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Virtual actuator fault tolerant control approach for Markovian jump linear systems[1]

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Abstract: In this paper, a fault tolerant control strategy for Markovian jump linear systems is proposed. The strategy is based on the virtual actuator approach for controller reconfiguration after actuator faults. The main advantage of this method is that the control loop is reconfigured such that any existing 'nominal' controller, designed for the fault-free system, can continue to be used in the presence of faults without the need of retuning it. The plant with the faulty actuator is modified by adding the virtual actuator block to 'mask' the fault. The fault tolerant controller is implemented as an output feedback controller and designed using linear matrix inequalities.

Keywords: Fault tolerant control, virtual actuator, Markovian jump systems.

1. INTRODUCTION

The deployment of sophisticated control systems in industrial processes has been stimulated by the increasing integration of intelligent sensors, actuators, controllers and microprocessors into automatic control systems as a result of the continuous development of new technologies. Faults in any of these many system components can affect the system performance and more severely, due to the high interconnection among components, can lead to the total collapse of the process operation. Therefore the demand for reliability, safety and fault tolerance increases by the same measure in such systems. Fault tolerant control (FTC) involves early fault detection and isolation (FDI) and a subsequent decision, which can involve the accommodation or reconfiguration of the system, to deal with the new situation. The topic of FDI/FTC has become the focus of increasing research over the past few decades, see, for example, the surveys Patton (1997) and Ding (2012). According to Blanke et al. (2006), FTC allows maintaining performance close to desirable levels and preserve stability conditions in the presence of component and/or instrument faults.

A type of controller reconfiguration based on the concept of *virtual actuators* has been recently proposed as a fault accommodation approach (see Blanke et al. (2006); Richter & Lunze (2009); Seron & De Doná (2009)). The main idea of the virtual actuator method is to reconfigure the faulty plant such that any existing 'nominal' controller, designed for the fault-free system, can continue to be used in the presence of faults without the need of retuning it. Thus, this approach has the advantage that it introduces minimal changes to the control loop when faults occur.

The virtual actuator approach is illustrated in Fig. 1. The scheme consists of a plant with input u, measured output y

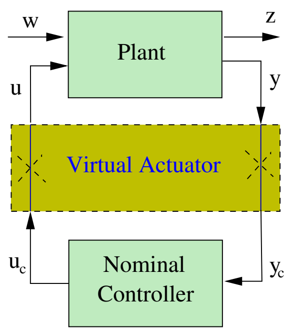

Fig. 1. Virtual actuator principle for control reconfiguration.

and performance output z, a nominal controller with input y_c and output u_c, and a virtual actuator with inputs y and u_c and outputs y_c and u. Plant and nominal controller are normally (in healthy operation) interconnected by setting $y_c = y$ and $u_c = u$, as indicated by the solid lines in Fig. 1. When actuator faults are diagnosed, the virtual actuator block is activated and becomes an interface between the plant and controller. When activated, the virtual actuator generates new interconnections (as indicated by the dashed crosses in Fig. 1) that produce, together with the faulty plant, for a given input u_c (approximately) the same output y_c as the nominal plant. Hence, the nominal controller, which remains unchanged in the loop, "sees" the same plant and reacts in the same way as in the absence of faults. The plant, on the other hand, receives the input u "reconfigured" by the virtual actuator. An important advantage of this approach is that any existing nominal controller which has been designed, fine-tuned and tested to satisfy desired specifications for the plant, can be used and kept in the loop at all times. In addition, the design of the virtual actuator is independent of the controller and is aimed at preserving specific closed-loop properties under fault such as closed-loop stability and robustness.

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In this paper we propose to extend the virtual actuator approach for FTC to Markovian jump linear systems (MJLS). The MJLS modelling framework has attracted increasing attention in the literature and the existing results cover a wide variety of problems, such as stochastic stability, stochastic H_∞ control, filtering problems, see Costa et al. (2005, 2013) and references therein. Although there is intensive research in FDI and MJLS, the design of FDI systems for MJLS has only recently been systematically addressed, with high emphasis on networked control system (NCS) applications. In Zhang et al. (1993), packet loss in NCS is modelled as a Markov process, and an FDI system is designed by means of linear matrix inequalities (LMI). Also the problem of random transmission delays in NCS, often addressed together with the data missing and packet loss problems, has been treated under the discrete MJS framework, see He et al. (2008, 2009); Sauter et al. (2009); Li et al. (2010). In Li et al. (2007), a fault detection system over noisy communication channels is also described via an MJLS. In Zhong et al. (2005), the design of an FDI system for MJLSs is formulated as an H_∞-filtering problem. FTC schemes based on a stochastic fault model where the modelling of faults allows to associate the stabilisation and robust control theory for MJSs, can be found in Aberkane et al. (2007, 2008); Peng et al. (2010). Fault detection in discrete-time MJLS was studied in Li et al. (2012), where a new reference model strategy was applied to construct a residual generator, robust against disturbances and sensitive to system faults. The problem of controller reconfiguration, however, has received less attention. In particular, to the best of the authors' knowledge, the virtual actuator approach for FTC of MJLS had not been studied to-date.

