

Robust Adaptive Synchronization of Unified Chaotic Systems

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Abstract: This paper proposes an adaptive methodology to estimate the unknown master system parameter and synchronize unified chaotic systems, even if bounded system disturbances are present. Based on both Lyapunov theory and Barbalat's Lemma, a robust adaptive scheme is proposed to make the synchronization error asymptotically null. Simulation results are provided to demonstrate the effectiveness and feasibility of the proposed synchronization method.

Keywords: Adaptive synchronization, adaptive control, chaotic systems, Lyapunov methods.

1. INTRODUCTION

Encouraged by the discovery of the chaotic dynamics by Lorenz in 1963 [1], chaos synchronization has been studied in the last thirty years by several researchers [2-12]. This is due to its potential applications in numerous engineering problems ranging from living system applications [5] to the non-living system applications [6]. For instance, chaos synchronization has been applied in electrical [7,8], biological [9], chemical [10], secure communication [11], and finance systems [12].

After Lorenz' model, several others chaotic systems such as Chua [13,14], Rössler [15,16], Lü [17,18], Chen [19,20], Liu [21,22], finance [23], unified [24], etc. have been proposed, and a great number of techniques, such as linear, nonlinear, passivity based, adaptive, backstepping, and sliding control, among other, have been introduced to achieve its synchronization. See, for instance, [25-29] and the references therein.

Despite the large number of existing techniques, few papers have been devoted to the chaotic synchronization in the presence of uncertain parameters and bounded disturbances. In most of these works [29-36], the main particularity is that the control law is not smooth, since the control law depends on a sign function which is discontinuous [29-34,36]. Also, the parameter adaptation law is not robust, because it lacks of a leakage term [29,31-34,36]. Moreover, the control law uses the master system parameters [29] and disturbances [35], and it is only ensured, that the synchronization error is uniformly bounded in the presence of uncertain parameters and disturbances [36].

Unfortunately, in spite of the relevance of the above mentioned works, they have some limitations. It is well known that the discontinuous feedback control raises

theoretical and practical issues [37]. From the theoretical point of view, standard existence and uniqueness of solutions of differential equations are, in general, not applicable [38]. Furthermore, the validity of the Lyapunov analysis will have to be examined in a framework that does not require the right-hand side of the state equation to be locally Lipschitz, in the state variables, and piece-wise continuous with respect to time [37-38]. Practical issues are associated with the imperfections of switching devices and delays leading to chattering, which result, for example, in low control accuracy and high heat losses in electrical power circuits [37]. On the other hand, it has been known since the early 1980s that nonrobust adaptive laws may suffer from parameter drift phenomenon [39], that is, the parameters drift to infinity with time. It is often due to the "pure" integral action of the adaptive law. Several leakage modifications to counteract this have been proposed since then [39,40]. Finally, it is a basic rule in chaos synchronization based cryptography that the details of the encryption algorithm are always known by the attacker [41]. Hence, the use of the master system parameters in the control law is controversial.

On the other hand; recently, a large number of studies have been focused on synchronization of unified chaotic systems for applications in secure communications [42]. The unified chaotic system is a three dimensional system that has a broad spectrum of chaotic behaviors, which is associated with a scalar parameter used in its model. Once the unified system can display hyperchaos [43] and the master system parameter can be used as a modulator element in chaotic parameter modulation schemes [11], it seems be adequate for chaotic communication applications, and, hence, this chaotic system will be employed to show the proposed design methodology.

Motivated by the previous facts, in this paper we propose an adaptive synchronization method for unified chaotic systems, which ensures asymptotic synchronization in the presence of

uncertain system parameter and bounded disturbances. The proposed controller has advantageous properties, when compared with previous works [29-36], since it is smooth, it does not require the master parameter for implementation purposes, and it uses a e-modification robust adaptive law [39] for adjusting the unknown parameter. Hence, it does not present chattering or parameter drift. The design methodology is based on the Lyapunov theory and the Barbalat' Lemma [39] and ensures the convergence of the synchronization error to zero, even in the presence of uncertain system parameter and bounded disturbances.

