Memorias del XVI Congreso Latinoamericano de Control Automático, CLCA 2014
Octubre 14-17, 2014. Cancún, Quintana Roo, México

Extremum-seeking Control for Anaerobic Digestion Process via Sliding Mode Approach

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Abstract: This paper proposes an extremum-seeking control approach with sliding mode to achieve the dynamic optimization of methane outflow rate in continuous anaerobic digestion processes. From the open-loop analysis for a two-population model, it is shown that there exist operational unstability due to the accumulation of volatile fatty acids (VFA) into the digester. Then, the controller is designed to achieve the regulation of VFA concentration around at an optimal value while maximizing the methane production. The control law comprises a variable-structure feedback to iteratively extremize the methane outflow rate and converges to the vicinity of the optimal value with sliding mode motion. In contrast with previous works on the extremum-seeking controllers with sliding mode, the control scheme includes a high-gain observer-based uncertain estimator which computes the unknown terms related to the growth kinetics and the inlet composition. Practical stabilizability for the closed-loop system around to an unknown optimal set-point is analyzed. Numerical experiments illustrate the effectiveness of the proposed control approach.

Keywords: Anaerobic digestion process, Extremum-seeking control, Sliding mode, Robust variable-structure control.

1. INTRODUCTION

Anaerobic digestion (AD) has gained considerable importance lately as a wastewater treatment technology to reduce organic matter in agro-food industries and municipal effluents. At the same time AD produces biogas, consisting firstly on methane and carbon dioxide and is widely used as a source of renewable energy. Nevertheless, its widespread application has been limited, because to the difficulties involved in achieving stable operation of the AD process Hess & Bernard (2008); Méndez-Acosta et al. (2008). From the power generation viewpoint, the optimization of methane outflow rate is one of the key issues in the operation of anaerobic processes. However, the optimal operation of AD process is complicated to reach, mainly due to: their highly nonlinear and unstable nature, inhibition by substrates or products and by the substantial unmodeled dynamics (Hess & Bernard, 2008; Sbarciog et al., 2010; Serhani et al., 2011; Shen et al., 2007). It is well known that the inhibition of the methanogenic bacteria growth by accumulation of volatile fatty acids (VFA) provokes the acidification of the anaerobic process (Hess & Bernard, 2008; Méndez-Acosta et al., 2005; Shen et al., 2007). Hence, AD process is suitable candidate for using optimal and robust stabilization schemes.

The control objective in some biotechnological processes is to optimize a performance function that can be a function of unknown parameters in order to keep a performance variable at its optimal value (Dochain et al., 2011). It is well common that the explicit form of the performance function in bio-processes is highly uncertain (e.g., the growth rate or the specific metabolites production). Self-optimizing control or extremum-seeking control are two techniques to handle these kinds of dynamic optimization problems (Ariyur & Krstic, 2003; Dochain et al., 2011). The goal of extremum seeking schemes is to find the operating set-points, a priori unknown, such that a performance function reaches their extremum value (Guay et al., 2004). An intensive research activity has been developed by Dochain et al. (see, e.g. Dochain et al. (2011)) in to design adaptive extremum seeking control schemes applied to biotechnological processes. However, the model-based adaptive extremum seeking algorithms require prior information about: (a) the models for the population's growth rates (as Haldane, Monod, or Cointois model) and (b) bounds of the parameters which, in most biochemical processes, is hard to obtain from available data (Dochain et al., 2011; Wang et al., 1999). On the other hand, the extremum-seeking control problem has been studied in the sliding mode control framework (see, e. g. Drakunov et al.

⋆ This work was partially supported by the project: "SPectroscopies for the Evaluation and ConTrol in Real time of biological cells physiological statE *(SPECTRE)*.

(1995); Haskara et al. (2000)). In such contributions, the main idea is to ensure that the desired output (or performance function) follow an increasing time signal as close as possible to its extremal value via discontinuous controls and sliding mode motions (Drakunov et al., 1995; Haskara et al., 2000; Pan et al., 2003; Utkin, 1992). Nevertheless, as far as we know, there is no extreme-seeking schemes based on sliding mode techniques for the dynamic optimization of AD process.