The remainder of the paper is organised as follows. Notation and basic results involving stability and the H_∞ norm of MJLS are introduced in Section 2. The virtual actuator approach is described in Section 3 and the main results are derived in Section 4. Section 5 presents an illustrative example and conclusions are given in Section 6.

2. NOTATION AND BASIC RESULTS

The set of natural (real) numbers is denoted by $\mathbb{N}$ ($\mathbb{R}$), $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$ and $\mathfrak{X} := \{1,2,\ldots,N\}$. Small letters denote vectors, capital letters denote matrices and $(')$ indicates matrix transpose. The notation $\mathrm{diag}(a_1,\ldots,a_n)$ represents a diagonal matrix with diagonal entries $a_1,\ldots,a_n$. The mathematical expectation is denoted by $E[\cdot]$. The squared norm of a stochastic signal $\xi(k)$, $k \in \mathbb{N}_0$, is defined as $\|\xi(k)\|_2^2 = \sum_{k=0}^{\infty} E[\xi'(k)\xi(k)]$. The class of all signals $\xi(k) \in \mathbb{R}^r$, $k \in \mathbb{N}_0$, such that $\|\xi(k)\|_2^2 < \infty$ is denoted by $\mathcal{L}_2^r$. We use the notation $X_{pi} = \sum_{j=1}^{N} p_{ij} X_j$, for all $i \in \mathfrak{X}$, for the convex combination of positive definite matrices $X_j \in \mathbb{R}^{n\times n}$ with weights p_{ij} for all $j \in \mathfrak{X}$.

2.1 The nominal system and the control loop

Consider the class of MJLS defined on the fundamental probability space $(\Omega, \mathfrak{F}, P)$,

$$\mathcal{S}: \begin{cases} x(k+1) = A(\theta_k)x(k) + B(\theta_k)u(k) + E(\theta_k)w(k), \\ \quad z(k) = C_z(\theta_k)x(k), \\ \quad y(k) = C_y(\theta_k)x(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the system state, $u(k) \in \mathbb{R}^m$ is the control input, $y(k) \in \mathbb{R}^p$ is the measured output, $w(k) \in \mathbb{R}^r$ is the disturbance input that belongs to $\mathcal{L}_2^r$ and $z(k) \in \mathbb{R}^q$ is the output to be controlled. $\{\theta_k; k \geq 0\}$ is a discrete-time homogeneous Markov chain with state space $\mathfrak{X}$ and transition probability matrix $\mathbb{P} = [p_{ij}]$ where

$$p_{ij} := P(\theta_{k+1} = j \mid \theta_k = i),\ \forall i,j \in \mathfrak{X},\ k \geq 0. \tag{2}$$

Whenever $\theta_k = i$ the system matrices of the ith mode are denoted by $A_i, B_i, E_i, C_{zi}, C_{yi}$, which are real known matrices of appropriate dimensions.

The system (1) in nominal ('fault-free') conditions is assumed to be controlled by a dynamic output-feedback controller which has the following mode-dependent structure

$$\mathcal{S}_c: \begin{cases} x_d(k+1) = A_d(\theta_k)x_d(k) + B_d(\theta_k)y(k), \\ \quad u(k) = C_d(\theta_k)x_d(k) + D_d(\theta_k)y(k), \end{cases} \tag{3}$$

where $x_d(k) \in \mathbb{R}^{n_d}$, $x_d(0) = 0$ and the matrices A_{di}, B_{di}, C_{di} and D_{di} for all $i \in \mathfrak{X}$ are of compatible dimensions. The control law assumes the complete access to the mode θ_k at time k. In addition we assume that the controller (3) stabilises the nominal process (1) in the sense of Definition 1 below. Defining $\tilde{x} = [x'\ x_d']'$ the closed-loop system (1) with (3) is given by

$$\begin{aligned} \tilde{x}(k+1) &= \tilde{A}(\theta_k)\tilde{x}(k) + \tilde{E}(\theta_k)w(k), \\ z(k) &= \tilde{C}(\theta_k)\tilde{x}(k), \end{aligned} \tag{4}$$

where, whenever $\theta_k = i$, $\tilde{A}_i := \tilde{A}(i)$, $\tilde{E}_i := \tilde{E}(i)$, $\tilde{C}_i := \tilde{C}(i)$ are given by

$$\tilde{A}_i = \begin{bmatrix} A_i + B_i D_{di} C_{yi} & B_i C_{di} \\ B_{di} C_{yi} & A_{di} \end{bmatrix}, \quad \tilde{E}_i = \begin{bmatrix} E_i \\ 0 \end{bmatrix}, \quad \tilde{C}_i = [C_{zi} \quad 0].$$

2.2 Stochastic stability and H_∞ performance

Results concerning stability and H_∞ performance for the MJLS class considered are given below.