2. PROBLEM FORMULATION

Consider the problem of control unified chaotic systems described by the following differential equation

$$\dot{x}_s = A(\beta)x_s + f_s(x_s) + d_s(x_s, t) + u \quad (1)$$

where $x_s \in \mathfrak{R}^3$ is the state of the slave system, $u \in \mathfrak{R}^3$ is the control input, $f_s(\cdot)$ is a known map, $d_s(\cdot)$ is an unknown disturbance and β is a known parameter,

$$A(\beta) = \begin{bmatrix} -25\beta - 10 & 25\beta + 10 & 0 \\ 28 - 35\beta & 29\beta - 1 & 0 \\ 0 & 0 & -\frac{8 + \beta}{3} \end{bmatrix} \quad (2)$$

and

$$f_s(x_s) = \begin{bmatrix} 0 \\ -x_{s1}x_{s3} \\ x_{s1}x_{s2} \end{bmatrix} \quad (3)$$

In order to have a well-posed problem, we assume that the right-hand side of (1) is piecewise continuous with respect to time and locally Lipschitzian with respect to x_s , such that (1) has a unique solution globally in time for any given initial condition.

We assume that the following can be established.

Assumption 1: On the region $\mathfrak{R}^3 \times [0, \infty)$

$$\|d_s(x_s, t)\| \leq d_{s0} \quad (4)$$

where d_{s0} is a positive constant, such that $d_{s0} < \bar{d}_s$ and \bar{d}_s is a known constant.

Remark 1: Assumption 1 is usual in synchronization of chaotic systems.

Remark 2: In case that $\beta = 0$, $\beta = 0.8$, and $\beta = 1$, system (1) becomes the Lorenz, Lü, and Chen systems, respectively, when perturbations are not present.

Remark 3: It should be noted that the conditions of existence and uniqueness of solutions of (1) are a prerequisite to the Lyapunov-type arguments to be used in the stability analysis.

Basically, it is necessary to show that the state trajectories do not escape to infinity in finite time [38].

We consider the master system as

$$\dot{x}_m = A(\alpha)x_m + f_m(x_m) + d_m(t, x_m) \quad (5)$$

where $x_m \in \mathfrak{R}^3$, α is a known parameter and $d_m(\cdot)$ is an unknown disturbance.

$$A(\alpha) = \begin{bmatrix} -25\alpha - 10 & 25\alpha + 10 & 0 \\ 28 - 35\alpha & 29\alpha - 1 & 0 \\ 0 & 0 & -\frac{8 + \alpha}{3} \end{bmatrix} \quad (6)$$

and

$$f_m(x_m) = \begin{bmatrix} 0 \\ -x_{m1}x_{m3} \\ x_{m1}x_{m2} \end{bmatrix} \quad (7)$$

It is assumed that the right-hand side of (5) is piecewise continuous with respect to time and locally Lipschitzian with respect to x_m , such that (5) has a unique solution globally in time.

Assumption 2: The parameter α is upper bounded by a known positive constant $\bar{\alpha}$, such that $\bar{\alpha} > \alpha$.

Assumption 3: On the region $\mathfrak{R}^3 \times [0, \infty)$

$$\|d_m(x_m, t)\| \leq d_{m0} \quad (8)$$

where d_{m0} is a positive constant, such that $d_{m0} < \bar{d}_{ms}$ and \bar{d}_{ms} is a known constant.

Hence, our aim is to design a smooth feedback control u , which depends on x_m , x_s , f_m , f_s , and t , but does not depend on α , such that the state x_s of the slave chaotic system (1) tracks of the state x_m of the master system (5), even in the presence of the unknown disturbances d_m and d_s .

