Maglev Tracking Control by a State-Feedback with Integral Action and Robust Velocity Reconstruction

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Abstract: In this work we present a design of a LQR control for a magnetic levitator system with integral term and full-state feedback with robust exact reconstruction of the velocity for regulation and tracking, considering external perturbations and non-modeled dynamics, as well as noisy measurements. Simulation and experimental results are presented in order to show the effectiveness of the presented scheme.

Keywords: Optimal control, Robust Exact Differentiatior, Maglev System, State-Feedback.

1. INTRODUCTION

The magnetic levitation system has been a widely used benchmark model prototype due to its unstable nature and therefore the challenging control design it represents, analyzed on both its nonlinear model and linear approximations, as indicated in Ollervides et al. (2005); Zhao et al. (1999), that even lead to more complex applications (Sinha and Pechev, 1999). The model typically consists on a metallic ball that is levitated by the magnetic flux generated by a coil that compensates the gravitational force, and usually three variables are considered: the position of the ball to the coil core face, its velocity and the electric current. Due to the nonlinearities inherent to the system, the noise in the data measurement and the instability of the equilibrium point the velocity measurement, the parameter uncertainties as well as the calculation of the derivative term in the proposed PID controls are some of the challenges that need to be overcame in a control design.

Several control strategies have been proposed in the literature, and generally some experimental challenges are in order, as only some variables, usually the ball position and the electric current are easily measured. Some approaches consider a Proportional-Integral-Derivative (PID), under the assumption of an exact knowledge of the parameters of the system, as well as state-feedback using a linearized model around an equilibrium point, as those reported by Barie and Chiasson (1996) and Ahmad and Javaid (2010), where an observer is used to reconstruct the velocity and compare it with a nonlinear state-feedback. Other works, such as the one reported by Morales et al. (2011), proposes a Generalized Proportional-Integral (GPI) controller plus an online parameter identification; however, the electric dynamics are neglected and simplified into an algebraic relation.

Other approach is given by Ortega (1998), where a passivity-based control is proposed considering the electric dynamics, taking into account that also the inductance of the electromagnet changes due to the position of the ball, leading to a more accurate model. However, as the current could lead to a singularity in the control scheme, it is changed so that it depends instead on the magnetic flux, and eventually only depending on the ball position, but depending on a knowledge of the system parameters. To deal with the parameter uncertainties, a backstepping technique has been proposed (Huang et al. (2000)), adaptive feedback and a nonlinear damping term (Yang and Tateishi (2001)), or sliding mode control (Al-Muthairi and Zribi (2004)).

However, the problem of speed reconstruction in electromechanical systems is a challenge that may limit the correct application of control techniques. Either by their lack of robustness, the existence of the picking phenomena or by their asymptotic convergence, some standard techniques cannot be effectively applied to solve the problem.

To overcome this limitation, high-gain differentiators (Atassi and Khalil (2000)) have been proposed to reconstruct the derivatives on signals and/or states, and are not exact with any fixed finite gain and feature the peaking effect with high-gains. The maximal output value during the transient grows infinitely as the gains tend to infinity (see, for example, Barbot et al. (2002), Poznyak (2003) and Xian et al. (2004)). Finite-time convergent techniques such as the first-order robust exact differentiator (Levant (1998)) can be used here, but its successive application is cumbersome and not effective.

The arbitrary-order robust exact sliding-mode-based differentiator (Levant (2003)) provides for the rth derivative accuracy proportional to the discretization step, and provides for the accuracy $\varepsilon^{1/(r+1)}$. Its straight-forward application requires the boundedness of the rth order derivative. In practice it means that velocities and/or accelerations should be bounded. As it is easy to understand, these restrictions are always satisfied in electro-mechanical systems.

In this work, in order to deal with the problem of the reconstruction of the velocity required in an usual state-space linear feedback control, we present a linear optimal control design to control the position of the maglev ball using the current and position measurements. We propose to reconstruct the velocity from robust sliding-mode based differentiator and an integral term to compensate the tracking error as well as the parameter and model uncertainties and non-modeled dynamics, with simulated and experimental results.

2. MAGNETIC LEVITATOR MODEL

The maglev system is shown in Fig. 1, where $V_c(t)$ is the control input to the coil, $i_c(t)$ its electric current, L_c its inductance, R_c its resistance, R_s the current sensor resistance and m the ball mass.

