**Memorias del XVI Congreso Latinoamericano
de Control Automático, CLCA 2014
Octubre 14-17, 2014. Cancún, Quintana Roo, México**

Computation of the safety ZMP zone for a biped robot based on error factors

Robin Wilmart * **Edmundo Rocha-Cózatl** ** **Octavio Narváez-Aroche** ***

* *Université de Mons, Mons, Belgique (e-mail: rwilmart@hotmail.com)*
** *Departamento de Ing. Mecatrónica, Facultad de Ingeniería, UNAM, Mexico City, Mexico (e-mail: e.rocha.cozatl@comunidad.unam.mx)*
*** *University of California, Berkeley (e-mail: ocnaar@berkeley.edu)*

Abstract: The biped Scout's gait stability is ensured by the ZMP concept. A lot of work has already been carried out to generate trajectories based on this criterion: ZMP should be placed within the convex hull of the footprint. In previous papers of the authors, in order to assure that this is fulfilled and due to reasons of computing time saving, an arbitrary rectangular safety zone (inside the footprint) was defined for trajectory planning. However, it remained the question about how could it be defined in a way that uncertainties in the model would be directly taken into account. This work aims at the computation of a less conservative ZMP safety zone relying on the robot's motion equations and the definition of error factors. Not only will implementing a better definition of the safety zone for the ZMP allow us to improve the process of trajectory planning, but also the dynamic stability of the biped during walking.

Keywords: Biped Robot, ZMP criterion, Trajectory planning.

1. INTRODUCTION

In order to ensure static stability in biped robotics, one has to make sure that the projection of the center of mass on the ground fits in the convex hull of the foot-support area, Goswami (1999). A biped-robot gait is said to be statically stable, Shih (1996), and a human posture is said to be balanced, Winter (1990), if the gravity line from its center of mass (or GCoM: Ground projection of the Center of Mass) falls within the convex hull of the foot-support area (henceforth called the support polygon). It is worth noting that a human being can almost always regain the upright posture as long as its feet are securely posed on the ground. The exit of the GCoM from the support polygon is then equivalent to the presence of an uncompensated moment on the foot, which causes it to rotate about a point in the polygon boundary, Goswami (1999). In the area of biped robot research, much progress has been made in the past few years. However, some difficulties remain to be dealt with, particularly about the implementation of fast and dynamic walking gaits, in other words anthropomorphic gaits, especially on uneven terrain, Sardain et al., (2004).

The zero-moment point (ZMP) is also known as a significant dynamic equilibrium criterion that was published in 1972 by Vukobratovic and Stepanenko and was first applied in mechatronics to control the WL-10RD robot developed in 1985 by Takanishi and Kato (Siciliano et al., (2008)). Other stability criteria have been developed as the Foot-Rotation Indicator Point (FRI) Goswami (1999), the Feasible Solution Wrench Point (FSW) Takao (2003) and a universal stability criterion of the Foot Contact of Legged Robots Hirukawa et al., (2006). None of these relatively new assessments have been explored in our work.

In the literature there are examples (López-García (2012), Vadakkepat (2008)) where a safety zone is defined inside the footprint in order to garantee stability and try to avoid that ZMP could be placed at the edge, what could imply also a marginal stability. The definition of this safety zone, rectangular for example, assures stable walk properties and makes some computations simpler. However, a rectangle is an arbitrary geometric form that could lead either to conservative or restrictive results depending on its dimensions.

There is no way to obtain this safety zone reported in the literature, therefore this work gives an answer to this question based on the knowledge of the kinematic and dynamic model of the biped robot, both programmed in Mathematica®. The results are of both theoretical and practical significance, since they can be used to conclude about the best positions for the pressure sensors on the soles of the feet.

In section 2 some concepts are defined to be used later in section 3 where the proposed methodology is described. The results are presented in section 4 as well as their discussion. Concluding remarks and future work are discussed in section 5.

2. BACKGROUND

2.1 Zero Moment Point

The ZMP is the point on the ground where the tipping moment acting on the biped, due to gravity and inertia forces, equals zero; the tipping moment being defined as the component of the moment that is tangential to the supporting surface.

It should be noted that the term ZMP is not a perfectly exact expression. Indeed, the normal component of the moment generated by the inertia forces acting on the biped is not necessarily equal to zero. If we bear in mind, however, that ZMP abridges the exact expression "zero tipping moment point", then the term becomes perfectly acceptable, Sardain et al., (2004).

