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Hybrid model predictive control for mineral grinding ⋆

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Abstract: The mining industry is in search of control strategies that allow the consideration of a global optimization of their processes, ensuring their stability. This has led to the application of recently developed control techniques on large-scale systems in the mining processes. One of the most important process of the mining industry is the mineral grinding, as the product particle size impacts, in a significant manner, the recovery rate of the valuable mineral in the separation stages. As a solution, a hybrid model predictive control (HMPC) is presented; this control approach maintains the product particle size on a defined range and minimizes the specific energy consumption, maintaining the grinding plant in a stable operation. As a comparison, a conventional model predictive control with an expert supervisor to handle discrete events was developed, showing HMPC is a suitable solution that can used in the grinding process, handling both discrete and continuous dynamics and events with one controller.

Keywords: Hybrid model predictive control; Dynamic modelling; Hybrid modelling; Hybrid identification.

1. INTRODUCTION

The mining industry has increased the search for methods and control algorithms to improve the energy efficiency of the mining process. This search is taking place due to the growth of energy consumption and energy prices on the last few years. The concentrator plant is one of the most energy-intensive process. A mining concentrator plant is divided in two processes, grinding and flotation. The objective of the grinding process is to prepare the ore, by reducing the particle size, to maximize the recovery rate of valuable mineral during the flotation process. The grinding circuits consume close to 60% of the energy in a concentrator plant (Rodriguez et al., 2003), for this reason the industry is searching for control strategies and techniques to reduce the energy consumption of the process, maintaining the required particle size of the ore to maximize the valuable mineral recovery rate.

A wide range of control technologies for grinding circuits are used by the actual mining industry: PID, multivariable, expert systems, fuzzy logic, neural networks, model predictive control (MPC), statistical process control, and others. The PID control technology is the most commonly used, with 63% (Wei and Craig, 2009), unlike the general process industries that use MPC control schemes (Bauer and Craig, 2008). The MPC technology is used only by 8% of the mining industries to control the grinding circuits (Wei and Craig, 2009), nevertheless the use of it has increased in the last few years, as it presents good performance in tracking and optimizing applications, allows the handling of restrictions and constraints, and presents a robust management of disturbances.

One of the disadvantages of the MPC control scheme is that it only allows the control of continuous dynamics and variables, and many industrial processes, as grinding circuits, involve the interaction of discrete elements and dynamics. A grinding circuit involves discrete dynamics and events, such as the state of: conveyors, hydrocyclones, grinding lines, and others. A solution to this problem is the use of the hybrid model predictive control technique to handle both the discrete and continuous nature of the process. The HMPC technique commonly only involves a hybrid modelling and optimization of the process. For nonlinear complex process, a hybrid identification technique is used during the development of the HMPC control strategy.

This article presents the development of a HMPC strategy involving hybrid identification using a piecewise autoregressive exogenous (PWARX) model. Also, it presents the evaluation of two control strategies: conventional MPC (CMPC) with an expert supervisor for the different operating modes, and a hybrid MPC (HMPC), showing a comparison between them when a discrete event occurs. The results will be evaluated by using the RMS error between the references and the controlled variables. Other aspects such as power consumption and the processed mineral tonnage will be taken into account. First the continuous and hybrid modelling are presented, followed by the hybrid identification process, control strategies and results.

2. METHODOLOGY

In this section the hybrid modelling of the grinding circuit will be presented, followed by the description of the hybrid identification process and the control strategies, MPC and HMPC.

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2.1 Plant Description

The grinding plant is divided in two stages, primary and secondary grinding. The primary grinding stage consists on a SAG mill, to reduce the particle size, and a vibratory screen, that classifies the output. The output is sent either to a pebble crusher, to reduce the particle size, or to a sump, if the size is small enough for the next grinding stage. The secondary grinding typically takes the ore from the sump to a battery of hydrocyclones that classifies the ore, by using centrifugal force, and sends it either to a ball mill or to the flotation process. It is important to notice that the secondary grinding can be divided into several grinding lines to make an efficient size reduction to the incoming ore, in this paper two secondary grinding lines are used.