Define the synchronization error $e(t) = x_s - x_m$. Then, from (1) and (5), we obtain the synchronization error equation

$$\dot{e} = A(\beta)e + f_s - f_m + d + u + \beta Bx_m + \alpha Cx_m \quad (9)$$

where $d = d_s - d_m$, $C = -B$ and

$$B = \begin{bmatrix} -25 & 25 & 0 \\ -35 & 29 & 0 \\ 0 & 0 & -1/3 \end{bmatrix} \quad (10)$$

Remark 4: For sake of simplicity, it was considered that $f_m(\cdot) = f_s(\cdot)$. However, in our formulation $f_m(\cdot)$ could be structurally different from $f_s(\cdot)$ to accomplish the synchronization between unrelated chaotic systems.

3. ADAPTIVE SYNCHRONIZATION

In this section, we considered the problem of asymptotic adaptive synchronization in the presence of unknown parameter and bounded disturbances. By using Lyapunov theory and Barbalat's Lemma, it is shown that the synchronization error converges asymptotically to zero. The proposed scheme is motivated by [40,44].

Theorem 1: Consider the slave (1) and master (5) chaotic systems, which satisfy Assumptions 1-3, the control law

$$u = -(\hat{\alpha}Cx_m + \beta Bx_m + f_s - f_m + A(\beta)e + Le + u_r) \quad (11)$$

with

$$u_r = \frac{\gamma_0 e}{\lambda_{\min}(K) \|e\| + \gamma_1 \exp(-\gamma_2 t)} \quad (12)$$

$$\dot{\hat{\alpha}} = -\alpha_0 [\alpha_1 \|e\| \hat{\alpha} - e^T KCx_m] \quad (13)$$

where

$$\begin{aligned} L^T P + PL &= -Q, \quad P = P^T > 0, \quad Q > 0 \quad K = P + P^T, \\ \gamma_0 &\geq 0, \quad \gamma_1 > 0, \quad \gamma_2 > 0, \quad \gamma_3 = \lambda_{\min}(Q), \\ \gamma_4 &= \|K\|_F \bar{d} + \alpha_1 \bar{\alpha}^2 / 2, \quad \alpha_0 > 0, \quad \alpha_1 > 0, \\ \bar{d} &= \bar{d}_s + \bar{d}_m, \text{ and } \|K\|_F \text{ is the Frobenius norm of } K. \end{aligned} \quad (14)$$

Then,

- i) If $\gamma_0 = 0$, the synchronization error $e(t)$ is uniformly ultimately bounded, with ultimate bound $\alpha_e = \gamma_4 / \gamma_3$.
- ii) If $\gamma_0 \geq \gamma_1 + \gamma_4$, the slave and master systems synchronize, i.e., $\lim_{t \rightarrow \infty} e(t) = 0$.

Proof: Consider the Lyapunov function candidate

$$V = e^T P e + \alpha_0^{-1} \tilde{\alpha}^2 / 2 \quad (15)$$

where $\tilde{\alpha} = \hat{\alpha} - \alpha$.

The time derivative of (15) results

$$\dot{V} = \dot{e}^T P e + e^T P \dot{e} + \tilde{\alpha} \alpha_0^{-1} \dot{\tilde{\alpha}} \quad (16)$$

On the other hand, by using (9) and (11), the closed-loop synchronization error can be written as

$$\dot{e} = -Le - \tilde{\alpha} Cx_m + d - u_r \quad (17)$$

By evaluating (16) along the trajectories of (13) and (17), we obtain

$$\begin{aligned} \dot{V} &= -e^T (L^T P + PL) e - e^T (P + P^T) Cx_m \tilde{\alpha} \\ &\quad - e^T (P + P^T) u_r + e^T (P + P^T) d \\ &\quad - \alpha_1 \|e\| \tilde{\alpha} \dot{\tilde{\alpha}} + e^T KCx_m \tilde{\alpha} \end{aligned} \quad (18)$$