The term zero-moment point (ZMP) was coined in Vukobratovic (1972). It can be stated as in Siciliano et al., (2008): "In Fig. 1 an example of force distribution across the foot is given. As the load has the same sign all over the surface, it can be reduced to the resultant force R, the point of attack of which will be in the boundaries of the foot. Let the point on the surface of the foot, where the resultant R passed, be denoted as the zero-moment point.

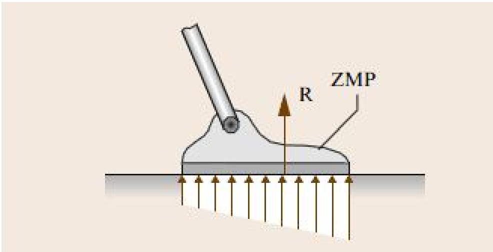

Fig. 1. Original definition of the zero-moment point (ZMP), (Siciliano et al., (2008)).

According to Sardain et al., (2004), the ZMP can indeed be associated to the center of pressure (CoP) of the floor reaction force.

2.2 Computation of ZMP for Full 3-D Dynamics

Assume that a robot consists of N rigid-body links (Fig. 2) and that all its kinematic information (the position of the CoM, link orientation, link velocity, etc.) has already been calculated by forward kinematics.

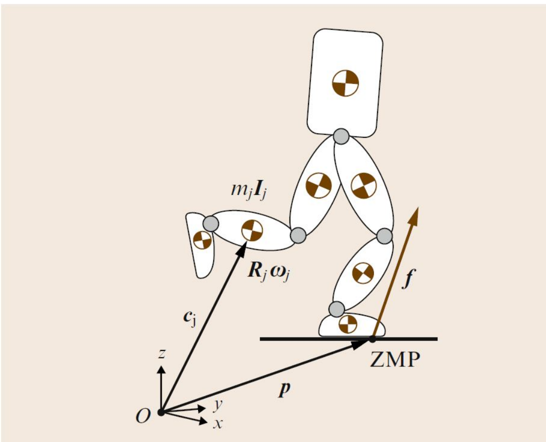

Fig. 2. Robot ZMP in 3D, (Siciliano et al., (2008)).

Let us introduce the nomenclature of the elements used in the computed ZMP equations.

m_j	:	Mass of the j-th body part.
M	:	Total mass.
$\mathbf{c}_j$	:	CoM of the j-th body part.
$\mathbf{c}$	:	CoM of the whole robot.
$\mathbf{R}_j$	:	3×3 rotation matrix of the j-th body part.
$\mathbf{I}_j$	:	3×3 inertia matrix of the j-th body part.
ω_j	:	Angular velocity of the j-th body part.
$\mathbf{f}$	:	External force applied to the robot by the ground.
τ	:	External moment around O applied to the robot by the ground.
τ_{ZMP}	:	Moment at the ZMP, whose first and second components are zero.
p_z	:	Height of the floor.

The total mass M and the center of mass $\mathbf{c}$ of the robot are:

$$M = \sum_{j=1}^{N} m_j, \qquad \mathbf{c} = \sum_{j=1}^{N} m_j \mathbf{c}_j / M$$

The total linear momentum $\mathbf{P}$ is given as: $\mathbf{P} = \sum_{j=1}^{N} m_j \dot{\mathbf{c}}_j$.

Then one can express the total angular momentum $\mathbf{L}$ with respect to the origin where $\mathbf{R}_j\ \mathbf{I}_j\ \mathbf{R}_j^T$ gives the inertial matrix in the ground-fixed frame.

$$\mathbf{L} = \sum_{j=1}^{N} \left[\mathbf{c}_j \times (m_j \dot{\mathbf{c}}_j) + \mathbf{R}_j\ \mathbf{I}_j\ \mathbf{R}_j^T \omega_j\right]$$

According to the Newton and Euler's law, one can write the external force $\mathbf{f}$ and the external moment τ where $\mathbf{g}$ is the vector of acceleration due to gravity:

$$\begin{aligned} \mathbf{f} &= \dot{\mathbf{P}} - M\mathbf{g} \\ \tau &= \dot{\mathbf{L}} - \mathbf{c} \times M\mathbf{g} \end{aligned} \tag{1}$$

