

## Sliding Mode Control for Biological Pest Control Problems

Hector Puebla\* America Morales-Diaz\*\*  
Alejandra Velasco Pérez\*\*\*

\**Universidad Autónoma Metropolitana Azcapotzalco, México, D.F. 02200*

*México (Tel: +52-5553189000; e-mail: hpuebla@correo.azc.uam.mx).*

\*\**CINVESTAV Saltillo, Grupo de Robótica y Manufactura Avanzada, Saltillo, México (e-mail:  
america.morales@cinvestav.edu.mx)*

\*\*\**Universidad Veracruzana, campus Cordova-Orizaba,  
(e-mail: alvelasco@uv.mx)*

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**Abstract:** In this work, the biological control problem is framed as a feedback control problem. The feedback control is addressed via a sliding mode control approach in conjunction with a recursive cascade control scheme. An example of a host-parasitoid model is used to illustrate the control design and performance. Numerical results show that the proposed controller is able to regulate pest population at the desired references via the manipulation of a biological control action.

*Keywords:* Pest management control, biological control, robust control, sliding mode control.

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## 1. INTRODUCTION

The study of pest management methods is of great practical value in several areas, including, ecology, epidemiology and agricultural production [1-3]. In the pest management, pest control problems have been addressed by many methods such as spraying pesticides (or chemical control) and releasing natural enemies (biological control) [3]. Biological control consists in the use of living organisms to suppress pest populations [1]. In this method, natural enemies of pests are released artificially when the pests have caused damages. Biological control includes the use parasites, pathogens and predators as natural enemies.

Several papers have been addressed biological pest control problems via optimal and impulsive control strategies [4-15]. Impulsive control designs are based on impulsive differential equations that need a more involved analysis than corresponding differential equations. The use of optimization methods and optimal control are limited due the assumption of a perfect model for design purposes.

To the best of the author's knowledge, very few papers have been addressed the biological control as a robust feedback control problem. Robust model based feedback is of upmost relevance for pest-control problems as it permits to deal with model uncertainties and incorporate the knowledge of the interactions of biological pest control captured by a simple mathematical model. In this paper, we introduce a robust feedback control approach based on sliding model control and a recursive cascade configuration. Sliding model control provides a framework for the systematic consideration of robustness against model uncertainties and also leads to simple control structure that can be implemented in practice.

This work is organized as follows: In Section 2 we discuss general ideas on biological pest control and we also present the case study that we are addressing in this work. In Section 3 we formulate the biological pest control problem as a robust feedback control problem. A novel cascade control configuration and the control design are also presented in Section 3. In Section 4 numerical studies shows the implementation and performance of the proposed robust feedback control approach. Finally, in Section 5 we provide some concluding remarks.

## 2. THE BIOLOGICAL PEST CONTROL PROBLEM

### 2.1 Biological pest control

Biological control consists in the use of living organisms to suppress pest populations [1]. Three main methods of biological control are [2]: (i) conservation of existing natural enemies, (ii) the introduction of new natural enemies establishing a permanent population and (iii) mass rearing and periodic release.

A pest outbreak occurs when the financial loss caused by the pest is greater than the cost to control the pest. The point where these costs equal is known as economic injury level (EIL) [1,2]. Thus, EIL is a useful concept because it quantifies the cost/benefit ratio underlying all pest control decisions. The EIL is usually expressed as a number of pests per unit area.

For the purpose of biological pest control, control actions (e.g. introduction and release of natural enemies) can be performed continuously or impulsively in a constant amount in periodic form [3-5,7,13,15].

### 2.2 A general model

We consider the following general class of mathematical models of pest control problems under biological control,

$$\begin{aligned} \bullet & \quad x_i = f_i(x) + g_i(x)x_{i+1} & 1 \leq i \leq j-1 \\ \bullet & \quad x_j = f_j(x) + g_j(x)u & j \leq n \\ \bullet & \quad x_{j+l} = f_{j+l}(x) & j+l = n, j < n \end{aligned} \quad (1)$$

where  $x$  are the states of the system,  $u$  is the controlled input and the pest population correspond to the first state in the above model, i.e. state  $x_1$ . Note that the first  $j$  equations in the model are given in a chained form.

### 2.3 The case study

A variety of pest-predator and epidemic pest models under biological control have been reported in the literature [6-11,15,16]. We consider as a case study, a host-parasitoid model of a lumped population of two soybean caterpillars (Rachiplusia nu and Pseudoplusia includes) and a parasitoid is considered [17]. The model is given by,

$$\begin{aligned} \bullet & \quad x_1 = (p_1 + p_2x_1)x_1 - p_3x_2 \\ \bullet & \quad x_2 = (-p_4 + p_5x_1)x_2 + u \end{aligned} \quad (2)$$

where  $p_i$  are model parameters and  $x_1$  and  $x_2$  are caterpillar and parasitoid densities, respectively. Moreover, the pest control strategy is through the introduction of natural enemies. The goal of the pest control strategy is to maintain the pest population at level  $x_{1,ref} = x_d$  by using the control  $u$ , where  $x_d$  is a pest population density below the economic injury level (EIL). It has been established [17] that densities above the value of  $x_d = 20$  pests/m<sup>2</sup> cause economic damages to big soybean caterpillar with the length more than 1.5 cm.

## 3. CONTROL DESIGN

### 3.1 The feedback control problem

The control objective is the regulation at or below the EIL of the pest population via a biological control action. In order to derive feedback control laws with minimum system information, the control problem description is completed by the following assumptions:

A1 States  $x_1$  to  $x_n$  are measured and available for control design purposes.

A2 Nonlinear function  $f(x)$  includes uncertain terms and terms including uncertain parameters, such that for control design purposes is considered completely unknown.

