Analog circuit implementation of a neuron with applications to communications

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Abstract: The mathematical model of a neuron is electronically implemented. This model describes the excitable oscillatory behavior of a single neuron. The designed circuit presents high resemblance with the original model and its dynamical properties. The system is electronically implemented by means of analog computation methods using operational amplifiers, resulting in a completely analog response which avoids any approximation error that may be caused by the common numerical integration methods. Furthermore, the designed circuit is connected through a self-synchronization method with potential application in neural network communication.

Keywords: Biological systems, neuron, electronic implementation, mathematical models, nonlinear dynamics, synchronization, communications.

1. INTRODUCTION

In the last decades, the behavior of biological systems has been thoroughly studied by different areas of science [M. Hájek et al. (1980)]. Today, mathematical models of those biological systems allow us to understand the dynamics of the body throughout different techniques.

One of the principal mathematical biological models is the one proposed by Hodgkin and Huxley (1953). It described the electrical excitation and propagation through the axon of the giant squid by the voltage–clamp technology. The model, which is represented by a four dimension system of ordinary differential equations, served as a watershed in the study of biological systems in different areas of science. However, a decade later R. FitzHugh (1961) using an approximation based on the Van der Pol equation for a relaxation oscillator, successfully reduced the model in the number of equations. This reduced model describes the electrical behavior of a neuronal membrane and the ion currents which are involved in the process. After this model was developed, a great number of experiments concerning the study of the electrical activity in neurons [see J. L. Hindmarsh and R. M. Rose (1984) and the references within] and from cells in the hole body started, for example in the analysis of the behavior of the β cells, which regulate glucose in the blood stream and exhibit a complex oscillatory membrane-potential pattern called Bursting Electrical Activity [see P. Meda et al. (1980) and P. M. Dean and E. K. Matthews (1968)].

Recently a new tendency on designing systems through analog continuous-time computation methods has emerged [P. Orponen (1997); H. T. Siegelmann and S. Fishman (1998); Y. Horio and K. Aihara (2008); Y. Xue (2015); J. Cabessa and A.E. Villa (2015)]. This method attends the needs of studying complex or difficult to experiment in real life dynamical systems, by means of their exact instead of their approximate responses. This is due to the fact that usually dynamical systems are studied and verified by numerical integration methods which are limited by discretization and memory limitations. One of the principal advantages of this method is that the system can be studied in a more physical and tangible way, and the random perturbations that affect them in different environments can also be reproduced, an example of this is the brain and the thermal noise [G. Trautter and G. Tamburrini (2007)]. Taking this in consideration, analog processes in axons and neural cells have been correlated in certain type of computation and decision–making processes [B. Scarpellini (2003)].

An important approach to be considered in this method of analog computing is by means of an analog electronic implementation of the system [K. M. Cuomo and A. V. Oppenheim (1993); S. Rajasekar et al. (1997); J. Zhao and

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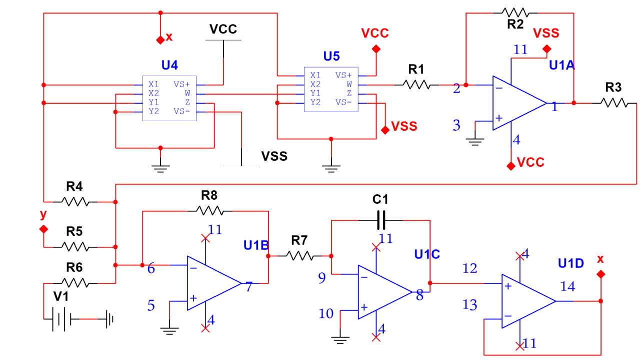

Fig. 1. Schematic diagram of the x state given by (1).