Suppose that the external force is acting on the ZMP located at $\mathbf{p}$, then:

$$\tau = \mathbf{p} \times \mathbf{f} + \tau_{ZMP} \tag{2}$$

Plugging (1) into (2) and given that the cross product is anticommutative ($\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$, $\forall \mathbf{a}, \mathbf{b} \in \mathbb{R}^n$) one obtains:

$$\tau_{ZMP} = \dot{\mathbf{L}} - \mathbf{c} \times M\mathbf{g} + \left(\dot{\mathbf{P}} - M\mathbf{g}\right) \times \mathbf{p} \tag{3}$$

If one considers the first and second rows of the expression (3), the latter can be rewritten as:

$$\tau_{ZMP,x} = \dot{L}_x + Mgy + \dot{P}_y p_z - \left(\dot{P}_z + Mg\right) p_y \tag{4}$$

$$\tau_{ZMP,y} = \dot{L}_y - Mgx - \dot{P}_x p_z + \left(\dot{P}_z + Mg\right) p_x \tag{5}$$

where $\tau_{ZMP} = [\tau_{ZMP,x}\ ,\ \tau_{ZMP,y}\ ,\ \tau_{ZMP,z}]^T$; $\mathbf{P} = [P_x\ ,\ P_y\ ,\ P_z]^T$; $\mathbf{L} = [L_x\ ,\ L_y\ ,\ L_z]^T$; $\mathbf{c} = [x\ ,\ y\ ,\ z]^T$.

Finally, the zero-moment point can be calculated from (4-5) using the definition of the ZMP that is to say $\tau_{ZMP,x} = \tau_{ZMP,y} = 0$:

$$p_x = \frac{Mgx + \dot{P}_x p_z - \dot{L}_y}{\dot{P}_z + Mg} \tag{6}$$

$$p_y = \frac{Mgy + \dot{P}_y p_z + \dot{L}_x}{\dot{P}_z + Mg} \tag{7}$$

2.3 Biped robot

Scout is a biped robot developed by Lynxmotion®. It is 23 (cm) tall and weighs 0.9 (kg). It is constituted by anodized aluminium links. Fig. 3 shows a CAD model that was used to obtain the kinematic and dynamic models.

Scout is constituted by two legs of six links. A central part allows the connection between the legs. This central part is called "torso" or "body" in reference to the human anatomy. In total it has 13 links connected to each other through rotational joints actuated by servomotors (shown in red). Since there exist 12 joints, but 18 generalized coordinates, the robot is said to be of 12 internal degrees of freedom (DoF), Chevallereau et al., (2009).

In order to identify unequivocally each link of the Scout, a specific nomenclature is adopted. The torso will be identified by the letter B (Body) and the links of the leg are referred by the subscript ni; where $1 \leq n \leq 6$ and $i = 1, 2$ depending on the leg that is concerned, the left or right one respectively. The axes-rotation of the servomotors is described by angles θ_{ni}.

The links that are in touch with the walking surface (feet) are shown in purple.

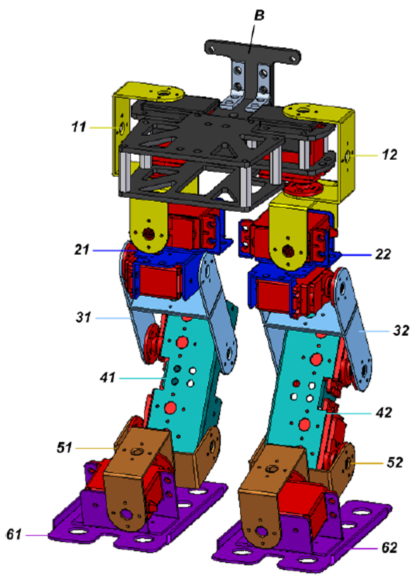

Fig. 3. CAD model of Scout, Narváez-Aroche (2010)

2.4 Nomenclature

In order to clarify the following methodology, let us define the nomenclature that is used in this work.

x_i, y_i, z_i, for $i = 1, 2$, denote cartesian position coordinates of left/right foot; $i = B$ denotes position of the body, $[mm]$.

$\dot{x}_i, \dot{y}_i, \dot{z}_i$, for $i = 1, 2$, denote velocity of left/right foot; $i = B$ denotes velocity of the body, $[mm/s]$.

$\ddot{x}_i, \ddot{y}_i, \ddot{z}_i$, for $i = 1, 2$, denote acceleration of left/right foot; $i = B$ denotes acceleration of the body, $[mm/s^2]$.

θ_i, ϕ_i, ψ_i, for $i = 1, 2$, denote orientation (Euler angles) of left/right foot; $i = B$ denotes orientation of the body, $[rad]$.