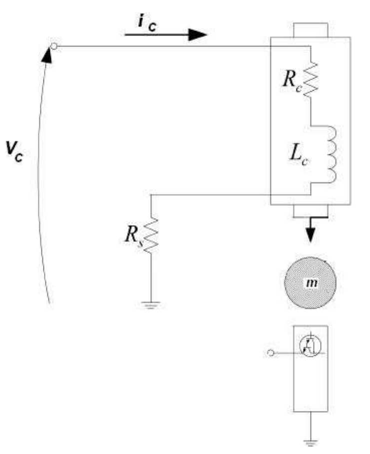

Fig. 1. Schematic diagram of the maglev system, taken from the Quanser User Manual

Applying Kirchhoff's Voltage Law, we obtain

$$V_c(t) = Ri_c(t) + L_c\frac{d}{dt}i_c(t) \tag{1}$$

where $R = R_c + R_s$. The magnetic force of attraction from the coil to the ball is given by the approximation $F_m = \frac{1}{2}\frac{K_m i_c^2}{z^2}$, where z is the ball position measured from the coil's face and K_m depends on the magnetic properties and geometry of the ball. This leads to the force equilibrium equation:

$$m\ddot{z} = -\frac{1}{2}\frac{K_m i_c^2}{z^2} + mg \tag{2}$$

where g the acceleration due to gravity. This leads to the nonlinear approximated model:

$$\dot{x} = \begin{bmatrix} -\dfrac{R}{L}x_1 + \dfrac{1}{L_c}V_c(t) \\ x_3 \\ g - \dfrac{K_m}{m}\left(\dfrac{x_1}{x_2}\right)^2 \end{bmatrix} \tag{3}$$

where $x = [i_c(t)\ z(t)\ \dot{z}(t)]^T$ is the state vector.

2.1 Linearized model

The equilibrium point of the system is given by

$$-\frac{1}{2}\frac{K_m I_{eq}^2}{Z_{eq}^2} + mg = 0 \tag{4}$$

where Z_{eq} and I_{eq} are the position and the electric current at the equilibrium, parameterized as:

$$I_{eq} = \sqrt{2}\sqrt{\frac{mg}{K_m}}Z_{eq} \tag{5}$$

It can be seen that the K_m can be obtained from:

$$K_m = \frac{2mgZ_{eq}^2}{I_{eq}^2} \tag{6}$$

Defining the translated system

$$\bar{x}(t) = \begin{bmatrix} \bar{i}(t) \\ \bar{z}(t) \\ v(t) \end{bmatrix} \begin{bmatrix} i(t) - I_{eq} \\ z(t) - Z_{eq} \\ v(t) \end{bmatrix},\ \bar{V}_c = V_c(t) - V_{c-eq} \tag{7}$$

we obtain the linearized system around the equilibrium point, where $V_{c-eq} = RI_{eq}$ is the input on the equilibrium point.

$$\dot{\bar{x}} = \begin{bmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & 0 & 1 \\ -\frac{K_m\sqrt{\frac{2\,m\,g}{K_m}}}{m\,Z_{eq}} & \frac{2\,g}{Z_{eq}} & 0 \end{bmatrix}\bar{x} + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix}\bar{V}_c \tag{8}$$

3. CONTROL DESIGN

The control is designed using an optimal control design for the system (8) such that minimizes the quadratic cost function

$$J = \int_0^\infty \bar{x}^T(\tau)Q\bar{x}(\tau) + R\bar{V}_c^2(\tau)d\tau \tag{9}$$

with the control $V_c(t) = -k\bar{x} + V_{c-eq} + u_i(t)$.

From the classic literature (Hendricks et al. (2008)), it is known that the solution is given by the Algebraic Ricatti Equation (ARE)

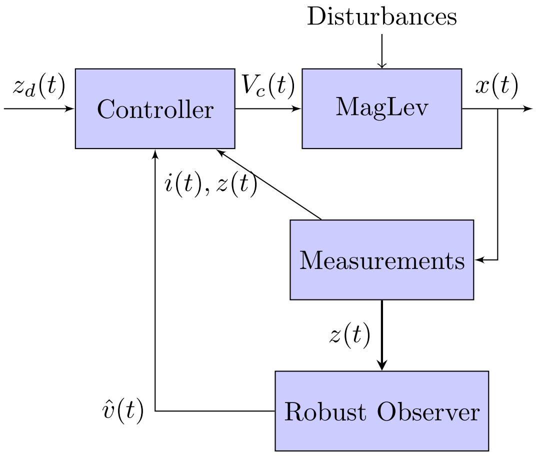

Fig. 2. Block diagram of the controller

$$A^T P + PA + Q - PBR^{-1}B^T P = 0, \tag{10}$$

and the gain is given by

$$k = R^{-1}B^T P \tag{11}$$

As the model presents parameter uncertainties as well as non-modelled dynamics, an integral term $u_i(t)$ in the control is considered