In this paper, we propose an extremum-seeking control scheme with sliding mode to achieve the dynamic optimization of methane outflow rate in anaerobic processes. The control law is designed to regulating the VFA concentration at the optimal value while maximizing the methane production. The VFA concentration and methane outflow rate are considered available for on-line measurement, and the dilution rate was taken as the control input. The first step in the controller design is an "ideal" optimum seeking controller which is found from the extremum-seeking control with sliding mode techniques (Haskara et al., 2000; Pan et al., 2003). The ideal control allows us to reach at extremum of the methane outflow rate and converges to the vicinity of the optimal VFA concentration with sliding mode motions and oscillations. In the second step, a highgain observer-based uncertainty estimator is used to approach the unknown terms in the ideal control law. Thus, the practical stabilizability in the neighborhood of the unknown optimal set-point is ensured when the estimator scheme and controller are coupled. The rest of the paper is organized as follows: In Section 2 the dynamic model for AD process is presented and some issues related with operational stability at the optimal conditions are shown, also the control problem is formulated. Section 3 contain the design of the extremum-seeking controller and the stability analysis of the closed-loop system is shown. Numerical experiments that illustrate the performance and robustness of the proposed control approach are shown in Section 4. Finally, some concluding remarks are discussed in Section 5.

2. MODEL DESCRIPTION AND PROBLEM STATEMENT

Throughout this paper the reduced version of the AD model developed by Bernard et al. (2001), is used in to design of the proposed control scheme. The underlying model assumes two main bacterial populations, the first one, called acidogenic bacterial X_1, consumes organic substrate S_1 (total soluble Chemical Oxygen Demand COD except Volatile Fatty Acids VFA) and produces VFA, that is considered as secondary substrate S_2 through an acidogenesis stage. The second population, known as methanogenic bacteria X_2, uses VFA as substrate in a methanization stage for growth and produces methane and carbon dioxide. From a mass balance in an ideal Continuous Stirred Tank Reactor (CSTR), the system dynamics is given by the four-dimensional dynamical system

$$\dot{S}_1 = D(S_{1f} - S_1) - k_1\mu_1(S_1)X_1 \quad (1)$$

$$\dot{X}_1 = \mu_1(S_1)X_1 - aDX_1 \quad (2)$$

$$\dot{S}_2 = D(S_{2f} - S_2) + k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2 \quad (3)$$

$$\dot{X}_2 = \mu_2(S_2)X_2 - aDX_2 \quad (4)$$

The state vector is defined as $\xi = [S_1X_1S_2X_2]' \in \Re^4$, S_{1f} and S_{2f} denotes the inlet concentration of the primary organic substrate and VFA, respectively. The dilution rate D, is defined as the ratio between the feeding flow F with respect to the digester volume V, while k_1, k_2, k_3 are constant yield coefficients. The parameter $a \in [0, 1]$ represents the proportion of the bacterial population that are affected by the digester dilution; so, $a = 0$ and $a = 1$ correspond to an ideal fixed bed reactor and to an ideal CSTR, respectively (Bernard et al., 2001; Méndez-Acosta et al., 2005). Because of the very low solubility of methane in the liquid phase, the concentration of dissolved methane is neglected, and the produced methane is assumed to go directly out of the digester, with the outflow rate of methane gas Q proportional to the reaction rate of the methanogenesis, (see, Bernard et al. (2001))

$$Q(\xi) = k_4\mu_2(S_2)X_2 \quad (5)$$

where k_4 is the yield for the methane production. With respect to the specific growth rates for the acidogenic and methanogenic populations, in Bernard et al. (2001) are assumed to be described by the Monod and Haldane expressions, respectively, i.e.,

$$\mu_1(S_1) = \frac{\mu_{1,max}S_1}{K_{S1} + S_1} \quad (6)$$

$$\mu_2(S_2) = \frac{\mu_{2,max}S_2}{K_{S2} + S_2 + S_2^2/K_{I2}} \quad (7)$$

where $\mu_{1,max}, K_{S1}, \mu_{2,max}, K_{S2}$, and K_{I2} are the maximum bacterial growth rate and the half-saturation constant associated to the substrate S_1, the maximum bacterial growth rate in the absence of inhibition, and the saturation and inhibition constants associated to substrate S_2, respectively.