Using now (4), (8), and (12)-(14), (18) implies

$$\dot{V} \leq -\gamma_3 \|e\|^2 + \gamma_4 \|e\| - \frac{\alpha_1 \|e\| \tilde{\alpha}^2}{2} - \frac{\gamma_0 \|e\|^2}{\|e\| + \gamma_1 \exp(-\gamma_2 t)} \quad (19)$$

At first, let us consider $\gamma_0 = 0$. Then, (19) can be rewritten as

$$\dot{V} \leq -\|e\| \left[\gamma_3 \|e\| + \frac{\alpha_1 \tilde{\alpha}^2}{2} - \gamma_4 \right] \quad (20)$$

Hence, $\dot{V} < 0$ outside the compact set $\Omega_1 = \left\{ x_a := \begin{bmatrix} e(t) \\ \tilde{\alpha}(t) \end{bmatrix} \in \mathfrak{R}^{n+1} \mid \|e(t)\| \leq \alpha_e \text{ or } \|\tilde{\alpha}(t)\| \leq \alpha_p \right\}$ where $\alpha_p = (2\gamma_4 / \alpha_1)^{1/2}$. Thus, since α_e and α_p are positive constants, by employing usual Lyapunov arguments [39], we concluded that $e(t)$ and $\tilde{\alpha}(t)$ are uniformly bounded. In addition, since γ_3 and α_1 can be arbitrarily selected according to (14), $e(t)$ is uniformly ultimately bounded with ultimate bound α_e .

In case $\gamma_0 \geq \gamma_1 + \gamma_4$, (19) implies

$$\dot{V} \leq -\gamma_3 \|e\|^2 - \gamma_1 \|e\| \left[\frac{\|e\| - \gamma_4 \exp(-\gamma_2 t)}{\|e\| + \gamma_1 \exp(-\gamma_2 t)} \right] \quad (21)$$

Define

$$\Omega_2 = \left\{ x_a := \begin{bmatrix} e(t) \\ \tilde{\alpha}(t) \end{bmatrix} \in \mathfrak{R}^{n+1} \mid \|e(t)\| \leq \gamma_4 \exp(-\gamma_2 t) \right\} \quad (22)$$

Note that the numerator in the bracket of (21) is greater than zero for $\|e\| > \gamma_4 \exp(-\gamma_2 t)$ (or $e \in \Omega_2^c$), hence

$$\dot{V} \leq -\gamma_4 \|e\|^2 \quad (23)$$

Further, since V is bounded from below and non-increasing with time, we have

$$\lim_{t \rightarrow \infty} \int_0^t \|e(\tau)\|^2 d\tau \leq \frac{V(0) - V_\infty}{\gamma_4} < \infty \quad (24)$$

where $\lim_{t \rightarrow \infty} V(t) = V_\infty < \infty$. Notice that, based on (17), with the bounds on $e, \tilde{\alpha}, d$ and u_r , \dot{e} is also bounded. Thus, \dot{V} is uniformly continuous. Hence, by applying the

Barbalat's Lemma [39, p. 76], we conclude that $\lim_{t \rightarrow \infty} e(t) = 0$ for all $e \in \Omega_2^c$.

Once the synchronization error $e(t)$ has entered Ω_2 , it will remain in Ω_2 forever, due to (22) and (23). Consequently, we conclude that $\lim_{t \rightarrow \infty} e(t) = 0$ holds in the large, i.e., whatever the initial value of $x_a(t)$ (inside or outside Ω_2).

Remark 5: Notice that in our formulation the control law does not depend on α . This peculiarity can be explored by analog chaos-based secure communications systems, since the master system parameter α is not transparent to the receiver.

Remark 6: It is interesting to notice that the six first terms in the parenthesis of the right-hand side of (11) ensure the convergence of the synchronization error to an arbitrarily neighborhood of the origin whose radius can be controlled, for instance, by the matrix L . The last term is robustifying one to make the residual synchronization error asymptotically null with convergence rate controlled by γ_2 .