Definition 1. The MJLS (1) with null control $u(k) \equiv 0$, null external input $w(k) \equiv 0$ for all $k \in \mathbb{N}_0$, and initial conditions $x(0) = x_0 \in \mathbb{R}^n$, $\theta_0 \in \mathfrak{X}$ is *stochastically stable* if for every initial conditions x_0 and θ_0

$$E\Big[\sum_{k\geq 0} x'(k)x(k) \,|\, \theta_0, x_0\Big] < \infty. \tag{5}$$

The following result from Ji & Chizeck (1990) provides a necessary and sufficient condition for stochastic stability of the system (1) with $u(k) \equiv 0$, $w(k) \equiv 0$, based on the existence of positive definite solutions to a set of coupled Lyapunov-like inequalities.

Lemma 1. The system $\mathcal{S}$ given in (1), with $u(k) \equiv 0$, $w(k) \equiv 0$ for all $k \in \mathbb{N}_0$ and transition probabilities (2) is stable if and only if there exist $P_i = P_i' > 0$ satisfying the coupled Lyapunov equations

$$A_i' P_{pi} A_i - P_i < 0, \quad \forall i \in \mathfrak{X}, \tag{6}$$

where P_{pi} is defined using the transition probabilities (2) as

$$P_{pi} \triangleq \sum_{j=1}^{N} p_{ij} P_j. \tag{7}$$

The formal definition of the H_∞ norm of the MJLS $\mathcal{S}$ with state space realisation given in (1) is as follows.

Definition 2. The H_∞ norm of a stable system $\mathcal{S}$ from the input w to the output z (with $u(k) \equiv 0$) is given by

$$\|\mathcal{S}\|_\infty^2 = \sup_{\theta_0 \in \mathfrak{X},\, 0 \neq w \in \mathcal{L}_2^r} = \frac{\|z\|_2^2}{\|w\|_2^2}.$$

Observe that in the deterministic case, characterised by $N = 1$, the previous definition reduces to the usual H_∞ norm of the linear time invariant discrete-time system $\mathcal{S}$.

The next result from Seiler & Sengupta (2003) is a bounded real lemma for the MJLS (1) and shows how its H_∞ norm can be calculated.

Lemma 2. The system $\mathcal{S}$ with $u(k) \equiv 0$ is stable and satisfies the norm constraint $\|\mathcal{S}\|_\infty^2 \leq \gamma$ if and only if there exist matrices $P_i = P_i' > 0$ such that

$$\begin{bmatrix} A_i & E_i \\ C_{zi} & 0 \end{bmatrix}' \begin{bmatrix} P_{pi} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_i & E_i \\ C_{zi} & 0 \end{bmatrix} - \begin{bmatrix} P_i & 0 \\ 0 & \gamma I \end{bmatrix} < 0, \tag{8}$$

where P_{pi} is defined in (7).

Notice that feasibility of (8) requires the existence of positive definite matrices P_i such that $A_i' P_{pi} A_i - P_i < -C_{zi}' C_{zi} \leq 0$ for all $i \in \mathfrak{X}$ which is possible if and only if $\mathcal{S}$ is stable, cf. Lemma 1.

Applying Schur complements to (8), the following result can be derived (see, e.g., Gonçalves et al. (2012)).

Lemma 3. The H_∞ norm of system $\mathcal{S}$, for $u(k) \equiv 0$, can be calculated as

$$\|\mathcal{S}\|_\infty^2 = \inf_{\gamma, P_i} \gamma, \tag{9}$$

where $P_i = P_i' > 0$ and $\gamma \in \mathbb{R}$, such that the matrix inequality

$$\begin{bmatrix} P_i & 0 & A_i' & C_{zi}' \\ 0 & \gamma I & E_i' & 0 \\ A_i & E_i & P_{pi}^{-1} & 0 \\ C_{zi} & 0 & 0 & I \end{bmatrix} > 0, \tag{10}$$

with P_{pi} defined in (7), is satisfied for all $i \in \mathfrak{X}$.