The dynamical characterization of the uncontrolled AD process (1-5) is based from the analysis developed in (Hess & Bernard, 2008; Sbarciog et al., 2010). Firstly, if we focused on the subsystem (1-2) it is clear that is closed to a classical two-dimensional chemostat model with monotonic kinetics (Monod-like). Then, the dynamical behavior of (1-2) remains simple as stated in the following Property:

Property 1. *(Hess & Bernard, 2008) If $aD < \mu_1(S_{1f})$, the System (1-2) with positive initial condition admit a single globally asymptotically stable equilibrium $(\bar{S}_1, \bar{X}_1) \in \Re^2_+$ in the positive orthant, and, consequently, after a transient time T, (1-2) satisfy the inequality $k_1\mu_1(S_1)X_1 \leq DS_{1f}$.*

Since the methanogenesis is slower and can be inhibited it turns out to be the limiting step (see, Hess & Bernard (2008); Sbarciog et al. (2010)). We now consider the methanogenic system given by (3-4) after a period greater that T (Property 1). From the Proposition 1, we can inferred that the total concentration of VFA available for the methanogenesis stage is $S_{2f} + \frac{k_2}{D}\mu_1(S_1)X_1 \leq S_{2f} + \frac{k_2}{k_1}S_{1f} = \tilde{S}_{2f}$. In order to study the methanogenesis stage as a stand-alone process, we consider $\tilde{S}_{2f}$ as a worst-case upper bound of the total concentration of VFA in the digester, i.e., $S_2(t) < \tilde{S}_{2f}$. Thus the methanogenic system (3-4) is reduced to a one-stage process independent of the acidogenic stage, whose dynamical behavior is given by (for more details see Hess & Bernard (2008))

$$\dot{S}_2 = D(\tilde{S}_{2f} - S_2) - k_3\mu_2(S_2)X_2 \quad (8)$$

$$\dot{X}_2 = \mu_2(S_2)X_2 - aDX_2 \quad (9)$$

It is easy to see that the system (8-9) is close to a generic two-dimensional chemostat model with a non-monotonic kinetics (Haldane-like).

Now, in regard to the optimal operating conditions for AD process, in the sense of maximizing the methane production Q, we have the following. The goal is to hold the concentration of VFA at the critical value S_2^*, in where the growth rate of methanogenic population reach their maximum (Properties 2 and 3). In this way, the following corollary establishes the structural properties of the methanogenic system (8-9) at $aD = \bar{\mu}_2$, that corresponds to the optimal equilibrium of the global AD process in $\bar{\xi}^* = [\bar{S}_1, \bar{X}_1, S_2^*, X_2^*]'$, with $X_2^* = k_3^{-1}(\tilde{S}_{2f} - S_2^*)$ (see Fig. 7).

Corollary 1. (Lara-Cisneros et al., 2012) The dynamical system (8-9) with positive initial condition, where μ_2 satisfies the Properties 2 and 3, is structurally unstable for open-loop configuration at $aD = \overline{\mu}_2$.

From the above result, we can see that the optimal operation of the AD process, in the sense to maximizing the methane production Q, occurs on structural unstable conditions in open-loop configuration. In other words, small changes in the AD environment or external disturbances for system (1-4) operating close to optimal conditions (i.e., in $\bar{\xi}^* = [\bar{S}_1, \bar{X}_1, S_2^*, X_2^*]'$) may be lead to undesirable acidification of the process. Thus, the goal is to design a robust extremum-seeking scheme to achieves the practical stabilization for (1-4) around of the optimum VFA concentration value S_2^* (a priori unknown or highly uncertain) while maximizing the methane production given by (5), despite uncertainty in the dynamic model and load disturbance in the influent concentration.

3. CONTROLLER DESIGN

In this section, a control approach to achieve the stabilization of the VFA concentration around of the optimal (and uncertain) setpoint S_2^* while maximizing the methane production is designed. The design of the extremum-seeking control scheme will proceed in different steps. As first designing step, an "ideal" optimum seeking controller is found. In order to design the optimum seeking controller the following assumptions are considered.

Assumption 2. Only the VFA concentration S_2 and the outflow rate of methane gas Q are available for on-line measurement.

Assumption 3. The influent concentration S_{1f} and S_{2f} are piecewise constants, bounded and uncertain functions (load disturbance).

Assumption 4. The growth kinetics for the acidogenic and methanogenic stages, μ_1, μ_2 are: smooth, bounded and uncertain functions (modeling errors).

Let us consider the following result developed by (Haskara et al., 2000; Pan et al., 2003).