$$u_i(t) = k_i \int_0^{\infty} (z(t) - z_d(t))dt \tag{12}$$

where $z_d(t) = Z_{eq} + r(t)$, $k_i \in \Re$, $r(t) \in \Re$ an bounded external reference signal. Finally, the obtained control strategy is given by:

$$V_c(t) = -k_1 i(t) - k_2 z(t) - k_3 v(t) + kX_{eq} + u_i(t) \tag{13}$$

where $X_{eq} = [I_{eq}\ Z_{eq}\ 0]^T$. As the system does not directly allow to measure the velocity $v(t)$, it is necessary to reconstruct it. The velocity of the system can be estimated by the application of the arbitrary-order robust exact differentiator (Levant (2003)) in finite-time, such that the control is

$$V_c(t) = -k_1 i(t) - k_2 z(t) - k_3 \hat{v}(t) + kX_{eq} + u_i(t),\ t \geq t_1 \tag{14}$$

with t_1 finite, that is discussed in the following subsection.

3.1 Arbitrary-order robust exact differentiator

With this aim, let us use in this subsection, z_i, v_i to denote the variables of the differentiator, and let f_0 be a Lebesgue-measurable signal to be differentiated. The nth order differentiator can be expressed in the following form:

$$\begin{aligned}
\dot{z}_0 &= v_0 = z_1 - \kappa_n |z_0 - f_0|^{\frac{n}{n+1}}\ sign(z_0 - f_o), \\
\dot{z}_1 &= v_1 = z_2 - \kappa_{n-1} |z_1 - v_0|^{\frac{n-1}{n}}\ sign(z_1 - v_0), \\
&\vdots \\
\dot{z}_i &= v_i = z_i - \kappa_{n-i} |z_i - v_{i-1}|^{\frac{n-i}{n-i+1}}\ sign(z_i - v_{i-1}), \\
&\vdots \\
\dot{z}_n &= -\kappa_1\ sign(z_n - v_{n-1})
\end{aligned} \tag{15}$$

for suitable positive constant coefficients κ_i to be chosen recursively large in the given order. Under the assumption that a constant M exists such that $|f_0^{(n)}| \leq M$, a possible selection of the given constant is $\kappa_1 = 1.1M$, $\kappa_2 = 1.5M^{1/2}$, $\kappa_3 = 2M^{1/3}$, $\kappa_4 = 3M^{1/4}$, $\kappa_5 = 5M^{1/5}$, $\kappa_6 = 8M^{1/6}$; however, different values can be used. The following equalities are true in the absence of measurement noise after a finite time transient process (Levant (2003)):

$$|z_i - f_0^{(i)}(t)| = 0 \qquad i = 0, ..., n \tag{16}$$

It was demonstrated by Levant (2003) that non-idealities like measurement noise and finite frequency commutation cause a bounded error in the estimated derivatives and, as a result, a bounded loss of accuracy for the controller that uses the "noisy" derivative estimates.

In particular, let consider $f(t)$ the signal, affected by noise, to be differentiated. If the input noise satisfy the inequality $|f(t) - f_0| \leq \epsilon$. Then the following inequalities are established after a finite time transient:

$$|z_i - f_0^{(i)}| \leq \mu_i \epsilon^{(n-i+1)/(n+1)}, \qquad i = 0, ..., n$$

for positive constants μ_i depending on the parameters of the differentiator.

It is interesting to note that the velocity reconstruction is independent from the dynamics of the rest of the variables of the system.

4. RESULTS

The proposed controller with a robust exact differentiator was implemented both on simulation as well as in an experimental framework, in order to check the performance on both *ideal* and real conditions.

4.1 Simulation results

The simulation was run using MATLAB 2013a software, with an i5-Core Intel processor PC. The considered parameters for the model are depicted in Table 1. In this case, the model was taken with reference to the equilibrium point given by $Z_{eq} = 6$ mm, resulting in the linearized system:

$$\Sigma_{(X_{eq},U_{eq})} : \left[\begin{array}{ccc:c} -0.2424 & 0 & 0 & 2.4242 \\ 0 & 0 & 1 & 0 \\ -23.19 & 3260 & 0 & 0 \\ \hdashline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right], \tag{17}$$

and the matrix in the cost function were taken as

$$Q = \begin{bmatrix} 10^3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ R = 1. \tag{18}$$

By solving the appropiate Ricatti equation $A^T P + PA + Q - PBR^-1B^T P = 0$, the resulting gain matrix

$$k = [\,147\ \ -34733\ \ -608.3\,]. \tag{19}$$

Is interesting to note how this control could correspond to a cascaded P control for the current with a gain k_1 and a PD control for the mechanical part given by $k_2 = k_p$ and $k_3 = k_d$.