$\dot{\theta}_i, \dot{\phi}_i, \dot{\psi}_i$, for $i = 1, 2$, denote angular velocity of left/right foot; $i = B$ denotes velocity of the body, $[rad/s]$.

$\ddot{\theta}_i, \ddot{\phi}_i, \ddot{\psi}_i$, for $i = 1, 2$, denote angular acceleration of left/right foot; $i = B$ denotes acceleration of the body, $[rad/s^2]$.

Finally, $\theta_{ni}, \dot{\theta}_{ni}, \ddot{\theta}_{ni}$ denote angular position/velocity/acceleration of each link of the robot, i.e. for $i = 1, 2$ and $n = 1, ..., 6$.

It should be noted that **j** is the direction of walking, **i** is the direction perpendicular to the walking direction (both in the ground plane), **k** denotes the direction perpendicular to the groud plane, all of them forming a right-handed coordinate system.

For details, see Narváez-Aroche (2010).

3. PROPOSED METHODOLOGY

According to the ZMP criterion, it is easy to understand that the definition of a safety zone for the ZMP is necessary to prevent possible errors between the theoretical model and the real one. Indeed, without a safety zone, it is difficult to calculate a ZMP on the limit of the support polygon.

Our idea for computing the ZMP safety zone is simple. First, it is about looking into the physical conditions (in terms of position, velocity and acceleration) of the robot so as to place the ZMP on the support contact perimeter. Finally, using the concept of "error factors", the parameters are distorted and ZMP position is recalculated for each point of the convex hull of the contact points (see Fig. 4). The main hypothesis is: by the definition of percentages of error in position, velocity or acceleration (which are physically comprehensible) one can obtain a less conservative definition of the ZMP safety zone Wilmart (2013).

Nothing indicates that the support polygon perimeter transformation is linear, i.e. it is not sure that the rectangle making up the convex hull of the contact points turns into another smaller rectangle within the bigger one. On the contrary, in general it can be assumed that the rectangle might become an irregular shape within the original one. Thus, the transformation might be a nonlinear one.

In a recent work, López-García (2012), trajectory planning was made by using a genetic algorithm that finds the best parameters for the torso trajectory so as to make the robot gait more robust. In order to perform this optimization, it was necessary to define a safety zone for the ZMP, which was arbitrarily defined. A less conservative definition of it would lead to a better trajectory planning.

The developed programs are based in a previous program included in Narváez-Aroche (2010), where all parameters as link lengths, masses, inertias, etc. are defined.

A detailed explanation of the methodology follows.

St 1. Consider Fig. 4.

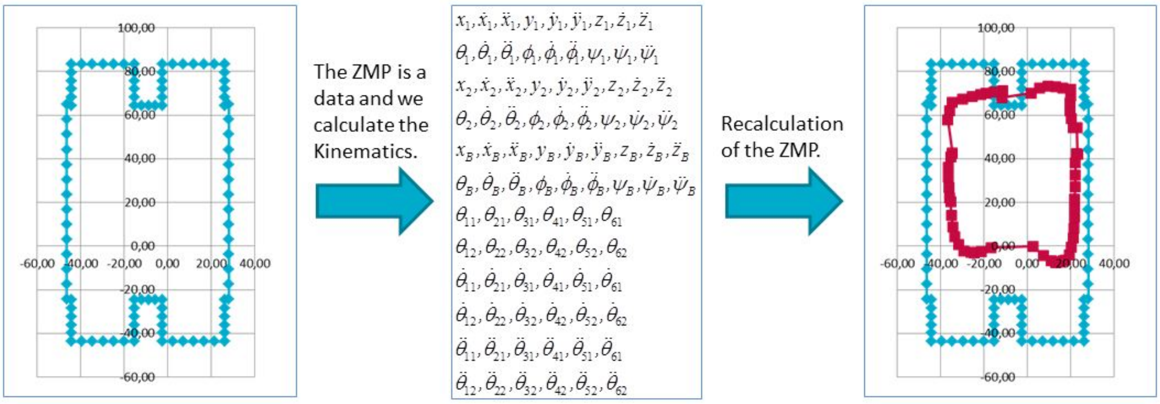

Fig. 4. Transformation of the foot (in blue) in the ZMP safety zone (in red).