Theorem 5. (Pan et al., 2003) Consider the SISO nonlinear dynamical systems $\dot{x} = f(x, u)$ with the performance function $y = F(x)$ where $x \in \Re^n$, $u \in \Re$ and $y = \Re$, where the static performance map at closed-loop equilibrium point $x_e(\theta)$ from θ to y, represented by $y = F(x_e(\theta)) = F(\theta)$ is smooth and has an unique maximum point at θ^*. Then the extremum-seeking controller

$$\dot{\theta} = \lambda sgn\left[\sin(\frac{\pi}{\alpha}z(t))\right] \quad (10)$$

with $\lambda, \alpha > 0$, and the switching function $z(t)$ is defined as $z(t) = y(t) - g(t)$ and $g(t)$ is a monotically increasing function satisfying $\dot{g} = \rho > 0$, will converge to the maximum θ^* with switching among the sliding modes $z(t) = c_i$ $(i = 0, 1, ...)$ while the parameter variable $\theta(t)$ oscillates inside the δ-vicinity $\Theta_\delta = \{\theta \in \Re | \theta^* - \delta \leq \theta \leq \theta^* - \delta\}$, and the performance function $y(t)$ oscillates within $y(\theta^*)$ and $y(\theta^* \pm \delta)$ if parameters of the extremum-seeking controller are chosen to satisfy

$$\tfrac{\rho}{\lambda} < \tfrac{\alpha}{2\delta}$$

Now with respect to AD process (1-4), in (2) has been shown that if the Property 1 holds, then (1-4) admit the equilibrium $\bar{\xi} = [\bar{S}_1, \bar{X}_1, \bar{S}_2, \bar{X}_2]'$. Also, since $(\bar{S}_1, \bar{X}_1) \in \Re_+^2$ is a globally asymptotically stable equilibrium for the subsystem (1-2), and, $\bar{X}_2$ depends of $\bar{S}_2$ as, $\bar{X}_2 = k_3^{-1}(\tilde{S}_{2f} - \bar{S}_2)$, then we can focus on to design a controller to extremize the VFA concentration at the optimal set-point S_2^*. In this way, using ideas from modeling-error compensation techniques (Álvarez-Ramírez., 1999; Méndez-Acosta et al., 2005), let us define a function for lumping the uncertainties in (3) as follows: $\eta(t) \equiv k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2 + \Delta S_{2f}D$, where ΔS_{2f} is an uncertain and bounded function that represents the variation of the influent composition around to a well-known nominal value of the inlet VFA concentration $\overline{S_{2f}}$, such that $S_{2f} = \overline{S_{2f}} + \Delta S_{2f}$ (Assumption 2). Then, once this function is defined, it is possible to rewrite the equation (3) as follows

$$\dot{S}_2 = D(\overline{S_{2f}} - S_2) + \eta \quad (11)$$

Now, based on the extremum-seeking control with sliding mode techniques (Haskara et al., 2000; Pan et al., 2003), the follows control law is proposed such that the system (1-4) converges to around of the optimal equilibrium $\bar{\xi}^* = [\bar{S}_1, \bar{X}_1, S_2^*, X_2^*]'$.

Proposition 6. Consider the AD system (1-4) and suppose that Property 1 and Assumptions 1-3 are satisfied. Then the extremum-seeking control law

$$D = \frac{1}{\overline{S_{2f}} - S_2}\left[-\eta + \lambda sgn\left(\sin(\frac{\pi}{\alpha}z)\right)\right]; \qquad \lambda, \alpha > 0 \quad (12)$$

where $z(t) = Q(t) - g(t)$ with $\dot{g}(t) = \rho > 0$, will converge to the maximum S_2^* with switching among sliding modes $z(t) = c_i$ $(i = 0, 1, ...)$ while the VFA concentration S_2 oscillates inside the δ-vicinity $S_{2\delta} = \{S_2 \in \Re | S_2^* - \delta \leq S_2 \leq S_2^* - \delta\}$, while $\frac{\rho}{\lambda} < \frac{\alpha}{2\delta}$.