A3 There exist rough estimates of nonlinear function  $g(x)$ .

A2 and A3 consider model uncertainties, or in the worst case the whole terms are unknown. The rough estimate of  $g(x)$  can be obtained using a base constant value of this term. These are realistic assumptions. Indeed, the same ecosystem which consists of different pests and its natural enemies can be modelled in different forms. Moreover, both interactions between species and model parameters (e.g. reproduction and mortality rates, carrying capacity of the environment, etc.) have some degree of uncertainties [18-20].

### 3.2 The recursive cascade control configuration

Based on the chained form of the pest control problem described by model (1), it is proposed a recursive cascade control configuration that transforms a high order relative degree control design to a one order relative degree control problem [20]. The design is recursive because the computation of  $u_{i+1}$  requires the computation of  $u_i$ . The cascade control configuration consist of a master controller driven by the regulation error,  $e = y - y_{ref}$ , that provides reference values to a chain of slave controllers, where the last one correspond to the real control input  $u$ . The recursive cascade control configuration reduces the control design to affine control system (i.e. relative one degree control problems).

### 3.3 Sliding mode control design

To the best of the author's knowledge, SMC approaches have not been applied to control pest population problems, and only have been proposed for few biological control problems [21-23]. The idea behind the SMC is to derive the system state trajectory onto a specified surface  $s(t)$  along which the process can slide to its desired final value. SMC design consist of two phases, namely the sliding phase with  $s(t) = 0$  and  $ds(t)/dt = 0$  and the reaching phase with  $s(t) \neq 0$ . Corresponding to the two phases, two types of control law, that is, continuous and discontinuous control laws can be derived separately.

In this work, the super twisting SMC (STSMC) has been chosen to evaluate the viability of the SMC in biological pest control problems. This is because the STSMC is suitable for relative degree one plants and provides robust stabilization,

finite time convergence to the sliding surface and chattering reduction even in the presence of model uncertainties and disturbances [22, 25]. The STSMC is given by,

$$u(t) = -\overline{g(x)}^{-1} \left[ -\delta_1 \int \text{sign}(s(t)) dt - \delta_2 |s(t)|^{1/2} \text{sign}(s(t)) \right] \quad (3)$$

The control law only depends on the measurable variable, namely the controlled pest population and the rough estimate of  $g(x(t))$ , and no state observer is required. Moreover, a simple controller structure is obtained resulting in low real time computational costs. The stability and finite time convergence characteristics of the STMC have been addressed in the literature and are beyond of the scope of this paper [22, 25].

## 4. NUMERICAL SIMULATIONS

As was discussed below, the control objective for the case study consists in the regulation of the pest population below the EIL via the release of parasitoids. In order to illustrate the implementation of the proposed control configurations, the explicit form of the controller is given as,

$$u_{v1}(t) = -\overline{p_3}^{-1} \left[ -\delta_1 \int \text{sign}(y_{1,ref} - y_1) dt - \delta_2 |y_{1,ref} - y_1|^{1/2} \text{sign}(y_{1,ref} - y_1) \right] \quad (4)$$

$$u(t) = - \left[ -\delta_3 \int \text{sign}(u_{v1}(t) - y_2) dt - \delta_2 |u_{v1}(t) - y_2|^{1/2} \text{sign}(u_{v1}(t) - y_2) \right]$$

The recursive cascade control configuration is obtained with the use of  $x_2 = u_{v1}$  as a virtual control input for the regulation of the pest population  $y_1 = x_1$ , which is driven by the regulation error  $e_1(t) = y_{1,ref} - y_1$ . The resulting virtual control input is the reference for drive the real control input  $u$ , i.e.  $e_2(t) = u_{v1} - y_2$ .

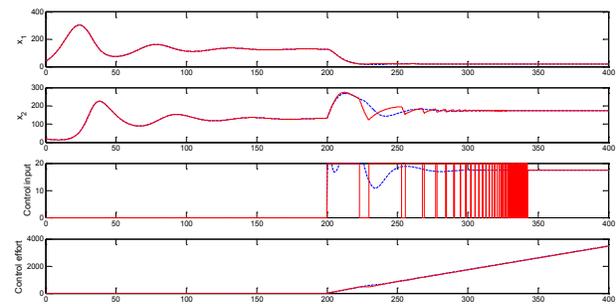


Figure 1: Performance of the STSMC controller.

Figure 1 shows the performance of proposed controller for the following controller design parameters:  $[\delta_1, \delta_2, \delta_3, \delta_4] = [75, 0.005, 35, 0.005]$ . A comparison with a linear controller based on modelling error compensation ideas is also shown in Figure 1. It can be observed that in order to regulate the pest population, the control input computed with the MEC approach consist in an initial high release of parasitoids which is stabilized after 100 time units. On the other hand, the control input computed with the STSMC approach

consists in a switching type control action. The control action achieves minimum and maximum constraints with decreasing time duration at these constraints. Once that the desired reference is achieved a continuous constant action is observed. Interesting, the accumulative amount of released parasitoids is the same for both control actions. Thus, proposed control approaches provide different alternatives of the release of same amount of parasitoids that can be applied to control the pest population at the desired reference.

## 5. CONCLUSIONS

In this paper we have presented the design of a sliding model control approaches to address biological pest control problems. First we have introduced a general class of pest population model under biological control actions given in a chained form, including prey-predator and SI type models. Next, based on the model structure a recursive cascade control configuration is proposed leading to simple SISO control problems. The robust control approach based on SMC ideas is then formulated. STSMC approach is endowed with a discontinuous action that can be appropriate for practical implementation of pest control problems.

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