limits) while the rest of the coordinates remain constant.

[2] Property 4 is satisfied *e.g.* by robots having only revolute joints (Kelly *et al.*, 2005, §4.3).

(a) $\varsigma\sigma(\varsigma) > 0$, $\forall\varsigma \neq 0$;
(b) $|\sigma(\varsigma)| \leq M$, $\forall\varsigma \in \mathbb{R}$.

If in addition

(c) $\sigma(\varsigma) = \varsigma$ when $|\varsigma| \leq L$,

for some positive constant $L \leq M$, σ is said to be a *linear saturation* for (L, M).

Lemma 1. Let $\sigma : \mathbb{R} \to \mathbb{R}$ be a generalized saturation with bound M and let k be a positive constant. Then

1. $\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) = 0$;
2. $\exists\sigma'_M \in (0, \infty)$ such that $0 \leq D^+\sigma(\varsigma) \leq \sigma'_M$, $\forall\varsigma \in \mathbb{R}$;
3. $|\sigma(k\varsigma + \eta) - \sigma(\eta)| \leq \sigma'_M k|\varsigma|$, $\forall\varsigma, \eta \in \mathbb{R}$;
4. $\frac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma'_M\varsigma^2}{2}$, $\forall\varsigma \in \mathbb{R}$;
5. $\int_0^\varsigma \sigma(kr)dr > 0$, $\forall\varsigma \neq 0$;
6. $\int_0^\varsigma \sigma(kr)dr \to \infty$ as $|\varsigma| \to \infty$;
7. if σ is strictly increasing, then
   (a) $\varsigma[\sigma(\varsigma + \eta) - \sigma(\eta)] > 0$, $\forall\varsigma \neq 0$, $\forall\eta \in \mathbb{R}$;
   (b) for any constant $a \in \mathbb{R}$, $\bar{\sigma}(\varsigma) = \sigma(\varsigma + a) - \sigma(a)$ is a strictly increasing generalized saturation function with bound $\bar{M} = M + |\sigma(a)|$.

Proof. Item 3 is a direct consequence of the Lipschitz-continuity of σ and item 2 of the statement (as analogously stated *e.g.* in (Khalil, 2002, Lemma 3.3) under continuous differentiability). The rest of the items are proven in (López-Araujo *et al.*, 2013a). □

3. THE PROPOSED CONTROL SCHEME

The proposed SPD-SI control law is defined as

$$u(q, \dot{q}, \phi) = -s_P(K_P\bar{q} + K_D\dot{q}) + s_I(K_I\phi) \tag{4}$$

where $\bar{q} = q - q_d$, for any constant desired equilibrium position vector $q_d \in \mathbb{R}^n$; $\phi \in \mathbb{R}^n$ is the output vector variable of the integral-action dynamics, defined as[3]

$$\dot{\phi} = -\dot{q} - \varepsilon K_P^{-1} s_P(K_P\bar{q}) \tag{5}$$

$K_P = \text{diag}[k_{P1}, \ldots, k_{Pn}]$, $K_D = \text{diag}[k_{D1}, \ldots, k_{Dn}]$ and $K_I = \text{diag}[k_{I1}, \ldots, k_{In}]$, with $k_{Di} > 0$, $k_{Ii} > 0$, $i = 1, \ldots, n$, and positive P gains such that

$$k_{Pm} \triangleq \min_i\{k_{Pi}\} > k_g \tag{6}$$

(see Property 4.2); for any $x \in \mathbb{R}^n$, $s_P(x) = \big(\sigma_{P1}(x_1), \ldots, \sigma_{Pn}(x_n)\big)^T$ and $s_I(x) = \big(\sigma_{I1}(x_1), \ldots, \sigma_{In}(x_n)\big)^T$, with $\sigma_{Pi}(\cdot)$, $i = 1, \ldots, n$, being *strictly increasing linear saturation functions* for (L_{Pi}, M_{Pi}), and $\sigma_{Ii}(\cdot)$, $i = 1, \ldots, n$, being *strictly increasing generalized saturation functions* with bounds M_{Ii}, such that

$$L_{Pi} > 2B_{gi} \tag{7a}$$
$$M_{Ii} > B_{gi} \tag{7b}$$
$$M_{Pi} + M_{Ii} < T_i \tag{7c}$$