A schematic of the simulated plant is shown in Figure 1, it consists of a typical primary grinding stage and a secondary grinding formed by two lines. Each hydrocyclone battery has 14 hydrocyclones and the pebble crusher feeds both of the ball mills of the secondary grinding stage, in addition to the material rejected by the hydrocyclone batteries.

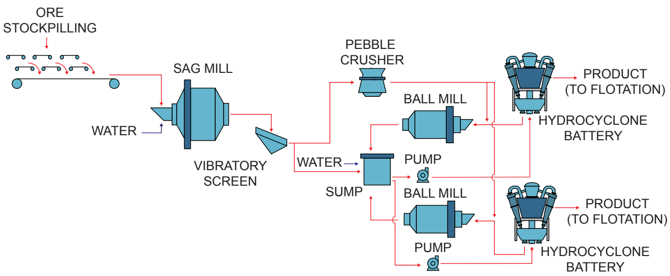

Fig. 1. Schematic of the simulated plant

2.2 Continuous Modelling

The original grinding plant model was developed in Orellana (2010), it uses mathematical models of the machinery and processes by introducing simplifications of Weymont (1979), Austin et al. (1987) and Barahona (1984). It describes both primary and secondary grinding by unit operations.

The first stage of the primary grinding is the SAG Mill, represented by the following variables and states:

$$\mathrm{X_S} = \begin{bmatrix} \mathrm{w_S} \\ w_{SA} \end{bmatrix} \quad \mathrm{U_S} = \begin{bmatrix} \mathrm{f_S} \\ q_{i_S} \end{bmatrix} \quad \mathrm{Y_S} = \begin{bmatrix} \mathrm{p_S} \\ q_{o_S} \end{bmatrix} \tag{1}$$

where $\mathrm{w_S}$ is the solids hold-up, w_{SA} is the water hold-up, $\mathrm{f_S}$ is the solids input flow, q_{i_s} water input flow, $\mathrm{p_S}$ is the solids output flow, and q_{o_s} is the output water flow. The outputs from the SAG mill are represented by (2):

$$\mathrm{Y_S} = \begin{bmatrix} (I-c)\dfrac{\delta_S}{\sqrt{\Sigma \mathrm{w_S}}}\mathrm{w_S} \\ \left(\kappa_{S_1} + \dfrac{\kappa_{S_2}}{(\Sigma \mathrm{w_S})^4}\right) w_{SA} \end{bmatrix} \tag{2}$$

where c is the rate of return of the output, δ_S is a discharge constant of the solids, and κ_{S_1} and κ_{S_1} are discharge constants of the water.

The output from the SAG mill are classified by the vibratory screen, the rejection rate of solids is dependent on the vibratory screen aperture. The water, as in Orellana (2010), is retained by 10% on the vibratory screen due to the humidity retained by the inside ore. The pebble crusher objective is to reduce the ore size of the rejected material of the vibratory screen. The crushing efficiency is dependent on the physical features of the pebble crusher and the input mass flow of material.

The sump retains the output from the vibratory screen that was not rejected, and water is added into the system to create a pulp. Therefore the input of the pump is represented by (3)

$$\bar{f}_P = \frac{F_p}{\rho_m} + q_{iP} + G_p \tag{3}$$

where F_p is the sum of the solids mass flow vector, ρ_m is the density of the material inside the sump, q_{iP} is the water mass flow from the vibratory screen, and G_p is the added mass flow of water. An important variable that of the pump is the pulp level inside the sump, this is represented by (4):

$$\frac{dh_p}{dt} = \frac{\bar{f}_P - \bar{p}_P}{A_P} \tag{4}$$

with $\bar{p}_P$ as the output mass flow y A_P as the sump transverse area. The output mass flow of water and solids are represented by (5) and (6):

$$P_p = \bar{p}_P \rho_{t_P} c_P \tag{5}$$

$$q_{oP} = \bar{p}_P \rho_{t_P} (1 - c_P) \tag{6}$$

where P_p is the solids output, q_{oP} is the water output, c_P is the solids rate inside the pulp, and ρ_{t_p} is the pulp density. This material output is pushed by pumps from the sump.