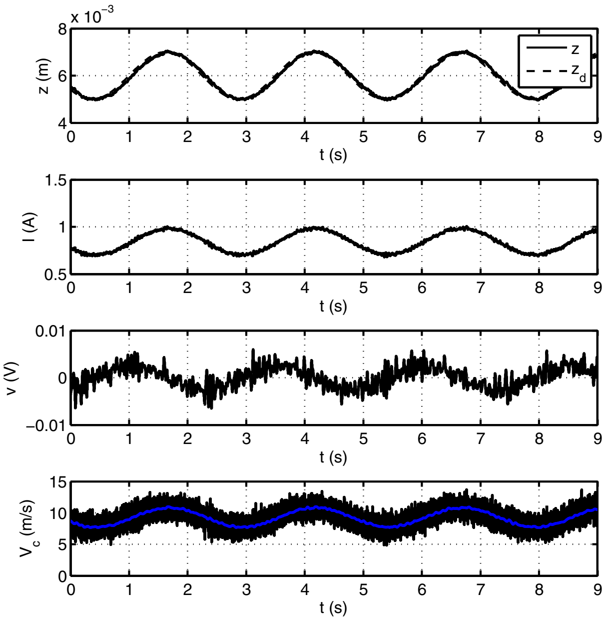

Fig. 3. Simulation results for a sine reference

The simulation was run not considering the integral term, with sine reference of 1 mm of amplitude and a frequency 0.4 Hz, including a measurement noise of zero mean, standard deviation $\sigma_i = 10^{-4}$ A for the current, $\sigma_z = 10^{-5}$ m for the position, and maximal amplitudes 2 A for the current and 12 mm for the position. A robust differentiator with $\kappa_1 = 5$, $\kappa_2 = 10$, $M = 1$ has been applied. The results are shown in Fig. 7, where it is interesting to notice how the robust derivative can reconstruct the actual velocity of the system. In Fig. 8 it is shown the performance of the simulated system with a pulse reference around the same equilibrium point, with an amplitude of 1 mm and also a frequency of 0.4 Hz. In Fig. 5 it is shown the specific performance of the robust differentiator.

Parameter	Value	Units
Inductance L	0.4125	H
Coil impedance R_c	11	Ω
Gravitational acceleration (Mexico City)	9.78	m/s^2
Ball mass	65.968	g
Magnetic constant K_m	$6.5308 \cdot 10^{-5}$	Nm^2/A^2
Max. $V_c(t)$	20	V

Table 1. Maglev parameters

4.2 Experimental results

The experiments were done on a Quanser Maglev system as shown in Fig. 6, using Matlab 2010a with data acquisition provided by a dSpace board controller with sampling time of 100 μs. In this case, as there exists a parametric as well as measurement uncertainties and noise, the integral term was given a gain of $k_i = -2 \cdot 10^5$.

The results are shown in Fig. 7 and Fig. 8 for a similar reference than the ones given in the simulation. It can be noticed how the velocities are well reconstructed, while