$i = 1, \ldots, n$; and ε (in (5)) is a positive constant satisfying

$$\varepsilon < \varepsilon_M \triangleq \min\{\varepsilon_1, \varepsilon_2\} \tag{8}$$

where

$$\varepsilon_1 \triangleq \sqrt{\frac{\beta_0\beta_P\mu_m}{\mu_M^2}} \quad , \quad \varepsilon_2 \triangleq \frac{f_m}{\beta_M + \frac{(f_M+\beta_D)^2}{4\beta_0 k_{Pm}}} < \frac{f_m}{\beta_M} \triangleq \varepsilon_3$$

with $\beta_0 \triangleq 1 - \max\left\{\frac{k_g}{k_{Pm}}, \max_i\left\{\frac{2B_{gi}}{L_{Pi}}\right\}\right\}$ (observe that by inequalities (6) and (7a): $0 < \beta_0 < 1$), $\beta_P \triangleq \min_i\left\{\frac{k_{Pi}}{\sigma'_{PiM}}\right\}$, $\beta_D = \max_i\{k_{Di}\sigma'_{PiM}\}$, $\beta_M \triangleq k_C B_P + \mu_M \sigma'_{PM}$, $B_P \triangleq \sqrt{\sum_{i=1}^n \left(\frac{M_{Pi}}{k_{Pi}}\right)^2}$, $\sigma'_{PM} \triangleq \max_i\{\sigma'_{PiM}\}$, σ'_{PiM} being the positive bound of $D^+\sigma_{Pi}(\cdot)$, in accordance to item 2 of Lemma 1, and μ_m, μ_M, k_C, f_m, f_M, B_{gi} and k_g as defined through Properties 1–4.

Remark 1. Let us note that inequalities (7) (stating conditions on the saturation function parameters) require the satisfaction of Assumption 1 with $\alpha = 3$. Similar conditions on the control input bounds have been required by other approaches where input constraints have been considered (Colbaugh *et al.*, 1997). Previous saturating PID-type schemes that do not explicitly include a similar or analog condition on the control input bounds are not always exhaustive in the search for the whole set of explicit conditions that support the developed closed loop analyses. Moreover, the way how such analyses are addressed lead to additional constraints on the control gains which complicate the tuning task and restrict the performance adjustment/improvement possibilities. Observe that the control gains in the approach proposed in this work are not tied to the satisfaction of any additional tuning restriction apart from inequality (6), and condition (8) concerning the integral-action-related parameter ε.

4. CLOSED-LOOP ANALYSIS

Consider system (1),(3) taking $u = u(q, \dot{q}, \phi)$ as defined through Eqs. (4)-(5). Observe that the satisfaction of (7c), under the consideration of (3), shows that

$$T_i > |u_i(q, \dot{q}, \phi)| = |u_i| = |\tau_i| \tag{9}$$

$i = 1, \ldots, n$, $\forall(q, \dot{q}, \phi) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$. Hence, the closed-loop dynamics takes the form

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q)$$
$$= -s_P(K_P\bar{q} + K_D\dot{q}) + \bar{s}_I(\bar{\phi}) + g(q_d) \tag{10a}$$
$$\dot{\bar{\phi}} = -\dot{q} - \varepsilon K_P^{-1} s_P(K_P\bar{q}) \tag{10b}$$

where $\bar{\phi} = \phi - \phi^*$ and

$$\bar{s}_I(\bar{\phi}) = s_I(K_I\bar{\phi} + K_I\phi^*) - s_I(K_I\phi^*) \tag{11}$$

with $\phi^* = (\phi_1^*, \ldots, \phi_n^*)^T$ such that $s_I(K_I\phi^*) = g(q_d)$, or equivalently $\phi_i^* = \sigma_{Ii}^{-1}\big(g_i(q_d)\big)/k_{Ii}$, $i = 1, \ldots, n$ (notice that their strictly increasing character renders the generalized saturation functions σ_{Ii} invertible). Observe that, by item 7b of Lemma 1, the elements of $\bar{s}_I(\bar{\phi})$ in Eq. (11), *i.e.* $\bar{\sigma}_{Ii}(\bar{\phi}_i) = \sigma_{Ii}(k_{Ii}\bar{\phi}_i + k_{Ii}\phi_i^*) - \sigma_{Ii}(k_{Ii}\phi_i^*)$, $i = 1, \ldots, n$, turn out to be strictly increasing generalized saturations.

Proposition 1. Consider the closed-loop system in Eqs. (10), under the satisfaction of Assumption 1 with $\alpha = 3$ and inequalities (7). Thus, for any positive definite diagonal matrices K_D, K_I and K_P such that inequality (6) is satisfied and any ε fulfilling inequality (8), global asymptotic stability of the closed-loop trivial solution $(\bar{q}, \bar{\phi})(t) \equiv (0_n, 0_n)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$.