The flow of pulp that passes through the pumps can be described as in (7)

$$\bar{f_{bc}} = \left[\frac{-1 + \sqrt{1 + 4g_{cp} * [\kappa_{B1} V_b - \kappa_{B2} \rho_{tP} g * (h_h - h_p)]}}{2 * g_{cP}} \right] \tag{7}$$

Where V_b is the pump speed, k_{B1} and k_{B2} are pump constants, g the gravity force, ρ_{tP} is the pulp density, h_h the height difference between the pump and the hydrocyclones, h_p is the pulp height inside the sump and g_{cP} is defined as (8)

$$g_{cP} = \kappa_{B2} * \left[\frac{\kappa_{H4}}{(1 - c_P)^{0.25}} + \kappa_{B3} \right] \tag{8}$$

with κ_{B3} representing a geometric parameters constant of the system, κ_{H4} a pressure constant in the input of the hydrocyclone battery, and c_P is the solids percentage in the pulp.

2.3 Hybrid Modelling

A piecewise representation of the model was developed(Estrada and Cipriano, 2014), this is achieved by adding activation variables to certain terms of the equations. Different hybrid behaviours were modelled by this methodology, such as the stockpile feeding process, ON/OFF state of secondary grinding lines, and quantity of active hydrocyclones. The continuous equations were adapted to introduce discrete variables to the process, allowing the use of activation terms to create a hybrid model.

On stockpile discharge process, the solids feeding the grinding process, $\mathbf{f_s}$, are mass flow vectors, which are separated into different intervals dependent of the ore size ranges (Orellana, 2010). This can be described as following:

$$\mathbf{f_s} = F_t * \mathbf{f_{size}} \tag{9}$$

Where F_t is the total mass flow and $\mathbf{f_{size}}$ is the vector dependent of the ore size distribution. $\mathbf{f_{size}}$ can take three values depending on the ore particle size: small, $\mathbf{f_{sma}}$, medium, $\mathbf{f_{med}}$, and coarse, $\mathbf{f_{coa}}$. This is shown in (10)

$$\mathbf{f_s} = \begin{cases} Ft * \mathbf{f_{sma}}, & g_a(t) = 1 \\ Ft * \mathbf{f_{med}}, & g_a(t) = 2 \\ Ft * \mathbf{f_{coa}}, & g_a(t) = 3 \end{cases} \tag{10}$$

where g_a allows the selection of the ore size distribution. This allows to simulate, in certain grade, the material discharge from the ore stockpiling to the process, since in practice the pile has the finest material in the centre and the coarsest granulometries on the sides, making the stockpile discharge first the finest materials, and forcing the miners to push the remaining material to the centre. The modelling of the primary feeding is shown in Figure 2.

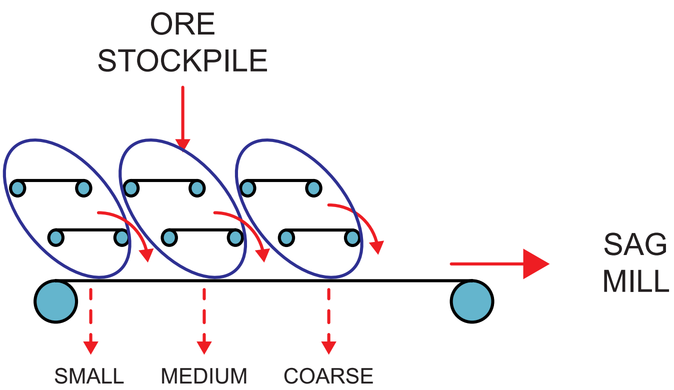

Fig. 2. Ore stockpile feeding modelling

The ON/OFF state of the conveyors is modelled by adding an activation term to the sum:

$$\mathrm{f}_{s_{\mathrm{total}}} = \alpha_1 * \mathrm{f}_{s_{\mathrm{sma}}} + \alpha_1 * \mathrm{f}_{s_{\mathrm{med}}} + \alpha_3 * \mathrm{f}_{s_{\mathrm{coa}}} \tag{11}$$

where the activation terms are $\alpha_r \in \{0,1\}$ with $r = \{1,2,3\}$. And $f_{s_{sma}}$, $\mathrm{f}_{s_{\mathrm{med}}}$ and, $\mathrm{f}_{s_{\mathrm{coa}}}$ represent each one of the conveyors. In this work six conveyors are used, but only three conveyors have this activation term: fine, medium, and coarse granulometries. The other three conveyors were set with a constant flow of ore, simulating that the materials mix with each other on the stockpile.

For the secondary grinding lines hybrid modelling, an activation term was added to the solid flow equations as in (12)

$$\bar{\mathbf{f_b}} = s_{ls} * \bar{\mathbf{f_{bc}}} \tag{12}$$

where $\mathbf{f_{bc}}$ is the solid flow output of the pump, and $s_{ls} \in \{0,1\}$, controls the ON/OFF state of the pump. For the water flow the activation term was introduced as in (13)

$$q_{oP} = s_{ls} * [\bar{p_P}\rho_{tP}(1 - c_P)] \tag{13}$$

where c_P represents the solids percentage in the pulp, ρ_{tP} is the pulp density and $\bar{p_P}$ is the material output flow from the sump.

The last hybrid modelling element of the grinding circuit is the hydrocyclones active in each battery. For this, a modification was introduced to the hydrocyclone battery model of Orellana (2010). Since the hydrocyclones pressure, d50 diameter, and water flow equations have a constant term, n_H, that corresponds to the number of active hydrocyclones on the battery, the modification in our model is that n_H is a sum of the activation variables of each hydrocyclone, allowing a dynamic change of the active number during the process evolution. This is represented as in (14)

$$n_H = \sum_{i=1}^{14} s_{H_i} \tag{14}$$

where s_{H_i} is the state of each hydrocyclone.

3. HYBRID IDENTIFICATION AND CONTROL STRATEGIES

The identification method used in this work is divided into two steps, the first step consists on generating an AutoRegresive eXogenous (ARX) for each of the plant modes. The ARX model is combined in a Linear Single PWARX model of the complete plant. The ARX model was obtained with the industrial data tuned simulator, the data uses a sample time of 3.6 seconds.

This identified variables were transformed manually into a mixed logical dynamic (MLD) model, as the MLD framework is a powerful tool for modeling discrete-time linear hybrid systems (Jost and Torrisi, 2002), and it is compatible with the Hybrid Systems DEscription Language (HYSDEL), created to formulate HMPC strategies.

Two control strategies will be implemented, the first one consists on a conventional MPC problem and the second on a HMPC solution. The continuous MPC problem can be solved as in (Maciejowski, 2002). The hybrid model predictive control problem uses the MLD representation of the plant.

An MLD system is completely represented by the following set of equations:

$$\begin{aligned} x(t+1) &= Ax(t)+B_1u(t)+B_2\delta(t)+B_3z(t) \\ y(t) &= Cx(t)+D_1u(t)+D_2\delta(t)+D_3z(t) \\ E_2\delta(t)+E_3z(t) &\leq E_1u(t)+E_4x(t)+E_5 \end{aligned} \tag{15}$$

where x, y and u are the state, output and input of the system expressed by:

$$x = \begin{bmatrix} x_c \\ x_\ell \end{bmatrix}, x_c \in \mathbb{R}^{n_c}, x_\ell \in \{0,1\}^{n_\ell} \tag{16}$$

$$y = \begin{bmatrix} y_c \\ y_\ell \end{bmatrix}, y_c \in \mathbb{R}^{p_c}, y_\ell \in \{0,1\}^{p_\ell} \tag{17}$$

$$u = \begin{bmatrix} u_c \\ u_\ell \end{bmatrix}, u_c \in \mathbb{R}^{m_c}, u_\ell \in \{0,1\}^{m_\ell} \tag{18}$$