3. VIRTUAL ACTUATOR FOR MJLS: SYNTHESIS PROBLEM

3.1 Faulty MJLS and reconfiguration principle

An actuator fault for a MJLS is modelled here as a change of the control input matrix from $B(\theta_k)$ to $B_f(\theta_k, f_k)$. More specifically, we adopt $B_f(\theta_k, f_k) = B(\theta_k)F(f_k)$ where $F(f_k) = \text{diag}(f_1(k), f_2(k), \ldots f_p(k))$, with $0 \leq f_s \leq 1$ representing the effectiveness of the s-th actuator, such that the extreme values $f_s(k) = 0$ and $f_s(k) = 1$ represent the total failure and the "healthy" situation of the s-th actuator at time k, respectively. For instance, for a (3×3) matrix $B(\theta_k)$, where at time k the first actuator is healthy ($f_1(k) = 1$), the second is totally lost ($f_2(k) = 0$) and the third has lost its effectiveness ($f_3(k) = 0.6$), we have

$$B_f(\theta_k, f_k) = \begin{bmatrix} b_{11}(\theta_k) & b_{12}(\theta_k) & b_{13}(\theta_k) \\ b_{21}(\theta_k) & b_{22}(\theta_k) & b_{23}(\theta_k) \\ b_{31}(\theta_k) & b_{32}(\theta_k) & b_{33}(\theta_k) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}.$$

It is assumed that the faulty process is still controllable and observable, which implies that an output feedback controller can be designed for the faulty system.

The system $\mathcal{S}$ thus changes to the faulty system $\mathcal{S}_f$

$$\mathcal{S}_f : \begin{cases} x_f(k+1) = A(\theta_k)x_f(k) + B_f(\theta_k, f_k)u_f(k) + E(\theta_k)w(k), \\ z_f(k) = C_z(\theta_k)x_f(k), \\ y_f(k) = C_y(\theta_k)x_f(k). \end{cases} \tag{11}$$

The input to the faulty system, denoted by $u_f(k)$, differs from $u(k)$ because, in general, the fault changes the loop behaviour. After a fault occurs, the closed-loop system formed by the faulty plant $\mathcal{S}_f$ and the nominal controller $\mathcal{S}_c$ is, in general, no longer adequate.

We are now in a position to be able to summarise the reconfiguration problem, which consists in finding a new controller $\mathcal{S}_{c,r}$ such that the reconfigured closed-loop system composed by $\mathcal{S}_f$ and $\mathcal{S}_{c,r}$ is stable and meets the original control goals as closely as possible.

The reconfiguration approach adopted here is the same as proposed in Richter & Lunze (2009) for a deterministic system. Basically, this approach consists in augmenting the faulty closed-loop system by means of a dynamical reconfiguration block $\mathcal{S}_A$, named *virtual actuator*, with internal state variable x_v (initial condition x_{v0}), which is interconnected with the faulty plant $\mathcal{S}_f$ and the nominal controller $\mathcal{S}_c$ by means of the signal pairs (u_f, y_f) and (u_c, y_c). Together with the faulty plant $\mathcal{S}_f$, the reconfiguration block $\mathcal{S}_A$ forms the *reconfigured system* $\mathcal{S}_r$ = ($\mathcal{S}_f$, $\mathcal{S}_A$), to which the nominal controller (3) is connected. Seen from the faulty plant, the reconfigured controller is given by the interconnection $\mathcal{S}_{c,r}$ = ($\mathcal{S}_A$, $\mathcal{S}_c$).

The next definition is adapted from Richter & Lunze (2009).

Definition 3. The reconfigured system $\mathcal{S}_r$ satisfies the *strict fault-hiding goal*, if for all $k > 0$ and any x_0 there exists an initial condition x_{v0} such that the following holds:

$$y_c(k) - y(k) = 0, \quad \forall\, u_c(k) \text{ and } \forall\, w(k).$$

If the strict fault-hiding goal is achieved, then from the controller perspective, the reconfigured system $\mathcal{S}_r$ and the nominal system $\mathcal{S}$ have the same behaviour.

3.2 Virtual actuator for MJLS

Here the concept of virtual actuator is extended to the class of MJLS (1).

Definition 4. Consider the faulty plant $\mathcal{S}_f$. The virtual actuator is defined as the system

$$\mathcal{S}_A : \begin{cases} x_v(k+1) = A_v(\theta_k)x_v(k) + B_v(\theta_k)u_c(k), \\ u_v(k) = M(\theta_k)x_v(k), \\ u_f(k) = N(f_k)u_c(k) - N(f_k)u_v(k), \\ y_c(k) = C_y(\theta_k)x_v(k) + y_f(k), \end{cases} \tag{12}$$

where the 'feedforward' matrix $N(f_k)$ is independent of θ_k and defined as

$$N(f_k) := \text{diag}(n_1(k), \ldots, n_p(k)),\ n_j(k) = \begin{cases} \dfrac{1}{f_j} & \text{if } f_j > 0, \\ 0 & \text{if } f_j = 0, \end{cases} \tag{13}$$

and the state space matrices are given by

$$A_v(\theta_k) := A(\theta_k) + B_f(\theta_k, f_k)N(f_k)M(\theta_k), \tag{14}$$

$$B_v(\theta_k) := B(\theta_k) - B_f(\theta_k, f_k)N(f_k). \tag{15}$$

The virtual actuator 'feedback' matrix $M(\theta_k)$ is a degree of freedom that will be used below to achieve specific performance goals for the reconfiguration task.