Y. B. Kim (2007)]. This physical realization can be obtained by operational amplifiers and linear and nonlinear electronic components. Some reports have been developed in the last decades regarding the electronic implementations of neuron models, for examples the ones described in [S. Rajasekar et al. (1997); J. Zhao and Y. B. Kim (2007); R. D., Pinto et al. (2000); A. Szücs, et al. (2000)]. However, the electronic implementation reported here is based on the method described by P. Orponen (1997), in which operational amplifiers (op-amps) are connected in different forms in order to obtain summations, integration and multiplication effects on the signals. Furthermore, the presented circuit of the neuron is implemented in a self-synchronization method in which the transmission and reception of a signal is possible in a neural network communication scheme.

This article is organized as follows: Section 2 presents the general theory that envelops the mathematical model of the neuron; Section 3 introduces the physical implementation of the biological system; Section 4 describes the coupling between two neurons and the communication scheme. Finally conclusions are drawn in Section 5.

2. MATHEMATICAL MODEL OF THE NEURON

The two dimensional model described by FitzHugh is based on the Van der Pol equation for a relaxation oscillator. The model is generalized by the addition of terms to produce a pair of non-linear differential equations with either a stable singular point or a limit cycle oscillator [R. FitzHugh (1961)], resulting in the following set of equations:

$$\begin{aligned} \dot{x} &= c(x + y - x^3/3 + I), \\ \dot{y} &= -(x - a + by)/c, \end{aligned} \tag{1}$$

where x and y are the state variables, x stands for the membrane potential and its excitability, while y represents the accommodation and refractoriness of the potential. The term $I \in \mathbf{R}$ corresponds to stimulus intensity depicted by the membrane current in the Hodgkin and Huxley model. The systems electrical activity changes oscillating regarding to a cathodal shock of the membrane current [R. FitzHugh (1961)]. Generally this term takes the value $I = 0.4$, while the parameters $a = 0.7, b = 0.8, c = 3$.

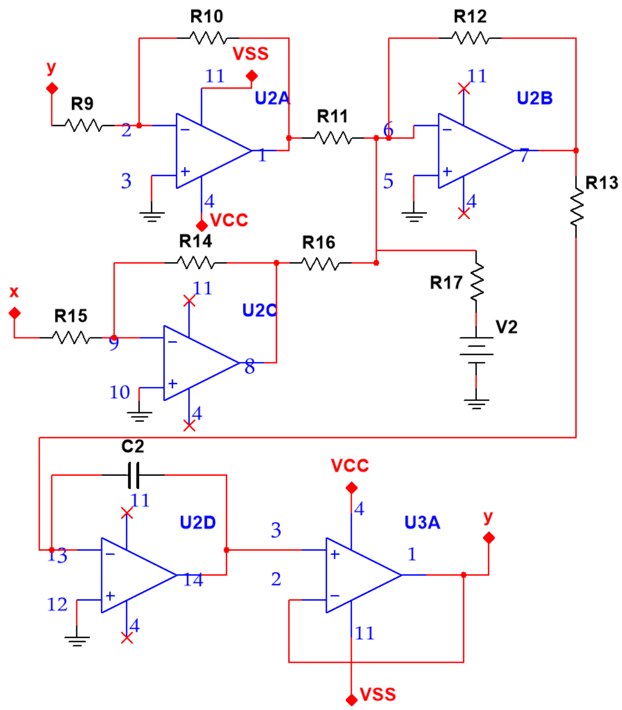

Fig. 2. Schematic diagram of the y state given by (1).

3. ELECTRONIC IMPLEMENTATION OF THE NEURON

The design of the system was performed considering the technic based on the op-amps [P. Orponen (1997)]. One of the main advantages of these electronic devices, is that this type of integrated circuits are not affected by the loss of information in an analog to digital conversion. The differential equation of a system is considered throughout the configurations of the amplifier (inverting amplifier, inverting adding and inverting integrator) as described next.