Proof. By (9), one sees that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $\forall t \geq 0$. This proves that, under the proposed scheme, the input saturation values, T_i, are never

[3] Under time parametrization of the system trajectories, the integral-action dynamics in Eqs. (5) adopts the (equivalent) integral-equation form $\phi(t) = \phi(0) - \int_0^t \left[\dot{q}(\varsigma) + \varepsilon K_P^{-1} s_P\left(K_P\bar{q}(\varsigma)\right)\right] d\varsigma$, for any initial vector values $\bar{q}(0), \dot{q}(0), \phi(0) \in \mathbb{R}^n$.

reached. Now, in order to carry out the stability analysis, a scalar function $V(\bar{q},\dot{q},\bar{\phi})$ is defined as follows [4]

$$V = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}H(q)\dot{q} + \mathcal{U}(q) - \mathcal{U}(q_d) - g^T(q_d)\bar{q} + \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr$$

where $\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i$, $\int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr = \sum_{i=1}^n \int_0^{\bar{\phi}_i} \bar{\sigma}_{Ii}(r_i)dr_i$ and recall that $\mathcal{U}$ represents the gravitational potential energy. Note, by recalling Eqs. (2), that the defined scalar function can be rewritten as

$$V = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}H(q)\dot{q} + \mathcal{U}_{\gamma_0}^c(\bar{q}) + \gamma_0\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr$$

where

$$\begin{aligned}\mathcal{U}_{\gamma_0}^c(\bar{q}) &= \int_{0_n}^{\bar{q}} \left[g(r+q_d) - g(q_d) + (1-\gamma_0)s_P(K_P r)\right]^T dr \\ &= \sum_{i=1}^n \int_0^{\bar{q}_i} \left[\bar{g}_i(r_i) - g_i(q_d) + (1-\gamma_0)\sigma_{Pi}(k_{Pi}r_i)\right] dr_i\end{aligned}$$

with

$$\begin{aligned}\bar{g}_1(r_1) &= g_1(r_1+q_{d1}, q_{d2}, \ldots, q_{dn}) \\ \bar{g}_2(r_2) &= g_2(q_1, r_2+q_{d2}, q_{d3}, \ldots, q_{dn}) \\ &\vdots \\ \bar{g}_n(r_n) &= g_n(q_1, q_2, \ldots, q_{n-1}, r_n+q_{dn})\end{aligned}$$

and γ_0 is a constant satisfying

$$\beta_0\frac{\varepsilon^2}{\varepsilon_1^2} < \gamma_0 < \beta_0 \tag{12}$$

(observe, from inequality (8) and the definition of β_0, that $0 < \beta_0\varepsilon^2/\varepsilon_1^2 < \beta_0 < 1$). Under this consideration, $\mathcal{U}_{\gamma_0}^c(\bar{q})$ turns out to be lower-bounded by

$$W_{10}(\bar{q}) = \sum_{i=1}^n w_i^{10}(\bar{q}_i) \tag{13a}$$

where

$$w_i^{10}(\bar{q}_i) \triangleq \begin{cases} \frac{k_{li}}{2}\bar{q}_i^2 & \text{if } |\bar{q}_i| \le \bar{q}_i^* \\ k_{li}\bar{q}_i^*\left(|\bar{q}_i| - \frac{\bar{q}_i^*}{2}\right) & \text{if } |\bar{q}_i| > \bar{q}_i^* \end{cases} \tag{13b}$$

with $0 < k_{li} \le (1-\gamma_0)k_{Pi} - k_g$ and $\bar{q}_i^* = [L_{Pi} - 2B_{gi}/(1-\gamma_0)]/k_{Pi}$ (note that by inequality (12) and the definition of β_0: $0 < (1-\gamma_0)k_{Pi} - k_g$ and $\bar{q}_i^* > 0$); this is proven in (Mendoza *et al.*, 2014, Appendix I). From this, Property 1 and item 4 of Lemma 1, we have

$$\begin{aligned}V &\ge \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| + W_{10}(\bar{q}) + \gamma_0\sum_{i=1}^n \frac{\sigma_{Pi}^2(k_{Pi}\bar{q}_i)}{2k_{Pi}\sigma'_{PiM}} + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr \\ &\ge W_{11}(\bar{q},\dot{q}) + W_{10}(\bar{q}) + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr \end{aligned} \tag{14}$$