Remark 4. The reconfigured control $u_f(k)$ in (12), is such that the strict fault-hiding goal of Definition 3 is reached, i.e., $y_c(k) = y(k)$ (see Blanke et al. (2006)).

Hereafter, we adopt the notations A_{vi}, B_{vi}, and M_i whenever $\theta_k = i$. Also, the notation $B_f(\theta_k, f_k) := B_{fi}(f_k)$ (the subindex i corresponds to $\theta_k = i$) is adopted when convenient, where $B_{fi}(f_k) = B_i F(f_k) = B_i \operatorname{diag}(f_1(k), f_2(k), \ldots f_p(k))$.

Remark 5. Observe that the matrices A_{vi} and B_{vi} in (14) and (15) do not depend on f_k since the matrix $N(f_k)$ eliminates the effect of partial faults for each $i \in \mathfrak{X}$, see (13).

Under nominal conditions, the system $\mathcal{S}$ is controlled by the output feedback controller (3). Under faulty conditions, the system is reconfigured via the virtual actuator so that $y(k)$ in (3) is replaced by the virtual actuator signal $y_c(k)$ given in (12). We thus have that the nominal controller dynamics satisfy

$$x_d(k+1) = A_{di}x_d(k) + B_{di}C_{yi}\big(x_v(k) + x_f(k)\big). \tag{16}$$

Let $B_i^* := B_{fi}(f_k)N(f_k)$. The augmented model consisting of $\mathcal{S}_f$ (with $w(k) \equiv 0$), $\mathcal{S}_A$ and (16) is then given by

$$\begin{bmatrix} x_d(k+1) \\ x_f(k+1) \\ x_v(k+1) \end{bmatrix} = \begin{bmatrix} A_{di} & B_{di}C_{yi} & B_{di}C_{yi} \\ 0 & A_i & -B_i^*M_i \\ 0 & 0 & A_i + B_i^*M_i \end{bmatrix} \begin{bmatrix} x_d(k) \\ x_f(k) \\ x_v(k) \end{bmatrix} + \begin{bmatrix} 0 \\ B_i^* \\ B_i - B_i^* \end{bmatrix} u_c(k). \tag{17}$$

Defining $\bar{x}(k) := x_f(k) + x_v(k)$ and letting $u_c(k) = C_{di}x_d(k)$ (that is, for simplicity, we take $D_d(\theta_k) = 0$ in (3)), the reconfigured closed-loop system (17) can be rewritten as

$$\begin{bmatrix} \bar{x}(k+1) \\ x_d(k+1) \\ x_v(k+1) \end{bmatrix} = \begin{bmatrix} A_i & B_iC_{di} & 0 \\ B_{di}C_{yi} & A_{di} & 0 \\ 0 & (B_i - B_i^*)C_{di} & A_i + B_i^*M_i \end{bmatrix} \begin{bmatrix} \bar{x}(k) \\ x_d(k) \\ x_v(k) \end{bmatrix}. \tag{18}$$

Now, letting $\xi(k) := [\bar{x}' \;\; x_d']'$, $\tilde{A}_i$ defined in (4) (with $D_{di} = 0$) and

$$\tilde{B}_i = \begin{bmatrix} 0 & (B_i - B_i^*)C_{di} \end{bmatrix},$$

(18) yields

$$\begin{bmatrix} \xi(k+1) \\ x_v(k+1) \end{bmatrix} = \begin{bmatrix} \tilde{A}_i & 0 \\ \tilde{B}_i & A_i + B_i^*M_i \end{bmatrix} \begin{bmatrix} \xi(k) \\ x_v(k) \end{bmatrix}. \tag{19}$$

Notice that a separation principle can be applied to the above model. That is, the set σ of the eigenvalues of the closed-loop system (19) consists of the set of eigenvalues of the nominal closed-loop system plus the virtual actuator eigenvalues, as follows

$$\sigma = \sigma\{\tilde{A}_i\} \cup \sigma\{A_i + B_i^*M_i\}. \tag{20}$$

Thus, the nominal controller and the virtual actuator can be designed independently.

Remark 6. From (19), the behaviour of the virtual actuator state $x_v(k)$ is affected by $B_i^*M_i$ through the submatrix $A_i + B_i^*M_i$. The matrices B_i^* are the same as the nominal input matrices B_i but with a column of zeros if the corresponding actuator has been lost, since $N(f_k)$ corrects any partial actuator fault (see (13)). Thus, the virtual actuator feedback matrix M_i has to compensate only for total losses of the actuators and the partial faults are handled by the feedforward matrix $N(f_k)$.

3.3 Reconfiguration problem

The next definition is adapted from Richter & Lunze (2009).