4. SIMULATIONS

In this section, we aim to illustrate the convergence and robustness properties of the proposed synchronization scheme in the presence of internal and external disturbances. The master and slave systems are considered to have unrelated dominant dynamics.

To synchronize the slave system (1) and the master system (5), the adaptive laws (11)-(13) were implemented. It should be noted that (11)-(13) do not depend on α . However, α is required to generate the master system state trajectory, which is used in the simulations.

The initial conditions for the master and slave systems in all simulations were $x_m(0) = [1.5 \ 2 \ 5]^T$ and $x_s(0) = [4 \ 8 \ 3]^T$ in order to evaluate the performance of the proposed synchronization algorithm also under adverse initial conditions. The other design parameters were selected as $\gamma_0 = 100$, $\gamma_1 = 0.1$, $\gamma_2 = 0.001$, $\alpha_0 = 0.05$, $\alpha_1 = 10$, $\hat{\alpha}(0) = 0$, $P = \text{diag}(0.001, 10, 5)$, and $L = \text{diag}(10, 10, 5)$.

To check the robustness of the proposed scheme, the synchronization between Chen-Lorenz systems, i.e. $\beta = 1$ and $\alpha = 0$, is considered. It is assumed that the chaotic circuits are affected by unmeasured disturbances (for instance due to noise, electronic component tolerance, aging or faults) of the form

$$d_m(x_m, t) = \begin{bmatrix} \sin(5t) + \cos(10t) \\ \sin(5t) \ln(0.5x_{m1}^2 + 2x_{m2}^2 + x_{m3}^2) \\ \sin(t) + \cos(3t) \end{bmatrix} \quad (25)$$

$$d_s(x_s, t) = \begin{bmatrix} 3 \sin(t) (x_{s1}^2 + x_{s2}^2 + 5x_{s3}^2)^{1/2} \\ 3 \sin(2t) \\ \cos(10t) \end{bmatrix} \quad (26)$$

Figures 1-2 show the performance of the synchronization method in the presence of the disturbances (25) and (26). It can be seen that the synchronization error converges to zero in approximately 10^{-4} seconds. Hence, the simulations confirm the theoretical results: the synchronization error converges asymptotically to zero, even in the presence of disturbances in the master and slave systems, provided that the conditions of Theorem 1 are satisfied.

5. CONCLUSIONS

In this paper, we propose a synchronization scheme to synchronize uncertain chaotic system subject to unknown parameter and bounded disturbances. Based on Lyapunov-like analysis using Barbalat's Lemma, an adaptive controller which is composed by two parts was introduced. The first part is devoted to force the state of the slave system to converge to an arbitrary neighborhood of the state of the master system. The other part accomplishes the asymptotic synchronization of the slave-master systems in spite of the presence of unknown parameter and bounded disturbances. Simulations were performed to show the performance of the proposed method.