[4] Note that, in the error variable space, $q = \bar{q} + q_d$. Consequently $H(q) = H(\bar{q}+q_d)$, $C(q,\dot{q}) = C(\bar{q}+q_d,\dot{q})$ and $g(q) = g(\bar{q}+q_d)$. However, for the sake of simplicity, $H(q)$, $C(q,\dot{q})$, and $g(q)$ are used throughout the paper. Moreover, the arguments of V and its derivative along the system trajectories, $\dot{V}$, will be dropped throughout the developments.

where

$$\begin{aligned}W_{11}(\bar{q},\dot{q}) &= \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| + \frac{\gamma_0\beta_P}{2}\|K_P^{-1}s_P(K_P\bar{q})\|^2 \\ &= \frac{1}{2}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}^T Q^{11}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}\end{aligned}$$

with $Q^{11} = \begin{pmatrix}\gamma_0\beta_P & -\varepsilon\mu_M \\ -\varepsilon\mu_M & \mu_m\end{pmatrix}$. By inequality (12), $W_{11}(\bar{q},\dot{q})$ is positive definite (since with $\varepsilon < \varepsilon_M \le \varepsilon_1$, in accordance to inequality (8), any γ_0 satisfying (12) renders Q^{11} positive definite) and note that $W_{11}(0_n,\dot{q}) \to \infty$ as $\|\dot{q}\| \to \infty$ while, from Eqs. (13) and items 5-6 of Lemma 1, it is clear that W_{10} and the integral term in the right-hand side of (14) are radially unbounded positive definite functions of $\bar{q}$ and $\bar{\phi}$ respectively. Thus, $V(\bar{q},\dot{q},\bar{\phi})$ is concluded to be positive definite and radially unbounded. Its upper right-hand derivative along the system trajectories, $\dot{V} = D^+V$ (Michel *et al.*, 2008, §6.1A), is given by

$$\begin{aligned}\dot{V} = &-\dot{q}^T F\dot{q} - \dot{q}^T\left[s_P(K_P\bar{q} + K_D\dot{q}) - s_P(K_P\bar{q})\right] \\ &- \varepsilon s_P^T(K_P\bar{q})K_P^{-1}F\dot{q} + \varepsilon\dot{q}^T C(q,\dot{q})K_P^{-1}s_P(K_P\bar{q}) \\ &- \varepsilon s_P^T(K_P\bar{q})K_P^{-1}\left[g(q) - g(q_d) + s_P(K_P\bar{q})\right] \\ &- \varepsilon s_P^T(K_P\bar{q})K_P^{-1}\left[s_P(K_P\bar{q} + K_D\dot{q}) - s_P(K_P\bar{q})\right] \\ &+ \varepsilon\dot{q}^T s'_P(K_P\bar{q})H(q)\dot{q}\end{aligned}$$

where $H(q)\ddot{q}$ and $\dot{\bar{\phi}}$ have been replaced by their equivalent expressions from the closed-loop dynamics in Eqs. (10), Property 2.2 has been used and $s'_P(K_P\bar{q}) \triangleq \mathrm{diag}[D^+\sigma_{P1}(k_{P1}\bar{q}_1), \ldots, D^+\sigma_{Pn}(k_{Pn}\bar{q}_n)]$. The resulting expression can be rewritten as

$$\begin{aligned}\dot{V} = &-\dot{q}^T\left[s_P(K_P\bar{q} + K_D\dot{q}) - s_P(K_P\bar{q})\right] - \dot{q}^T F\dot{q} \\ &- \varepsilon s_P^T(K_P\bar{q})K_P^{-1}F\dot{q} + \varepsilon\dot{q}^T C(q,\dot{q})K_P^{-1}s_P(K_P\bar{q}) \\ &- \varepsilon\mathcal{W}_{\gamma_1}(\bar{q}) - \varepsilon\gamma_1 s_P^T(K_P\bar{q})K_P^{-1}K_P K_P^{-1}s_P(K_P\bar{q}) \\ &- \varepsilon s_P^T(K_P\bar{q})K_P^{-1}\left[s_P(K_P\bar{q} + K_D\dot{q}) - s_P(K_P\bar{q})\right] \\ &+ \varepsilon\dot{q}^T s'_P(K_P\bar{q})H(q)\dot{q}\end{aligned}$$

where

$$\begin{aligned}&\mathcal{W}_{\gamma_1}(\bar{q}) \\ &= s_P^T(K_P\bar{q})K_P^{-1}\left[(1-\gamma_1)s_P(K_P\bar{q}) + g(q) - g(q_d)\right] \\ &= \sum_{i=1}^n \left[\frac{(1-\gamma_1)}{k_{Pi}}\sigma_{Pi}^2(k_{Pi}\bar{q}_i) + \frac{\sigma_{Pi}(k_{Pi}\bar{q}_i)}{k_{Pi}}[g_i(q) - g_i(q_d)]\right]\end{aligned}$$

and γ_1 is a constant satisfying

$$\beta_0\frac{\varepsilon}{\varepsilon_2}\left[\frac{\varepsilon_3 - \varepsilon_2}{\varepsilon_3 - \varepsilon}\right] < \gamma_1 < \beta_0 \tag{15}$$