Also, $\delta \in \{0,1\}^{r_\ell}$ is the auxiliary logical variable and $z \in \mathbb{R}^{r_c}$ the auxiliary continuous variable.

The hybrid model predictive control problem can be stated as follows (Bemporad and Morari, 1999):

$$\begin{aligned} \min_{\{u_0^{N-1},\delta_0^{N-1},z_0^{N-1}\}} & \sum_{k=0}^{N-1} \|u(k)-u_e\|^2_{\mathcal{Q}_1} + \|\delta(k|t)-\delta_e\|^2_{\mathcal{Q}_2} \\ & + \|z(k|t)-z_e\|^2_{\mathcal{Q}_3} + \|x(k|t)-x_e\|^2_{\mathcal{Q}_4} + \|y(k|t)-y_e\|^2_{\mathcal{Q}_5} \end{aligned} \tag{19}$$

subject to:

$$\begin{aligned} x(T|t) &= x_e \\ x(k+1|t) &= Ax(k|t)+B_1u(k)+... \\ & ...+B_2\delta(k|t)+B_3z(k|t) \\ y(k|t) &= Cx(k|t)+D_1u(k)+... \\ & ...+D_2\delta(k|t)+D_3z(k|t) \\ E_2\delta(k|t)+E_3z(k|t) &\leq E_1u(k)+E_4x(k|t)+E_5 \end{aligned} \tag{20}$$

where $u_0^{N-1} = \{u(0),\ldots,u(N-1)\}$, $\mathcal{Q}_1 = \mathcal{Q}_1' > 0$, $\mathcal{Q}_2 = \mathcal{Q}_2' \geq 0$, $\mathcal{Q}_3 = \mathcal{Q}_3' \geq 0$, $\mathcal{Q}_4 = \mathcal{Q}_4' > 0$, $\mathcal{Q}_5 = \mathcal{Q}_5' \geq 0$, $\|x\|^2\mathcal{Q} = x'\mathcal{Q}x$, $x(k|t) \triangleq x(t+k, x(t), u_0^{k-1}, \delta_0^{k-1}, z_0^{k-1})$, and $\delta(k|t)$, $z(k|t)$, $y(k|t)$ are defined in similar fashion. The prediction $x(k|t)$ represents the state future value at $t+k$ given the state information at time t and the future values of the optimization variables. The control problem can be solved using mixed integer quadratic programming (MIQP). From (20) we have

$$\begin{aligned} x(k|t) = A^k x_0 + \sum_{i=0}^{k-1} A^i & [B_1u(k-1-i|t) \\ & + B_2\delta(k-1-i|t)+B_3z(k-1-i|t)] \end{aligned} \tag{21}$$

Adding (21) to the objective function and the constraints and defining the vectors,

$$\Omega \triangleq \begin{bmatrix} u(0) \\ \vdots \\ u(T-1) \end{bmatrix}, \Psi \triangleq \begin{bmatrix} \delta(0) \\ \vdots \\ \delta(T-1) \end{bmatrix}, \\ \Xi \triangleq \begin{bmatrix} z(0) \\ \vdots \\ z(T-1) \end{bmatrix}, \mathcal{V} \triangleq \begin{bmatrix} \Omega \\ \Psi \\ \Xi \end{bmatrix} \tag{22}$$