First, consider the state x given by (1) with its corresponding set of parameters. Integrating with respect of time both sides of the equation, the system results in $x = c\int(x + y - x^3/3 + I)dt$. Now the equation can be implemented in the configurations of the op-amps as is described by the circuits of Figure 1. If standard node analysis technique is applied to the circuit in the output terminal of U1D, it will result in an equation that governs the behavior of the output voltage of the voltage follower in the U1D component. Therefore the resulting state will be given by:

$$\begin{aligned} x = \frac{-1}{R7 \cdot C1} \int \Bigg(& \frac{R2 \cdot R8}{100 \cdot R1 \cdot R3} x^3 \\ & - \frac{R8}{R4} x \\ & - \frac{R8}{R5} y \\ & - \frac{R8}{R6} V1 \Bigg) dt - V_{C1_0}. \end{aligned} \tag{2}$$

The cubic term was implemented through the four quadrant multiplier AD633AN depicted as the U4 and U5 com-

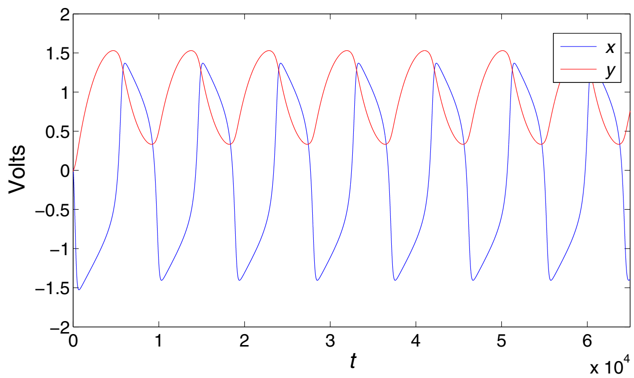

Fig. 3. Voltage output of the electronic circuit depicted in Figure 1 and 2.

ponents. In this case the U1 is arranged in the inverting amplifier configuration, U1B corresponds to the inverting adding amplifier, the integration is carried out by U1C while U1D stands only for a voltage follower to maintain the output signal.

By replacing the values of the resistor according to the parameters, the equation will result in the same form as the one in the variable state in Eq. (1). The term V_{C1_0} corresponds to the initial voltage in the capacitor C1. The values of all components are depicted in Table 1.

Similar considerations can be made to the states y of the equation (1). The integration of both sides will result in $y = -1/c \int (x - a + by)dt$, which implemented with the op-amp configuration will result in the circuit of Figure 2. Considering the node analysis in the output of U3A, the voltage will result in the following equation:

$$y = \frac{-1}{R13 \cdot C2} \int \Bigg(\frac{R12 \cdot R14}{R15 \cdot R16} x + \frac{R10 \cdot R12}{R9 \cdot R11} y - \frac{R12}{R17} V2 \Bigg) dt - V_{C2_0}. \tag{3}$$

The values of the resistors and voltages are depicted in Table 1. The term V_{C2_0} corresponds also to the initial voltage of the capacitor C2. The equation (3) results in the same as described before by integrating both sides Eq. (1). The oscillating output of the system is depicted in Figure 3 which takes the same form as if the system was solved numerically by any integration method.

4. SYNCHRONIZATION METHOD AND COMMUNICATION SCHEME

One of the principal characteristics among the neurons is that certain neurons in the mammalian brain have been long known to be joined by gap junctions, which are the most common type of electrical synapse [M. V. Bennett and R. S. Zukin (2004)]. The gap junctions are known to be capable of synchronizing electrical activity and may subserve metabolic coupling and chemical communication as well. Taking this in consideration, and in the same spirit as [L. M. Pecora and T. L. Carroll (1990); L. M. Pecora et al. (1997)] a synchronization method is considered regarding the neuron circuit.