(from inequality (8) and the definition of β_0, one verifies, after simple developments, that $0 < \beta_0\varepsilon(\varepsilon_3 - \varepsilon_2)/[\varepsilon_2(\varepsilon_3 - \varepsilon)] < \beta_0 < 1$; in particular $\varepsilon\varepsilon_2/\varepsilon_3 < \varepsilon < \varepsilon_2 \iff \varepsilon\varepsilon_2 < \varepsilon\varepsilon_3 < \varepsilon_2\varepsilon_3 \iff 0 < \varepsilon(\varepsilon_3 - \varepsilon_2) < \varepsilon_2(\varepsilon_3 - \varepsilon) \iff 0 < \varepsilon(\varepsilon_3 - \varepsilon_2)/[\varepsilon_2(\varepsilon_3 - \varepsilon)] < 1$). Under this consideration, $\mathcal{W}_{\gamma_1}(\bar{q})$ turns out to be lower-bounded by

$$W_{20}(\bar{q}) = \sum_{i=1}^n w_i^{20}(\bar{q}_i) \tag{16a}$$

where

$$w_i^{20}(\bar{q}_i) = \begin{cases} a_i \bar{q}_i^2 & \text{if } |\bar{q}_i| \leq L_{Pi}/k_{Pi} \\ \varpi_i(\bar{q}_i) & \text{if } |\bar{q}_i| > L_{Pi}/k_{Pi} \end{cases} \tag{16b}$$

with $\varpi_i(\bar{q}_i) = \frac{b_i}{k_{Pi}}\left(|\sigma_{Pi}(k_{Pi}\bar{q}_i)| - L_{Pi}\right) + a_i\left(\frac{L_{Pi}}{k_{Pi}}\right)^2$, $b_i = (1-\gamma_1)L_{Pi} - 2B_{gi}$, $a_i = \min\left\{d, \frac{b_i k_{Pi}}{L_{Pi}}\right\}$ and $d = (1-\gamma_1)k_{Pm} - k_g$ (notice, from inequality (15) and the definition of β_0, that $b_i > 0$ and $d > 0$, hence $a_i > 0$); this is proven in (Mendoza *et al.*, 2014, Appendix II). From this, Properties 1, 2.1 and 3, items 2 and 3 of Lemma 1 and (b) of Definition 1, and the positive definite character of K_P, we have that

$$\begin{aligned} \dot{V} \leq & -\dot{q}^T\left[s_P(K_P\bar{q} + K_D\dot{q}) - s_P(K_P\bar{q})\right] - f_m\|\dot{q}\|^2 \\ & + \varepsilon f_M \|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| + \varepsilon k_C B_P \|\dot{q}\|^2 \\ & - \varepsilon\gamma_1 k_{Pm}\|K_P^{-1}s_P(K_P\bar{q})\|^2 + \varepsilon\mu_M \sigma'_{PM}\|\dot{q}\|^2 \\ & + \varepsilon\beta_D \|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| - \varepsilon\mathcal{W}_{\gamma_1}(\bar{q}) \\ \leq & -\dot{q}^T\left[s_P(K_P\bar{q} + K_D\dot{q}) - s_P(K_P\bar{q})\right] \\ & - \varepsilon W_{21}(\bar{q},\dot{q}) - \varepsilon W_{20}(\bar{q}) \end{aligned} \tag{17}$$

where

$$\begin{aligned} W_{21}(\bar{q},\dot{q}) =& \ \gamma_1 k_{Pm}\|K_P^{-1}s_P(K_P\bar{q})\|^2 + \left(\frac{f_m}{\varepsilon} - \beta_M\right)\|\dot{q}\|^2 \\ & - (f_M + \beta_D)\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| \\ =& \begin{pmatrix} \|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\| \end{pmatrix}^T Q^{21} \begin{pmatrix} \|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\| \end{pmatrix} \end{aligned}$$