The problem formulated by (19) and (20) can be rewritten as in (Bemporad and Morari, 1999)

$$\begin{aligned} & \min_{\mathcal{V}} \mathcal{V}'S_1\mathcal{V} + 2(S_2 + x_0'S_3)\mathcal{V} \\ & \text{subject to: } F_1\mathcal{V} \leq F_2 + F_3x_0 \end{aligned} \tag{23}$$

The objective of both controllers is to maintain the output particle size of mesh 65 under a certain reference and minimize the specific energy consumption of the grinding plant. This is represented by the following objective function:

$$\begin{aligned} J = \sum_{i=0}^{7} \Big(& \|\mathbf{y}(k+i+1) - \mathbf{y}_{\text{ref}}(k+i+1)\|^2_{\mathbf{Q}_y} \\ & + \|\mathbf{\Delta u}(k+i)\|^2_{\mathbf{R}} + \mathbf{L}_y^\top \mathbf{y}(k+i+1) \Big) \end{aligned} \tag{24}$$

$$\mathbf{Q}_y = \begin{bmatrix} 0 & 0 \\ 0 & 9 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}, \quad \mathbf{L}_y^\top = [1 \ \ 0] \tag{25}$$

subject to:

$$\begin{aligned} x(t+1) &= Ax(t)+Bu(t)+B_3z(t) \\ y_r(t) &= Cx(t) \\ E_3z(t) &\leq E_\Delta \Delta u(t)+E_4x(t)+E_6u(t-1)+E_5 \\ \underline{y} &\leq y_i(t) \leq \overline{y} \\ s_i &\in \{0,1\} \end{aligned} \tag{26}$$

where $\hat{y}_r \in \mathbb{R}^2$ is the output reference, $y = [S_p' \ \ y_p']'$ with S_p as the specific energy consumption, and y_p as the output particle size, $u = [\mathbf{f_s}' \ \ V_{sag}' \ \ G_p]'$ with $\mathbf{f_s}$ the solids feeding the process, V_sag the SAG mill speed, and G_p as the water added to the sump. $\Delta u(t) = u(t) - u(t-1)$, $z = [K_e]'$ with K_e as the ore hardness. $\mathbf{Q_y}$ and $\mathbf{R}$ are weighted matrices for the quadratic terms, $\mathbf{L}_y^\top$ is the weighted vector for linear optimization, and $\{\underline{y}, \ \overline{y}\}$ are the manipulated variable limits.

4. RESULTS AND DISCUSSION

Two different scenarios were be evaluated, the first one consists on simulating the stockpile discharge dynamics by switching between different granulometries for the mineral feed. The second scenario consists on disabling one secondary grinding line. For both control strategies the controller objective will be to maintain the output particle size on the mesh 65 on a determined reference, and minimize the specific energy consumption.

With the conventional MPC, one controller is used for the first scenario, not being able to follow the reference particle size on the three granulometries due to the non hybrid identification, the results are shown in Figure 5. For the second scenario, two controllers are used, one for each plant mode, and an expert supervisor coordinates the operation of both controllers, as in Figure 3. The results are shown in Figure 6. With the hybrid MPC strategy, one controller is used for both scenarios, as in Figure 4. The results for the first scenario are found in Figure 7, and for the second scenario in Figure 8.

It is important to notice that all the controller weights are the same, both for CMPC and HMPC. As an evaluation measurement, the RMS error between the desired reference for the output particle size was used, and for the specific energy consumption we calculate the average consumption for the whole experiment. The results are found in Table 1 and Table 2.