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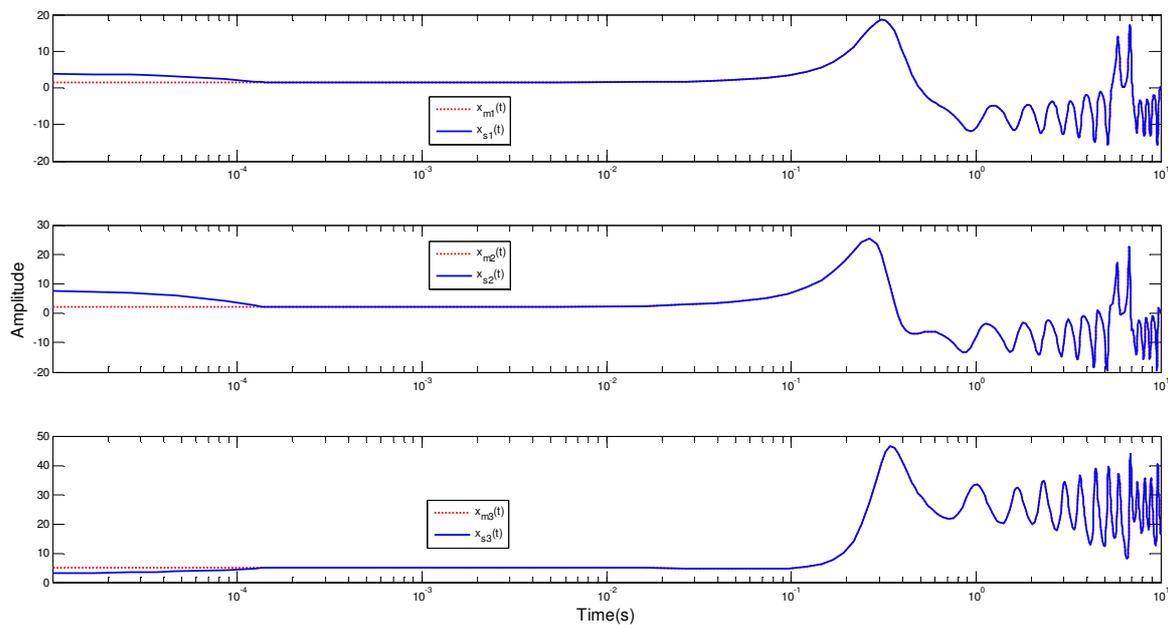


Figure 1. Performance in the synchronization between Chen slave–Lorenz master systems in the presence of unknown parameter and disturbances.

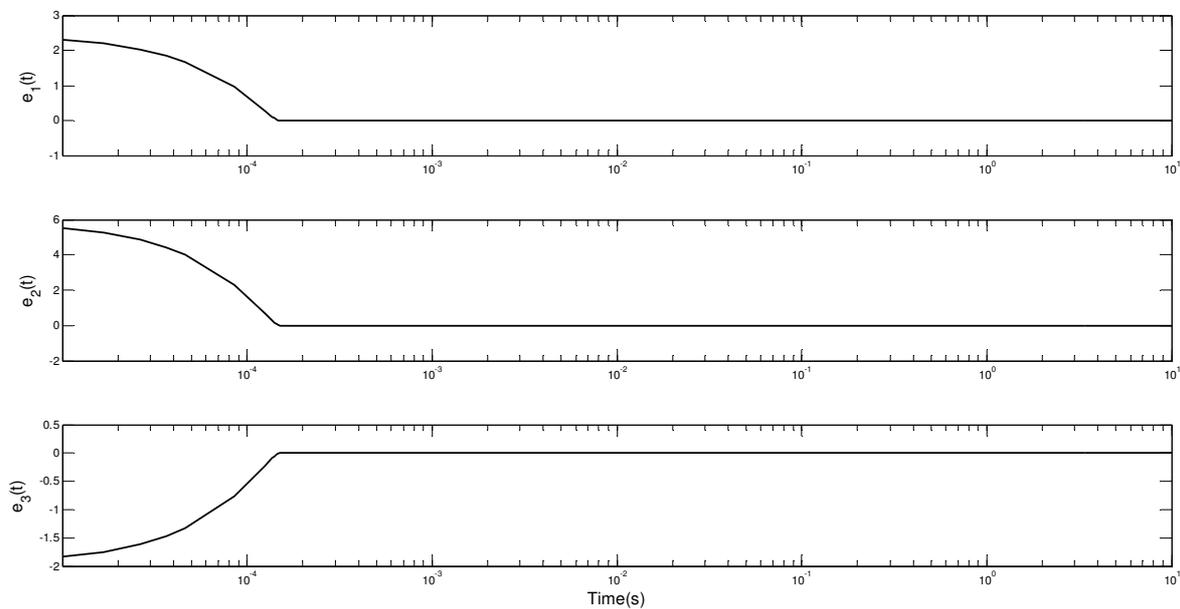


Figure 2. Synchronization error of the Chen slave–Lorenz master systems in the presence of unknown parameter and disturbances.