with

$$\begin{aligned} Q^{21} =& \begin{pmatrix} \gamma_1 k_{Pm} & -\dfrac{f_M + \beta_D}{2} \\ -\dfrac{f_M + \beta_D}{2} & \dfrac{f_m}{\varepsilon} - \beta_M \end{pmatrix} \\ =& \begin{pmatrix} \gamma_1 k_{Pm} & Q_{12}^{21} \\ Q_{12}^{21} & \beta_M\left(\dfrac{\varepsilon_3 - \varepsilon}{\varepsilon}\right) \end{pmatrix} \end{aligned}$$

with $Q_{12}^{21} = -\sqrt{\beta_0\beta_M k_{Pm}\left(\frac{\varepsilon_3 - \varepsilon_2}{\varepsilon_2}\right)}$. By inequality (15), $W_{21}(\bar{q},\dot{q})$ is positive definite (since with $\varepsilon < \varepsilon_M \leq \varepsilon_2 < \varepsilon_3$, in accordance to inequality (8), any γ_1 satisfying (15) renders Q^{21} positive definite) while, from Eqs. (16), it is clear that W_{20} is a positive definite function of $\bar{q}$ and, from item 7a of Lemma 1, the first term in the right-hand side of (17) is negative definite with respect to $\dot{q}$ (uniformly in $\bar{q}$). Hence, $\dot{V}(\bar{q},\dot{q},\bar{\phi}) \leq 0$ with $\dot{V}(\bar{q},\dot{q},\bar{\phi}) = 0 \iff (\bar{q},\dot{q}) = (0_n, 0_n)$. Further, from the closed-loop dynamics in Eqs. (10), we see that $\bar{q}(t) \equiv \dot{q}(t) \equiv 0_n \implies \ddot{q}(t) \equiv 0_n \implies \bar{s}_I(\bar{\phi}(t)) \equiv 0_n \implies \bar{\phi}(t) \equiv 0_n$ (at any $(\bar{q},\dot{q},\bar{\phi})$ on $Z = \{(x,y,z) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n : x = y = 0_n\}$ with $\bar{\phi} \neq 0_n$, the resulting unbalanced force term $\bar{s}_I(\bar{\phi})$ acts on the closed-loop dynamics forcing the system trajectories to leave Z). Therefore, by the invariance theory (Michel *et al.*, 2008, §7.2) —more precisely by (Michel *et al.*, 2008, Corollary 7.2.1)—, the closed-loop trivial solution $(\bar{q},\bar{\phi})(t) \equiv (0_n, 0_n)$ is concluded to be globally asymptotically stable, which completes the proof. □

5. EXPERIMENTAL RESULTS

In order to corroborate the efficiency of the proposed SPD-SI control scheme, real-time tests were implemented using a 2-DOF direct-drive robot manipulator. The experimental setup is a 2-revolute-joint robot arm located at the *Instituto Tecnológico de la Laguna*, Mexico, previously used in (López-Araujo *et al.*, 2013a). The robot actuators are direct-drive brushless servomotors operated in torque mode, *i.e.* they act as torque sources and receive an analog voltage as a torque reference signal. Joint positions are obtained using incremental encoders on the motors. In order to get the encoder data and generate reference voltages, the robot includes a motion control board based on a DSP 32-bit floating point microprocessor. The control algorithm is executed at a 2.5 millisecond sampling period on a PC-host computer.

For the experimental manipulator, Properties 1–4 are satisfied with $\mu_m = 0.088$ kg·m^2, $\mu_M = 2.533$ kg·m^2, $k_C = 0.1455$ kg·m^2, $f_m = 0.175$ kg·m^2/s, $f_M = 2.288$ kg·m^2/s, $B_{g1} = 40.29$ Nm, $B_{g2} = 1.825$ Nm and $k_g = 40.373$ Nm/rad. The maximum allowed torques (input saturation bounds) are $T_1 = 150$ Nm and $T_2 = 15$ Nm for the first and second links respectively. From these data, one easily corroborates that Assumption 1 is fulfilled with $\alpha = 3$.