Definition 5. Let $\mathcal{S}_A^*$ and $\mathcal{S}_A$ be two arbitrary virtual actuators that give rise to stable reconfigured closed-loop systems $(\mathcal{S}_f, \mathcal{S}_A^*, \mathcal{S}_c)$ and $(\mathcal{S}_f, \mathcal{S}_A, \mathcal{S}_c)$, with the controlled outputs denoted by z_f^* and z_f, respectively. The reconfigured closed-loop system with the virtual actuator $\mathcal{S}_A^*$, achieves the *optimal approximate stable trajectory recovery goal* if $(\mathcal{S}_f, \mathcal{S}_A^*, \mathcal{S}_c)$ is stable and, in addition, for any initial condition x_0 the following inequality holds for all nonzero $u_c(\cdot)$:

$$\frac{\|z - z_f^*\|_2}{\|u_c\|_2} < \frac{\|z - z_f\|_2}{\|u_c\|_2}.$$

Reconfiguration Problem 1. (Stable optimal performance recovery). Given the system (1), the nominal controller (3), and the faulty system (11), find an optimal reconfiguration block $\mathcal{S}_r^*$ that satisfies (i) the strict fault-hiding goal and (ii) the optimal approximate stable trajectory recovery goal.

Reconfiguration Problem 2. (Stable minimum input amplification). Given the system (1), the nominal controller (3), and the faulty system (11), find an optimal stabilising reconfiguration block $\mathcal{S}_r^*$, producing a control input u_f^* that (i) satisfies the strict fault-hiding goal, and (ii) guarantees that for all nonzero $u_c(k)$

$$\frac{\|u_f^*\|_2}{\|u_c\|_2} < \frac{\|u_f\|_2}{\|u_c\|_2}, \tag{21}$$

where u_f is the control input produced by any other reconfiguration block.

Considering $u_f(k)$ defined in (12), we observe that the feedforward matrix $N(f_k)$ is fixed once the fault and associated parameters f_k have been diagnosed. On the other hand, the feedback component $u_v(k)$ in (12) can be freely designed to reduce the input amplification introduced by the virtual actuator. We thus consider:

Reconfiguration Problem 3. Same as the Reconfiguration Problem 2 with (21) replaced by

$$\frac{\|u_v^*\|_2}{\|u_c\|_2} < \frac{\|u_v\|_2}{\|u_c\|_2}. \tag{22}$$

For simplicity, we use the 'system state-space' notation

$$\mathcal{S}_z := (A_{vi}, B_{vi}, C_{zi}, 0) \quad \text{and} \quad \mathcal{S}_u := (A_{vi}, B_{vi}, -M_i, 0).$$

Problem 1 can then be formulated using the H_∞-norm as (see Richter & Lunze (2009) for details)

$$\gamma_z := \min_{M_i} \|\mathcal{S}_z\|_\infty^2.$$

In this case we consider the H_∞ norm of the stable system $\mathcal{S}_A$ from the input u_c to the output $z_v := C_{zi}x_v$.

Similarly, Problem 3 is equivalent to minimising the H_∞-norm of $\mathcal{S}_u$, that is,

$$\gamma_u := \min_{M_i} \|\mathcal{S}_u\|_\infty^2.$$

For this case we consider the H_∞ norm of the stable system $\mathcal{S}_A$ from the input u_c to the output u_v.

4. MAIN RESULTS

The results here are based on Theorem 1 in Gonçalves et al. (2012). The following result presents the solution of Problem 1.

Theorem 1. Consider the optimisation problem

$$\min_{X_i,Y_i,H_i,Z_{ij}} \gamma_z \quad \text{subject to:}$$

$$\begin{bmatrix} X_i & 0 & (A_iX_i+B_i^*Y_i)' & (C_{zi}X_i)' \\ 0 & \gamma_z I & (B_i-B_i^*)' & 0 \\ A_iX_i+B_i^*Y_i & B_i-B_i^* & H_i+H_i'-Z_{pi} & 0 \\ C_{zi}X_i & 0 & 0 & I \end{bmatrix} > 0 \quad (a)$$

$$\begin{bmatrix} Z_{ij} & H_i' \\ H_i & X_j \end{bmatrix} > 0 \quad (b)$$

where $B_i^* := B_{fi}(f_k)N(f_k)$, $Z_{pi} := \sum_{j=1}^N p_{ij}Z_{ij}$ and $X_i = X_i' > 0$. The virtual actuator (12)–(15) with $M_i = Y_iX_i^{-1}$ is the unique solution to the Reconfiguration Problem 1.

Proof. The goal is to find M_i in order to minimise $\|\mathcal{S}_z\|_\infty$. We will follow Theorem 1 in Gonçalves et al. (2012) by showing that $\|\mathcal{S}_z\|_\infty^2 < \gamma_z$ if and only if inequalities (a) and (b) hold, and in the affirmative case $M_i = Y_iX_i^{-1}$ is the desired virtual actuator feedback gain.