Proof : By replacing D from (12) in (11) yields

$$\dot{S}_2 = \lambda sgn\left[\sin(\frac{\pi}{\alpha}z(t))\right] \quad (13)$$

Thus, from the open-loop analysis (2) and the Theorem 1, it is direct to see that the control law (12) achieves the convergence of the VFA concentration at the δ-vicinity $S_{2\delta}$. $\square$

The extremum-seeking control (12) requires perfect knowledge of the uncertain function $\eta(t)$. However, the kinetic model for the growth bacterial population is highly uncertain for most of real biotechnological processes, as well as the load disturbances in the influent composition. To solve this drawback, in this design step an estimation scheme consisting of a high-gain observer-based uncertainty estimator is proposed. Departing from the ideas previously reported by Álvarez-Ramírez et al. (see, e.g. Álvarez-Ramírez. (1999)), the uncertain term η is seen as a new state whose dynamics can be reconstructed from measurements of the input and the output signals. From (11) it is seen that the lumping uncertainty term η can be expressed as a function of the control input D, the nominal value $\overline{S_{2f}}$, the measurable signal S_2 and their time derivative $\dot{S}_2$, as $\eta = \dot{S}_2 - D(\overline{S_{2f}} - S_2)$, which evidences a kind of strong observability (Álvarez-Ramírez., 1999). Hence, η can be estimated by the following high-gain Luenberger-type observer

$$\dot{\hat{S}}_2 = D(\overline{S_{2f}} - \hat{S}_2) + \hat{\eta} + \kappa_1 L(S_2 - \hat{S}_2) \qquad (14)$$

$$\dot{\hat{\eta}} = \kappa_2 L^2 (S_2 - \hat{S}_2) \qquad (15)$$

where $\hat{S}_2$, $\hat{\eta}$ denotes the estimate for VFA concentration and the lumping uncertainty state η, respectively. The observer parameters $\kappa_{1,2}$ are chosen such that the polynomial $P(\lambda) = \lambda^2 + \kappa_1\lambda + \kappa_2$ is Hurwitz, and $L > 0$ is a positive parameter (high-gain observer). Finally, the robust extremum-seeking control with sliding mode is given by

$$D = \frac{1}{\overline{S_{2f}} - S_2}\left[-\hat{\eta} + \lambda sgn\left(\sin(\frac{\pi}{\alpha}z)\right)\right]; \qquad \lambda, \alpha > 0 \quad (16)$$

In this way, the resulting extremum-seeking scheme comprises the variable structure feedback (16), where the uncertain function η is computed by the dynamic uncertain estimator (14-15). The next result, departing from the concept of practical stabilizability, proves that the robust control scheme (14-16) can take the trajectories of the system (1-4) arbitrarily close to the optimal set-point S_2^*.

Proposition 7. Consider the AD system (1-4) and suppose that Property 1 and Assumptions 1-3 are satisfied and $\frac{\rho}{\lambda} < \frac{\alpha}{2\delta}$ is holds. Then, the variable-structure control scheme (14-16) achieves the practical stabilization around to the optimal set-point S_2^*.

Proof : From Proposition 1, the control law (12) achieves the convergence of VFA concentration at the δ-vicinity $S_{2\delta} = \{S_2 \in \Re | S_2^* - \delta \le S_2 \le S_2^* - \delta\}$ and, consequently, achieves the substrate stabilization to the optimal set-point S_2^*. However, it cannot be directly implemented due to Assumption 2 and 3. Then, a dynamic uncertainty estimation method (14-15) is proposed. Let $e(t) = [S_2 - \hat{S}_2, \eta - \hat{\eta}]'$ be the estimation error vector; since the observer parameters $\kappa_{1,2}$ are chosen such that the polynomial $P(\lambda) = \lambda^2 + \kappa_1\lambda + \kappa_2$ is Hurwitz, with $L > 0$ a positive parameter (high-gain observer). Then, is follows that the estimator scheme (14-15) guarantees the convergence of $e(t)$ to an arbitrary small neighborhood of the origin (i.e., $e(t) \to \epsilon$ as $t \to \infty$, with an estimation error of order $O(1/L)$). This implies that the performance of the robust controller (16) will tend asymptotically at "ideal" control law (12) as the time tends to infinity. And moreover, the controller (14-16) leads the trajectories of the system (1-4), arbitrarily close to the δ-vicinity $S_{2\delta}$. As a consequence, under the Property 1 Assumption 1-3, and the condition $\frac{\rho}{\lambda} < \frac{\alpha}{2\delta}$ is holds, the extremum-seeking controller (14-16) will be able to achieve the practical substrate stabilization around to the optimal concentration S_2^*. $\square$

4. NUMERICAL IMPLEMENTATION

The aim of this section is to illustrate the performance of the extremum-seeking controller (14-16) by numerical experiments performed using the nominal AD model proposed by Bernard et al. (2001). he parameter values for the model coefficients are shown in Table 1. The controller parameters are set to the following values: $\lambda = 0.8$, $\alpha = 1.0$, $\rho = 0.15$, $\kappa_1 = 1$, $\kappa_2 = 2$ and $L = 0.7$.