Define a number of points NP on the foot perimeter and obtain (using the kinematic model) the following corresponding position-velocity-acceleration data:

$*$ 18 positions and orientations of left foot

$$(x_1, y_1, z_1), \ (\dot{x}_1, \dot{y}_1, \dot{z}_1), \ (\ddot{x}_1, \ddot{y}_1, \ddot{z}_1),$$
$$(\theta_1, \phi_1, \psi_1), \left(\dot{\theta}_1, \dot{\phi}_1, \dot{\psi}_1\right), \left(\ddot{\theta}_1, \ddot{\phi}_1, \ddot{\psi}_1\right)$$

All of them are defined as zero since left foot is considered fixed (single support phase), except for $x_1 = 44.54(mm)$.

$*$ 18 positions and orientations of right foot

$$(x_2, y_2, z_2), \ (\dot{x}_2, \dot{y}_2, \dot{z}_2), \ (\ddot{x}_2, \ddot{y}_2, \ddot{z}_2),$$
$$(\theta_2, \phi_2, \psi_2), \left(\dot{\theta}_2, \dot{\phi}_2, \dot{\psi}_2\right), \left(\ddot{\theta}_2, \ddot{\phi}_2, \ddot{\psi}_2\right)$$

The considered walking pattern is that the swinging foot remains parallel to the ground, that is angles θ_2, ϕ_2, ψ_2 are zero as well as their first and second time derivatives. For x_2, y_2, z_2 the following function is considered for $0 \leq t \leq T = 3.5(seg)$:

$$f(t) = \left(\frac{2(f_0 - f_1) + T(df_0 + df_1)}{T^3}\right) t^3 + ...$$
$$... + \left(\frac{3(f_1 - f_0) - T(2df_0 + df_1)}{T^2}\right) t^2 + ...$$
$$... + df_0 t + f_0$$

that is, for coordinate x the trajectory was defined with $f_0 = 0, f_1 = 5, df_0 = 0, df_1 = 0$; for coordinate y the trajectory was defined with $f_0 = 0, f_1 = 40, df_0 = 0, df_1 = 0$; for coordinate z the trajectory was defined with $f_0 = 0, f_1 = 50, df_0 = 0, df_1 = 0$. Velocities and accelerations were defined with the corresponding analytical derivatives. Finally, x_2, y_2, z_2 were set to be those when the velocity is maximal, that is

$$(x_2, y_2, z_2) = (47.04, 20, 25)\ [mm],$$
$$(\dot{x}_2, \dot{y}_2, \dot{z}_2) = (2.1429, 17.1429, 21.4286)\ [mm/s],$$
$$(\ddot{x}_2, \ddot{y}_2, \ddot{z}_2) = (0, 0, 0)\ [mm/s^2]$$

$*$ 18 positions and orientations of the body

$$(x_B, y_B, z_B) \ (\dot{x}_B, \dot{y}_B, \dot{z}_B), \ (\ddot{x}_B, \ddot{y}_B, \ddot{z}_B),$$
$$(\theta_B, \phi_B, \psi_B) \left(\dot{\theta}_B, \dot{\phi}_B, \dot{\psi}_B\right), \left(\ddot{\theta}_B, \ddot{\phi}_B, \ddot{\psi}_B\right)$$

The considered walking pattern is that the body does not tilt, that is angles θ_B, ϕ_B, ψ_B are zero as well as their first and second time derivatives, because we do not want the robot to bend over. Finally, seven restrictions about the torso are arbitrarily added.

$$(x_B, z_B) = (-50, 215)\ [mm]$$
$$(\dot{x}_B, \dot{y}_B, \dot{z}_B) = (2, 1, 2), [mm/s]$$
$$(\ddot{x}_B, \ddot{z}_B) = (2, 2)\ [mm/s^2]$$

$*$ 12 angular positions (one per link)

$$\theta_{11}, \ \theta_{21}, \ \theta_{31}, \ \theta_{41}, \ \theta_{51}, \ \theta_{61}$$
$$\theta_{12}, \ \theta_{22}, \ \theta_{32}, \ \theta_{42}, \ \theta_{52}, \ \theta_{62}$$

$*$ 12 angular velocities (one per link)

$$\dot{\theta}_{11}, \ \dot{\theta}_{21}, \ \dot{\theta}_{31}, \ \dot{\theta}_{41}, \ \dot{\theta}_{51}, \ \dot{\theta}_{61}$$
$$\dot{\theta}_{12}, \ \dot{\theta}_{22}, \ \dot{\theta}_{32}, \ \dot{\theta}_{42}, \ \dot{\theta}_{52}, \ \dot{\theta}_{62}$$