For the necessity part, applying Lemma 3 with the matrices A_i and E_i in (10) replaced by $A_i + B_i^*M_i$ and $B_i - B_i^*$, assume that there exist matrices $P_i = P_i' > 0$ such that

$$\begin{bmatrix} P_i & 0 & (A_i+B_i^*M_i)' & C_{zi}' \\ 0 & \gamma_z I & (B_i-B_i^*)' & 0 \\ A_i+B_i^*M_i & B_i-B_i^* & P_{pi}^{-1} & 0 \\ C_{zi} & 0 & 0 & I \end{bmatrix} > 0, \qquad (23)$$

where $P_{pi} = \sum_{j=1}^N p_{ij}P_j$. Defining $X_i := P_i^{-1}$, $Y_i = M_iX_i$ and multiplying (23) on the right by $\text{diag}[X_i, I, I, I]$ and on the left by its transpose we have

$$\begin{bmatrix} X_i & 0 & (A_iX_i+B_i^*Y_i)' & (C_{zi}X_i)' \\ 0 & \gamma_z I & (B_i-B_i^*)' & 0 \\ A_iX_i+B_i^*Y_i & B_i-B_i^* & X_{qi} & 0 \\ C_{zi}X_i & 0 & 0 & I \end{bmatrix} > 0, \qquad (24)$$

where $X_{qi} := \left(\sum_{j=1}^N p_{ij}X_j^{-1}\right)^{-1}$. Also defining $Z_{pi} := \sum_{j=1}^N p_{ij}Z_{ij}$, for $Z_{ij} = H_i'X_j^{-1}H_i + \varepsilon I$ with $\varepsilon > 0$ and $H_i = X_{qi}$, by Schur complement the inequality (b) is verified. We also have that

$$\begin{aligned} H_i + H_i' - Z_{pi} &= H_i + H_i' - \sum_{j=1}^N p_{ij}\left(H_i'X_j^{-1}H_i + \varepsilon I\right) \\ &= H_i + H_i' - H_i'\Big(\sum_{j=1}^N p_{ij}X_j^{-1}\Big)H_i - \varepsilon I \\ &= H_i + H_i' - H_i'X_{qi}^{-1}H_i - \varepsilon I = X_{qi} - \varepsilon I \end{aligned}$$

where we have used the fact that $\sum_{j=1}^N p_{ij} = 1$. Since, from the above equality, $X_{qi} = H_i + H_i' - Z_{pi} + \varepsilon I$, taking ε sufficiently small inequality (a) is implied by (24). From the solution to this set of LMIs, the required gains M_i are obtained.

For the sufficiency part, suppose that LMIs (a) and (b) are valid. We will show that these inequalities ensure that $\|\mathcal{S}_z\|_\infty^2 < \gamma_z$. From (b), applying Schur complement it follows that $Z_{ij} > H_i'X_j^{-1}H_i$, which implies $Z_{pi} = \sum_{j=1}^N p_{ij}Z_{ij} > \sum_{j=1}^N p_{ij}(H_i'X_j^{-1}H_i)$. Thus,

$$\begin{aligned} H_i + H_i' - Z_{pi} &< H_i + H_i' - \sum_{j=1}^N p_{ij}\left(H_i'X_j^{-1}H_i\right) \\ &= H_i + H_i' - H_i'X_{qi}^{-1}H_i \\ &= X_{qi} - (H_i - X_{qi})'X_{qi}^{-1}(H_i - X_{qi}) \le X_{qi}. \end{aligned} \qquad (25)$$

Now, from (25), replacing $H_i + H_i' - Z_{pi}$ by X_{qi} in (a) the obtained inequality remains true. Multiplying this inequality on the right by $\text{diag}[X_i^{-1}, I, I, I]$ and on the left by its transpose we obtain (23) with $P_i = X_i^{-1}$ and consequently $\|\mathcal{S}_z\|_\infty^2 < \gamma_z$ from Lemma 3 completing the proof. □

The next theorem solves the Reconfiguration Problem 3.

Theorem 2. Consider the optimisation problem

$$\min_{\bar{X}_i,\bar{Y}_i,G_i,\bar{Z}_{ij}} \gamma_u \quad \text{subject to:}$$

$$\begin{bmatrix} \bar{X}_i & 0 & (A_i\bar{X}_i+B_i^*\bar{Y}_i)' & -\bar{Y}_i' \\ 0 & \gamma_u I & (B_i-B_i^*)' & 0 \\ A_i\bar{X}_i+B_i^*\bar{Y}_i & B_i-B_i^* & G_i+G_i'-\bar{Z}_{pi} & 0 \\ -\bar{Y}_i & 0 & 0 & I \end{bmatrix} > 0 \quad (c)$$

$$\begin{bmatrix} \bar{Z}_{ij} & \star \\ G_i & X_j \end{bmatrix} > 0 \quad (d)$$

where $B_i^* := B_{fi}(f_k)N(f_k)$ and $X_i = X_i' > 0$. The virtual actuator (12)–(15) with $M_i = \bar{Y}_i\bar{X}_i^{-1}$ solves the Reconfiguration Problem 3.