Letting

$$\sigma(\varsigma) = \begin{cases} \varsigma & \forall|\varsigma| \leq L \\ \rho(\varsigma; L, M) & \forall|\varsigma| > L \end{cases}$$

where $\rho(\varsigma; L, M) = \text{sign}(\varsigma)L + (M - L)\tanh\left(\frac{\varsigma - \text{sign}(\varsigma)L}{M-L}\right)$, for $0 < L < M$, the saturation functions used for the implementation were defined as $\sigma_{Pi}(\varsigma) = \sigma(\varsigma; L_{Pi}, M_{Pi})$ and $\sigma_{Ii}(\varsigma) = \sigma(\varsigma; L_{Ii}, M_{Ii})$, $i = 1, 2$. Note that with these saturation functions, we have $\sigma'_{PiM} = \sigma'_{IiM} = 1$, $\forall i \in \{1, 2\}$. The saturation parameters were selected in order to satisfy inequalities (7) as (all of them expressed in Nm): $M_{P1} = 90$, $M_{P2} = 10$, $L_{Pi} = 0.9M_{Pi}$, $M_{I1} = 41$, $M_{I2} = 2$ and $L_{Ii} = 0.9M_{Ii}$, $i = 1, 2$.

For comparison purposes, additional experiments were implemented using the algorithm proposed in (Su *et al.*, 2010) (choice made taking into account the analog nature of the compared algorithms: globally stabilizing in a bounded-input context, and the recent appearance of (Su *et al.*, 2010)), *i.e.*

$$u = -K_P\text{Tanh}(\bar{q}) - K_D\text{Tanh}(\dot{q}) - K_I\text{Tanh}(\phi)$$
$$\dot{\phi} = \eta^2\dot{q} + \eta\text{Tanh}(\bar{q})$$

where η is a (sufficiently large) positive constant and $\text{Tanh}(x) = (\tanh x_1, \ldots, \tanh x_n)^T$ for any $x \in \mathbb{R}^n$. For the sake of simplicity, this algorithm is subsequently referred to as the S10 controller.

At all the experiments, the desired configuration was fixed at $q_d = (q_{d1}, q_{d2})^T = (\pi/4, \pi/2)^T$ [rad]. The initial conditions were $q(0) = \dot{q}(0) = 0_2$ and $\phi(0) = 0_2$ was *conventionally* taken for both controllers for the sake of fairness. The control gains for both tested schemes were adjusted so as to get the best possible responses avoiding overshoot. In the case of the proposed algorithm, this was done taking care that inequalities (6) and (8) be fulfilled. For the S10 controller, no acceptable closed-loop trajectories could be obtained through the tuning conditions reported in (Su *et al.*, 2010), so in order to get the best possible responses such tuning conditions were disregarded except for the saturation avoidance inequalities. The resulting tuning values were: $K_P = \text{diag}[7000, 450]$ Nm/rad, $K_D = \text{diag}[150, 15]$

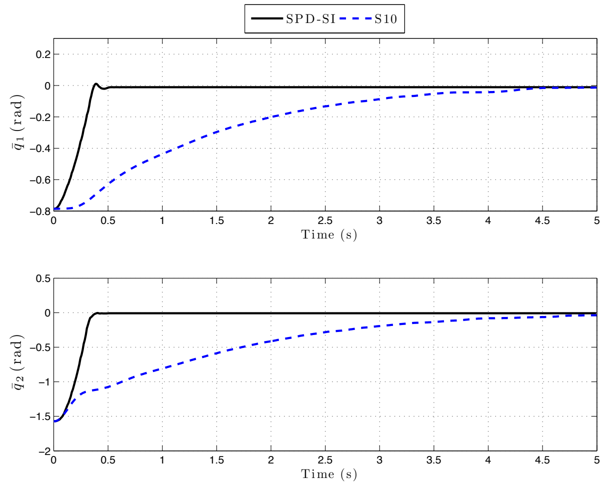

Fig. 1. Position errors

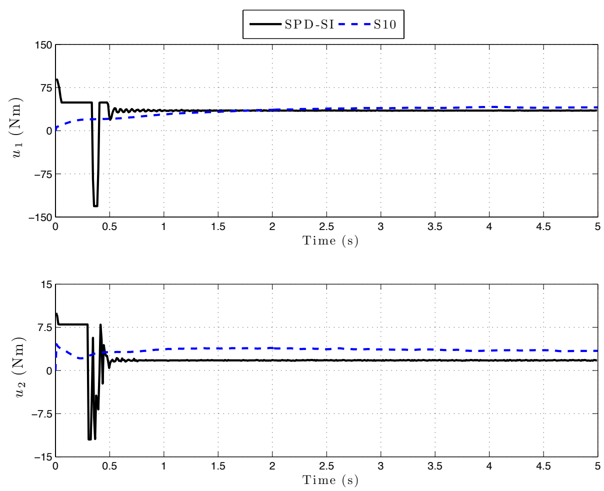

Fig. 2. Control signals

Nms/rad, $K_I = \text{diag}[1500, 500]$ Nm/rad and $\varepsilon = 7.03 \times 10^{-5}$ s^{-1} for the proposed SPD-SI scheme, and $K_P = \text{diag}[10, 5]$ Nm, $K_D = \text{diag}[8, 1]$ Nm, $K_I = \text{diag}[130, 8.5]$ Nm and $\eta = 0.9$ s/rad for the S10 controller.