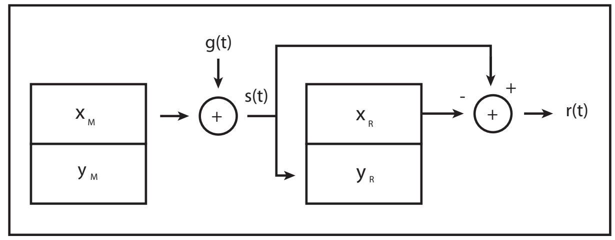

Fig. 4. Synchronization and communication scheme.

First consider a master and a slave system coupled in the following way. The slave system will be connected in the y_S state by substituting the x_S state of the slave system by a coupling function $s(t)$ generated from the x_M state of the master system and signal to be transmitted in the communication process. The state x_S will be left autonomously as the synchronization scheme depicts in Figure 4. The coupled system is described as follows:

$$\begin{aligned} \dot{x}_M &= c(x_M + y_M - x_M^3/3 + I), \\ \dot{y}_M &= -(x_M - a + by_M)/c, \\ \dot{x}_S &= c(x_S + y_S - x_S^3/3 + I), \\ \dot{y}_S &= -(s(t) - a + by_S)/c, \end{aligned} \tag{4}$$

where x_M, y_M and x_S, y_S correspond to the state variables of the master system and the states of the slave system, respectively. The master system will be the circuit depicted in Figure 1 and 2, the slave circuit will take the same form as the master circuit, except that the input in the op-amp U2C connected through R15 instead of being connected to x will be connected to a node call s. To take advantage of the synchronization of the systems and considering the information transmission that occurs electrically through the neurons, a sinusoidal signal $g(t) = 2sin(2 \times 10^3 \pi t)$ is transmitted with the x_M state of the master system. The signal is added with the x_M state, and recovered from the slave system in the case that $\lim_{t\to\infty} |x_M - x_S| \to 0$. The transmitted and recovered signals take the following form:

$$\begin{aligned} s(t) &= x_M + g(t), \\ r(t) &= s(t) - x_S. \end{aligned} \tag{5}$$

This synchronization scheme, is simple enough to implement it electronically in the electrical circuit schematics depicted in Figure 5, the terms x, g, s, r correspond to $x_M, g(t), s(t), r(t)$ respectively, while xr correspond to x_S.

Table 1. Values and components of the electronic implementation. Any resistance not included here is consider with a value of 10kΩ.

Component	Value or name
R1,R3-R7,R12,R13,R21-R23,R29,R30	1kΩ
R8,R11,R16,R17,R25,R28,R33,R34	3kΩ
R10,R27	8kΩ
R2,R19	100kΩ
C1,C2	1μF
U1-U3,U6-U8	TL084IN
U4,U5,U9,U10	AD633AN
VCC	12V
VSS	-12V
V1	-0.4V
V2	0.7V
V_{C1_0}, V_{C2_0}	0V

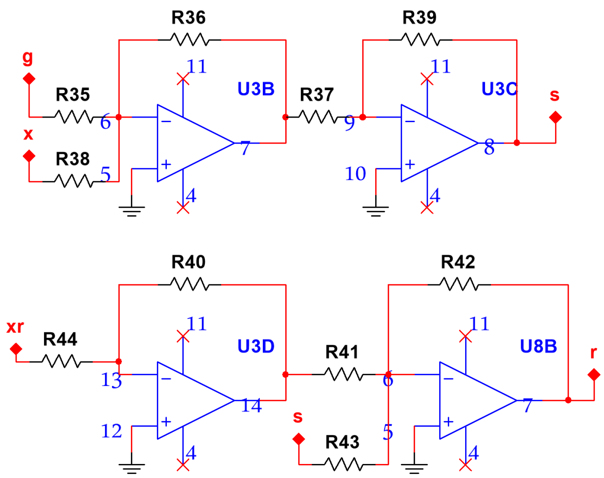

Fig. 5. Electronic implementation of the signal $s(t)$ and $r(t)$ from Eq. 5

The Euclidian distance of both signals, the information $g(t)$ and the recovered $r(t)$, was measured in the following way:

$$err(t) = \sqrt{|g(t)^2 - r(t)^2|}. \tag{6}$$

And in order to determine the functionality of the method and the synchronization among the circuits, Figure 6 a) depicts the projection of the output of the master and slave circuits onto the plane (x_M, x_S), it can be appreciated that both states are synchronized after some transient states. Since the systems are synchronized, the signal recovery can be implemented as mentioned before through eqs. (5). In Figure 6 b) the Euclidean distance in terms of time is shown, where it can be appreciated that throughout the time the error between them is oscillating below the value of 0.05. This oscillations occurs due to the external signal applied to the slave systems, which makes both systems almost identical. If the perturbation or the signal end, the systems will be identically synchronized again.

5. CONCLUSIONS

Electronic implementation of mathematical models are an interesting tool to prove the feasibility of systems. An important feature of this type of circuit implementation is that it does not use the parameter relation through inductors or capacitors as many other implementations do. This results in an easier and more direct parameter relation if further adjustment is necessary.

The FitzHugh approximation model of the neuron based on the Van der Pol equation for a relaxation oscillator, results in a simple and reduced option to design electronic implementations. Furthermore, the circuit presented here proved synchronization in a state substituting coupling which results suitable for a communication scheme. One possible application of the circuit can be extended in the computation processes through neuro-dynamics [Y. Horio and K. Aihara (2008)], which are a network of analog chaotic-neuron integrated circuits. Although the neuron circuit presented here does not present chaotic properties, the induction of a chaotic behavior can be implemented. The report of this may be studied elsewhere.

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REFERENCES

M. V. Bennett and R. S. Zukin. Electrical coupling and neuronal synchronization in the mammalian brain. *Neuron*, volume 41, number 4, pages 495-511, 2004.

J. Cabessa and A.E. Villa. On Super-Turing Neural Computation. *In Advances in Cognitive Neurodynamics (IV)*, pp. 307–312. Springer Netherlands, (2015).

K. M. Cuomo and A. V. Oppenheim. Circuit implementation of synchronized chaos with applications to communications. *Physical Review Letters*, volume 71, number 1, 1993.

P. M. Dean and E. K. Matthews. Electrical activity in pancreatic islet cells. *Nature*, volume 219, pages 389-390, 1968.

R. FitzHugh. Impulses and physiological states in theoretical models of nerve membrane. *Biophysical Journal*, volume 1, pages 445–466, 1961.

M. Hájek, J. Potůček, V. Brodan. A silicon pancreatic beta cell for diabetes. *Automatica*, volume 16, pages 191195, 1980.

J. L. Hindmarsh and R. M. Rose. A model of neuronal bursting using three coupled first order differential equations. *Proceedings of the Royal Society of London B: Biological Sciences* , volume 221.1222, pp. 87–102. (1984).

A. L. Hodgkin, A. F. Huxley. The dual effect of membrane potential on sodium conductance in the giant axon of Loligo. *Journal of Physiology*, volume 116, pages 497-506, 1952

Y. Horio and K. Aihara, Analog computation through high-dimensional physical chaotic neuro-dynamics. *Physica D: Nonlinear Phenomena*, volume 237(9), pages 1215–1225, 2008.

P. Meda, A. Perrelet and L. Orci. Gap junctions and β-cell functions. *Journal of Hormone and Metabolic Research*, volume 10, pages 157-162, 1980.

P. Orponen. A survey of continuous-time computation theory. *In Advances in algorithms, languages, and complexity*, pages 209-224, Springer US. Chicago, 1997.

L. M. Pecora and T. L. Carroll. Synchronization in chaotic systems. *Physical review letters*, volume 64, number 8, page 821, 1990.

L. M. Pecora, T. L. Carroll, G. A. Johnson and D. J. Mar. Fundamentals of synchronization in chaotic systems, concepts, and applications. *CHAOS*, volume 7, pages 520-543, 1997.