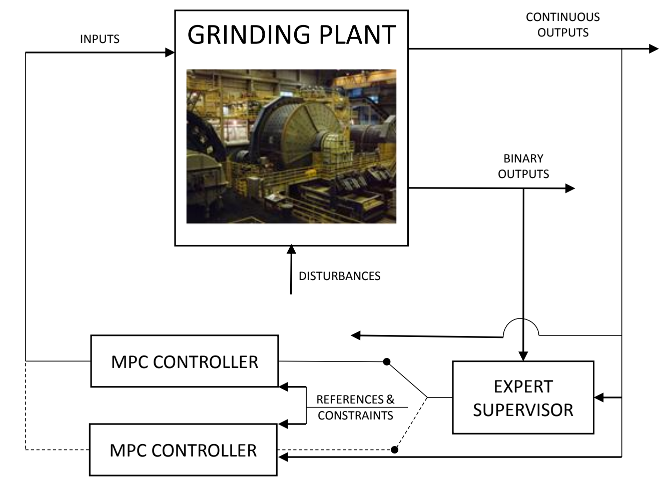

Fig. 3. MPC control strategy diagram

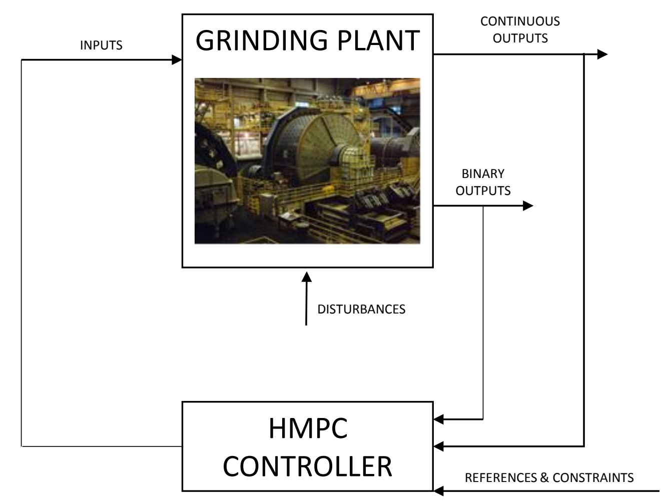

Fig. 4. HMPC control strategy diagram

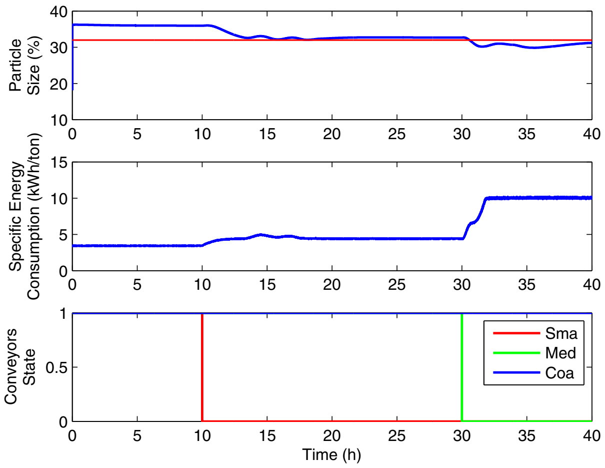

Fig. 5. Particle distribution state change CMPC

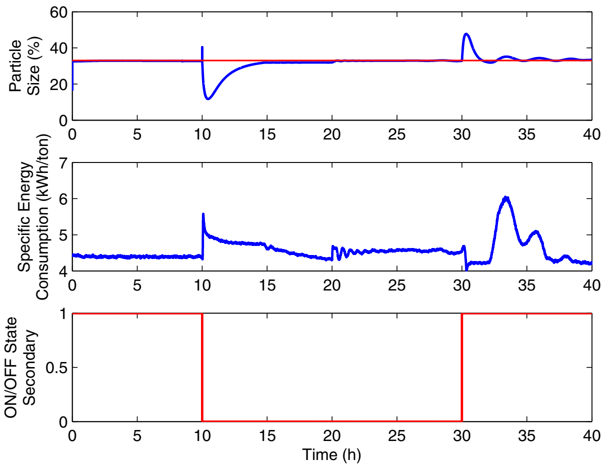

Fig. 6. Secondary line state change CMPC

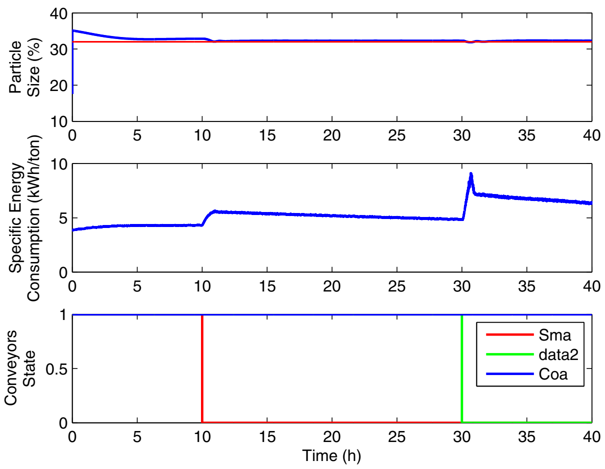

Fig. 7. Ore distribution state change HMPC

5. CONCLUSION

This paper presents the design of a hybrid model predictive control strategy for a grinding plant. It uses hybrid mod-

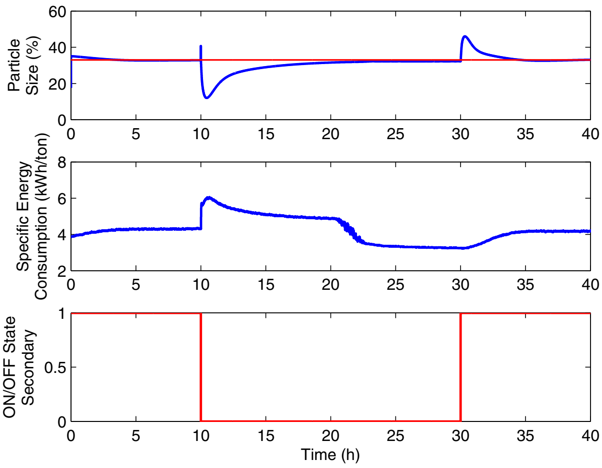

Fig. 8. Secondary line state change HMPC

Table 1. Scenario 1: Ore size distribution change

Control Strategy	RMS Error (%)	Mean Energy Consumption (kWh/ton)
CMPC	2.05	5.43
HMPC	0.73	5.34

Table 2. Scenario 2: Secondary grinding line state change

Control Strategy	RMS Error (%)	Mean Energy Consumption (kWh/ton)
CMPC	4.13	4.56
HMPC	4.48	4.24

elling, identification, optimization to achieve the results. The hybrid processes taken into account are the ON/OFF state of conveyors with different particle distribution sizes, and the ON/OFF state of a secondary grinding line.

As a comparison, a CMPC was designed to control the same plant, For the first scenario one MPC controller is used, and for the second scenario two MPC controllers, coordinated by an expert supervisor, are used. Each of the controllers, HMPC and MPC, were tuned with same weights. The disturbances that affect the different evaluated scenarios were the same on each case.

As it is seen on the first scenario, the ore size distribution change, the HMPC presents a much better solution than the CMPC, both the RMS error and the mean specific energy consumption are lower as it is seen on Table 1. On the second scenario, the ON/OFF state change of a secondary grinding line, the RMS is a little higher with HMPC compared to the CMPC, but the mean specific energy consumption is lower, this results can be seen on Table 2.

This results show that HMPC control strategy is a suitable solution for grinding processes. It offers a systematized methodology for the analysis, modelling, and development of the hybrid controllers, and it achieves better or similar results to commonly used control strategies as CMPC. Notice that the CMPC requires a tuning of every one of their controllers, and an expert supervisor to coordinate them, on certain cases each controller must be tuned individually. In the case of the HMPC, only one controller needs tuning in case of plant changes.

As future work, distributed hybrid model predictive controllers will be developed and evaluated, since large scale systems as a concentrator plant can use them to increase the robustness of the controller.

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