Figs. 1 and 2 show the position errors and control signals experimentally obtained. Note that while both controllers accomplished the regulation objective avoiding input saturation, the stabilization time achieved through the proposed scheme is considerably shorter than that of the S10 controller. The structural and tuning characteristics of the tested algorithms are concluded to state notorious differences on the consequent system responses in an authentic global regulation framework.

6. CONCLUSIONS

A bounded PID-type global regulator for robot manipulators with constrained inputs was presented. Its SPD-SI structure, constructed through *generalized* saturation functions, has proven to generate control signals that make a better use of the available input range than analog (PID-type) controllers with different structural features. Furthermore, its very simple control-gain tuning criterion, the simplest hitherto obtained in the considered analytical framework, has proven to notoriously simplify the implementability of the algorithm and considerably enhance its regulation authority in the search for closed-loop performance improvements. Experimental tests corroborated the analytical results.

REFERENCES

Alvarez-Ramirez, J., Kelly, R., and Cervantes, I. (2003). Semiglobal stability of saturated linear PID control for robot manipulators. *Automatica*, 39, 989–995.

Alvarez-Ramirez, J., Santibáñez, V., and Campa, R. (2008). Stability of robot manipulators under saturated PID compensation. *IEEE Transactions on Control Systems Technology*, 16, 1333–1341.

Colbaugh, R., Barany, E., and Glass, K. (1997). Global regulation of uncertain manipulators using bounded controls. In *Proc. 1997 IEEE Int. Conf. on Robotics and Automation*, Albuquerque, NM, 1148-1155.

Gorez, R. (2009). Globally stable PID-like control of mechanical systems. *Syst. & Ctl. Letters*, 38, 61–72.

Kelly, R. (1995). A tuning procedure for stable PID control of robot manipulators. *Robotica*, 13, 141-148.

Kelly, R. (1998). Global positioning of robot manipulators via PD control plus a class of nonlinear integral actions. *IEEE Transactions on Automatic Control*, 43, 934–938.

Kelly, R. , Santibáñez, V., and Loría, A. (2005). *Control of Robot Manipulators in Joint Space*. Springer, London.

Khalil, H.K. (2002). *Nonlinear Systems*. 3rd edition, Prentice-Hall, Upper Saddle River, NJ.

Krikelis, N.J., and Barkas, S.K. (1984). Design of tracking systems subject to actuator saturation and integrator wind-up. *International Journal of Control*, 39, 667–682.

López-Araujo, D.J., Zavala-Río, A., Santibáñez, V., and Reyes, F. (2013a). A generalized scheme for the global adaptive regulation of robot manipulators with bounded inputs. *Robotica*, 31, 1103–1117.

Mendoza, M., Zavala-Río, A., Santibáñez, V., and Reyes, F. (2014). A PID-type global regulator with simple tuning for robot manipulators with bounded inputs. In *Proc. 53rd IEEE Confernce on Decision and Control*, Los Angeles, CA, 6335–6341.

Meza, J.L., Santibáñez, V., and Hernández, V.M. (2005). Saturated nonlinear PID global regulator for robot manipulators: passivity-based analysis. In *Proc. 16th IFAC World Congress*, Prague, Czech Republic.

Michel, A.N., Hou, L., and Liu, D. (2008). *Stability of Dynamical Systems*. Birkhäuser, Boston.

Rocco, P. (1996). Stability of PID control for industrial robot arms. *IEEE Trans. on Rob. and Aut.*, 12, 606–614.

Santibáñez, V., and Kelly, R. (1996). Global regulation for robot manipulators under SP-SD feedback. In *Proc. 1996 IEEE International Conference on Robotics and Automation*, Minneapolis, MN, 927–932.

Santibáñez, V., Kelly, R., Zavala-Río, A., and Parada, P. (2008). A new saturated nonlinear PID global regulator for robot manipulators. In *Proc. 17th IFAC World Congress*, Seoul, Korea, 11690–11695.

Su, Y., Müller, P.C., and Zheng, C. (2010). Global asymptotic saturated PID control for robot manipulators. *IEEE Trans. on Ctl. Syst. Technology*, 18, 1280–1288.