R.D. Pinto, P. Varona, A.R. Volkovskii, A. Szcs, H.D. Abarbanel and M.I. Rabinovich. Synchronous behavior of two coupled electronic neurons. *Physical Review E*, volume 62(2), page 2644, 2000.

S. Rajasekar, K. Murali and M. Lakshmanan. Control of chaos by nonfeedback methods in a simple electronic circuit system and the FitzHugh-Nagumo equation. *Chaos, Solitons & Fractals*, volume 8, number 9, pages 1545-1558, 1997.

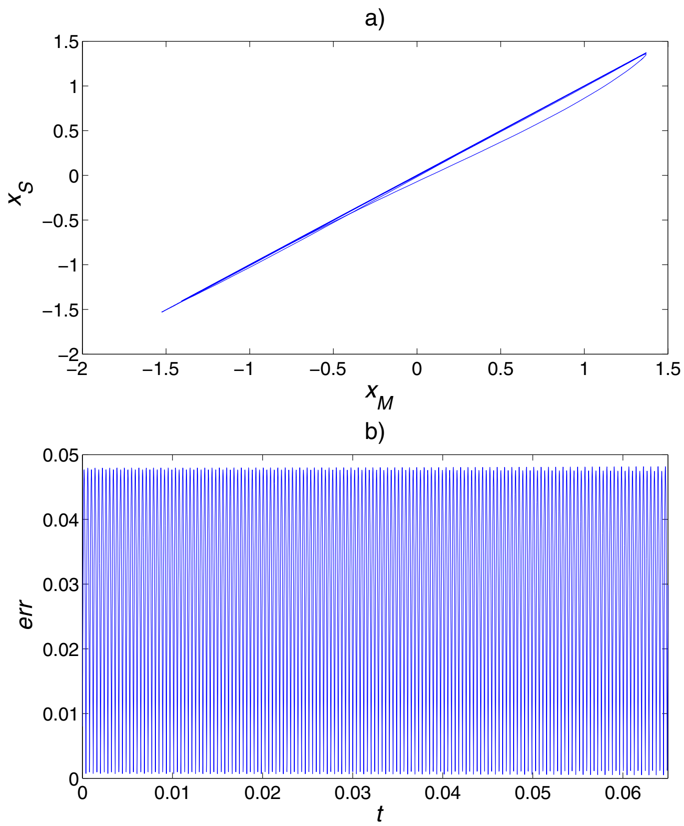

Fig. 6. a) Projection of the output of the master and slave circuits onto the plane (x_M, x_S). b) Euclidean distance as described in 6 .

B. Scarpellini. Comments on two undecidable problems of analysis, *Minds and Machines*, volume 13, pages 7985, 2003.

H. T. Siegelmann and S. Fishman. Analog computation with dynamical systems. *Physica D: Nonlinear Phenomena*, volume 120, number 1, pages 214–235, 1998.

G. Trautteur and G. Tamburrini. A note on discreteness and virtuality in analog computing. *Theoretical Computer Science*, volume 371, number 1, pages 106-114, 2007.

A. Szücs, P. Varona, A.R. Volkovskii, H.D. Abarbanel, M.I. Rabinovich and A.I. Selverston. Interacting biological and electronic neurons generate realistic oscillatory rhythms. *Neuroreport*, volume 11(3), pp. 563–569, 2000.

Y. Xue. Recent Development in Analog Computation-A Brief Overview. *arXiv preprint*, volume arXiv:1504.00450, 2015.

P. Young. Data-based mechanistic modelling of environmental, ecological, economic and engineering systems. *Environmental Modelling & Software*, volume 13, pages 105122, 1998.

J. Zhao and Y. B. Kim. Circuit implementation of FitzHugh–Nagumo neuron model using field programmable analog arrays. In Circuits and Systems (MWSCAS2007), 50th Midwest Symposium, pages 772-775, 2007.