Proof. The goal is to minimise $\|\mathcal{S}_u\|_\infty$. The proof follows the same idea of Theorem 1. Here, Lemma 3 is applied with the matrices A_i, E_i and C_{zi} in (10) replaced by $A_i + B_i^*M_i$, $B_i - B_i^*$ and $-M_i$, respectively. □

Reconfiguration Problems 1 and 3 can be conflicting since achieving trajectory recovery as in Problem 1 may require large control actions. The following result presents a method to achieve a tradeoff between these problems.

Corollary 1. Let $\lambda \in [0,1]$ be a tuning parameter. Then the virtual actuator (12)–(15) with gains $M_i = Y_iX_i^{-1}$ obtained by solving the optimisation problem

$$\min_{X_i,Y_i,H_i,G_i,Z_{ij},\bar{Z}_{ij}} \lambda\gamma_z + (1-\lambda)\gamma_u$$

subject to the LMIs (a), (b), (c) and (d), with $\bar{X}_i = X_i$ and $\bar{Y}_i = Y_i$ achieves a tradeoff between Reconfiguration Problems 1 and 3.

5. NUMERICAL EXAMPLE

In this section, we give a numerical example that illustrates the solution of Corollary 1. Let us consider a 2-dimensional dynamical system that has 3 modes with the following data:

Mode 1:

$$A_1 = \begin{bmatrix} 0.9753 & 0 \\ 0.0244 & 0.9753 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.0988 & -0.0494 \\ 0.0012 & 0.0488 \end{bmatrix};$$

Mode 2:

$$A_2 = \begin{bmatrix} 0.9876 & 0 \\ 0.0123 & 0.9876 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0497 & -0.0248 \\ 0.0003 & 0.0247 \end{bmatrix};$$

Mode 3:

$$A_3 = \begin{bmatrix} 0.9938 & 0 \\ 0.0062 & 0.9938 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.0249 & -0.0125 \\ 0.0001 & 0.0124 \end{bmatrix};$$

and $C_{z1} = C_{z2} = C_{z3} = \begin{bmatrix} 0 & 1 \end{bmatrix}$. The above matrices correspond to zero-order-hold discretisations of the linearised model of an interconnected two-tank system with 3 different sampling periods, as used in a 'controller driven sampling' framework (see Osella et al. (2013) for details). In the latter framework, a supervisory-type controller selects, e.g., randomly, the sampling period to be used at each time amongst a finite set of possible periods. The resulting discrete-time switched system fits the framework of the current paper when the sequence of sampling periods is a Markov chain. For this example we take the following transitions probabilities:

$$\mathbb{P} = \begin{bmatrix} 0.4 & 0.6 & 0 \\ 0.5 & 0 & 0.5 \\ 0.2 & 0.6 & 0.2 \end{bmatrix}.$$

We consider the fault matrix

$$F(\gamma) = \begin{bmatrix} 0.8 & 0 \\ 0 & 0 \end{bmatrix},$$

which corresponds to 20% loss of effectiveness of the first actuator and outage of the second actuator. Accordingly, the virtual actuator feedforward matrix (13) is taken as

$$N(f_k) = \begin{bmatrix} 1/0.8 & 0 \\ 0 & 0 \end{bmatrix}.$$

Solving the optimisation problem of Corollary 1 with $\lambda = 0.5$ yields $\gamma_z = 0.5045$, $\gamma_u = 0.3834$, the virtual actuator feedback gains

$$M_1 = -\begin{bmatrix} 8.6384 & 18.4711 \\ 0 & 0 \end{bmatrix}, M_2 = -\begin{bmatrix} 4.1730 & 8.6592 \\ 0 & 0 \end{bmatrix}, M_3 = -\begin{bmatrix} 0.9115 & 1.1563 \\ 0 & 0 \end{bmatrix},$$

and associated positive definite matrices

$$X_1 = \begin{bmatrix} 0.2689 & -0.1186 \\ -0.1186 & 0.0547 \end{bmatrix}, X_2 = \begin{bmatrix} 0.2572 & -0.1148 \\ -0.1148 & 0.0536 \end{bmatrix}, X_3 = \begin{bmatrix} 0.2529 & -0.1127 \\ -0.1127 & 0.0525 \end{bmatrix}.$$

6. CONCLUSIONS

We have proposed an FTC strategy for Markovian jump linear systems based on the virtual actuator approach for controller reconfiguration after actuator faults. A key advantage of this method is that the control loop is reconfigured such that any existing 'nominal' controller, designed for the fault-free system, can continue to be used in the presence of faults without the need of retuning it. The FTC controller is implemented as an output feedback controller and the virtual actuator parameters are designed using linear matrix inequalities.

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