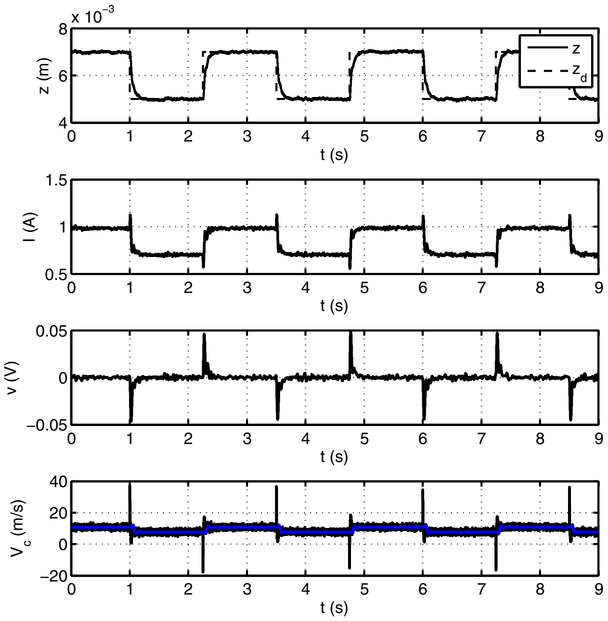

Fig. 4. Simulation results for a pulse reference

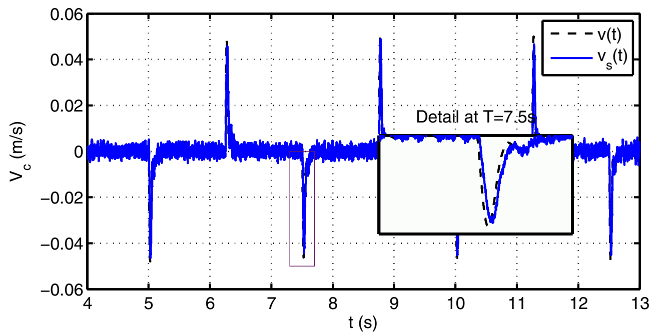

Fig. 5. Velocity reconstruction by the robust differentiator in simulation

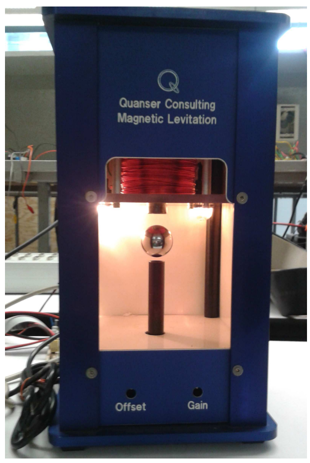

Fig. 6. Quanser $^{\mathrm{TM}}$Magnetic Levitator

keeping the system stable and tracking the desired reference. For presentation purposes, the control output V_c is filtered to show its mean value.

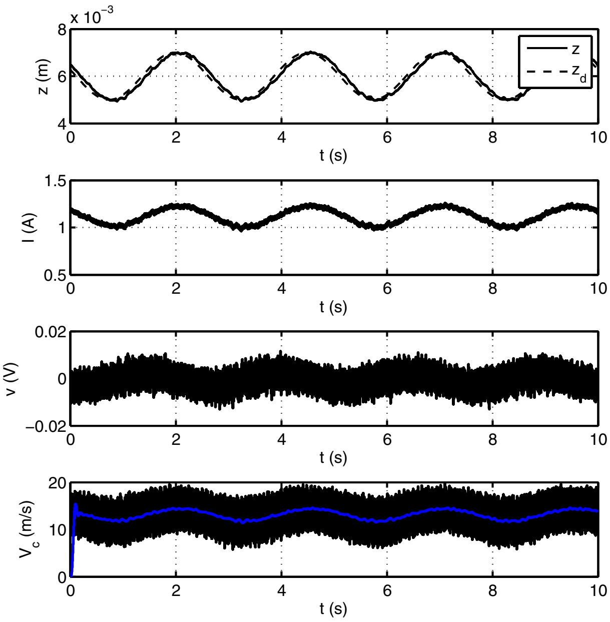

Fig. 7. Experimental results for a sine reference

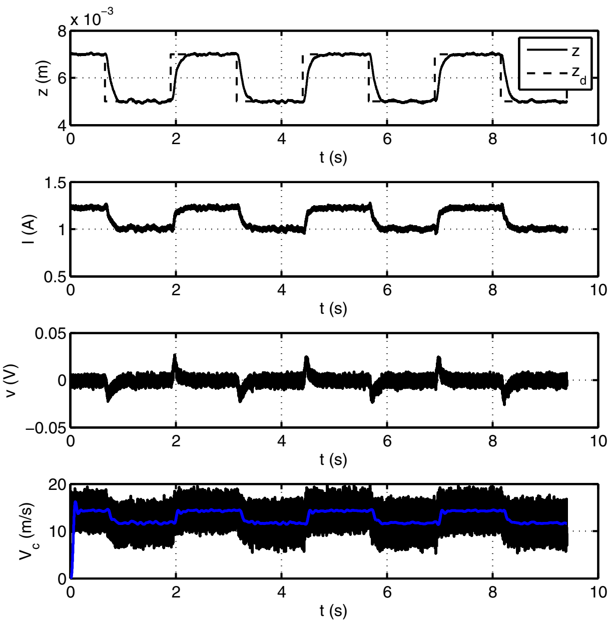

Fig. 8. Experimental results for a pulse reference

5. CONCLUSIONS

In this work we have presented simulation and experimental results of a LQR control for a magnetic levitator system with integral term and robust exact reconstruction of the velocity for regulation and tracking, considering external perturbations and non-modeled dynamics, as well as noisy measurements. It was shown that, even when the velocity observer is used in the control loop, it does reconstruct this state and helps to achieve an stable control, even in the presence of disturbances and measurement noise. Future work involves a the design and implementation of experimental nonlinear model control schema and higher derivatives reconstruction for a wider operation set.

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