Table 1. Nominal parameter values used in the numerical simulations (Bernard et al., 2001).

Parameter	Value	Units
k_1	42.14	g/g
k_2	116.5	$mmol/g$
k_3	268	$mmol/g$
k_4	453	$mmol/g$
μ_{1max}	0.05	h^{-1}
μ_{2max}	0.031	h^{-1}
K_{S1}	7.1	g/L
K_{S2}	9.28	$mmol/L$
K_{I2}	16	$mmol/L$
a	0.5	$--$
$\overline{S_{1f}}$	10	g/L
$\overline{S_{2f}}$	80	$mmol/L$

The performance of the control scheme (14-16) has been tested under different operating conditions (Figures 1-3). Firstly, the stabilization to the δ-vicinity $S_{2\delta}$ for distinct initial conditions is shown in Figure 1. It is observed that the trajectories of the closed-loop system are confined to δ-vicinity of the optimal set-point S_2^*, while the outflow rate of methane Q lies on the neighborhood of their maximum value. Global performance of (14-16) is shown in Figure **??** where the VFA concentration converges around to the unknown optimal set-point S_2^* in a finite time interval. It is shown from Fig. **??** that the closed-loop system has a high chattering with sliding mode motions and oscillations, that is common on extremum-seeking controllers with sliding mode (Haskara et al., 2000; Utkin, 1992). Here it is important to remark that the outflow rate of methane is carried to their maximum with oscillations (see Fig. **??**b). In below side, we can see that the corresponding control input signal presents a large control effort (Fig. **??**c). The performance of the extremum-seeking approach may be improved by including smooth sliding mode methods or by the low-pass filter techniques, this issue might be addressed in future contributions. As is known, the high-gain observers may destabilize the closed-loop system when the observer gain L tends to infinity. The peaking phenomenon has been studied by means the well-known peaking avoidance technique based on saturation functions, as well as with antireset windup schemes in Méndez-Acosta et al. (2005).

In order to illustrate the robust stabilization at the δ-vicinity despite changes in the influent composition, in the Fig. **??** is shown the controller performance in these conditions. As expected, the VFA concentration is regulated at the optimal set-point despite the load disturbances in the inlet composition. One of the central issue in extremum-seeking schemes is the speed of convergence. In the proposed approach the convergence speed may be modulated by means of tuning the control parameters λ and ρ taking into account the restriction $\frac{\rho}{\lambda} < \frac{\alpha}{2\delta}$.

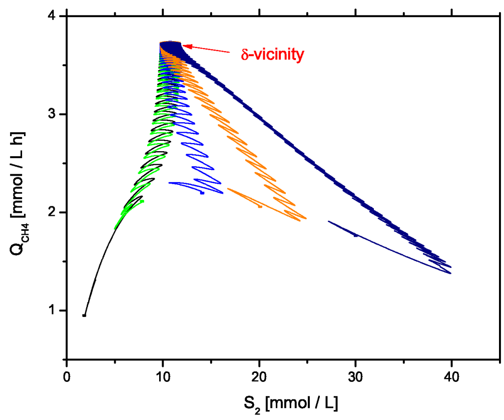

Fig. 1. Stabilization to the δ-vicinity $S_{2\delta}$ for distinct initial conditions.

5. CONCLUSION

An extremum-seeking control approach with sliding mode has been developed to ensure practical stabilization around at the uncertain optimal VFA concentration, while maximizing the methane outflow rate in anaerobic digestion (AD) processes. The practical stabilizability of the AD process around to the optimal set-point has been analyzed for the closed-loop system. The proposed control scheme achieves the optimal convergence despite the modeling-errors in the growth rates and the influent composition. Numerical experiments showed the effectiveness of the proposed extremum-seeking approach. The design of this kind of extremum-seeking controllers for optimal stabilization of biotechnological processes needs to be looked into more deeply in the context of experimental implementation.

ACKNOWLEDGEMENTS

This work was partially supported by the project: "SPectroscopies for the Evaluation and ConTrol in Real time of biological cells physiological statE *(SPECTRE)*. G. Lara-Cisneros thanks to CONACyT by the postdoctoral research grant.

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