$*$ 12 angular accelerations (one per link)

$$\ddot{\theta}_{11}, \ \ddot{\theta}_{21}, \ \ddot{\theta}_{31}, \ \ddot{\theta}_{41}, \ \ddot{\theta}_{51}, \ \ddot{\theta}_{61}$$
$$\ddot{\theta}_{12}, \ \ddot{\theta}_{22}, \ \ddot{\theta}_{32}, \ \ddot{\theta}_{42}, \ \ddot{\theta}_{52}, \ \ddot{\theta}_{62}$$

These angular positions, velocities and accelerations are obtained from the inverse kinematics equations, which are:

$*$ 12 position equations

$$\mathbf{T}_{0,B}\mathbf{T}_{B,1i}\mathbf{T}_{1i,2i}\mathbf{T}_{2i,3i}\mathbf{T}_{3i,4i}\mathbf{T}_{4i,5i}\mathbf{T}_{5i,6i}\mathbf{T}_{6i,i} = \mathbf{T}_{0,i}$$

$*$ 12 velocity equations

$$\mathbf{v}_B^0 + \mathbf{v}_{0i}^0 + \mathbf{v}_{1i}^0 + \mathbf{v}_{2i}^0 + \mathbf{v}_{3i}^0 + \mathbf{v}_{4i}^0 + \mathbf{v}_{5i}^0 + \mathbf{v}_{6i}^0 = \mathbf{v}_i^0$$
$$\omega_i^0 = \omega_{6i}^0$$

$*$ 12 acceleration equations

$$\mathbf{a}_B^0 + \mathbf{a}_{0i}^0 + \mathbf{a}_{1i}^0 + \mathbf{a}_{2i}^0 + \mathbf{a}_{3i}^0 + \mathbf{a}_{4i}^0 + \mathbf{a}_{5i}^0 + \mathbf{a}_{6i}^0 = \mathbf{a}_i^0$$
$$\mathbf{a}_i^0 = \mathbf{a}_{6i}^0$$

All previous equations $\forall i = 1 \ , \ 2$.

In summary, since there are 90 unknowns and 52 restrictions, 38 equations are needed: 36 equations of inverse kinematics plus 2 equations for ZMP computation, (6)-(7).

St 2. Distort all data obtained from step 1 and recalculate the ZMP. That is, a ZMP that was originally calculated on the perimeter footprint will be shifted to another point within the support polygon as the result of the distortion of positions, velocities and accelerations, as we can see in Fig. 4 (each point in blue is shifted to a point in red).

The main idea is that an *a priori* distortion on those variables make physically more sense than an *a priori* distortion directly on the ZMP. Nevertheless, performing this idea verbatim would require to distort 90 variables in all possible combinations, thus demanding a lot of computational effort. For simplifying purposes the following three considerations are made.

2.1 First of all, only fifty-four parameters are used so as to recalculate a ZMP with equations (6)-(7). These are: positions, velocities an accelerations of the body (x_B, y_B, z_B and θ_B, ϕ_B, ψ_B and their first and second time derivatives) and the angular positions, velocities and accelerations of the links (θ_{ij}, for $i = 1, ..., 6$ and $j = 1, 2$, and their first and second time derivatives).

Each position-velocity-acceleration was modified by using one error factor for each one, that is $\hat{V} = s_v V$, where $\hat{V}$ is the distorted variable, V the original one (computed in St 1) and s_v the corresponding error factor.

The definition of the error factors is related to the error as $s_v = \frac{100 \pm \varepsilon(\%)}{100}$ that is, if one propose 10% of error (ε) in the corresponding variable, the error factor will be either 0.9 or 1.1.

Since these factors distort all variables, the ZMP will be as well distorted after recalculating it. As there exist two values for each error factor, the objective is to find the one which will allow calculating the best ZMP. As we can see in Fig. 5, a point of the support polygon perimeter can be shifted differently depending on the error factor value. The best point is considered to be the one that is nearest to the foot center. The modified points are shown as red crosses and their position vectors relative to the center of the footprint are in green. In Fig. 5 four possible configurations among an infinity are presented, just to illustrate the influence of error factors.

This first consideration reduces the computational effort, but is not enough since one has to find the right combination among fifty-four factors with two possible values. This means to choose the best combination among 2^{54}, i.e. 18,014,398,509,481,984. Even for a computer and especially for Mathematica®, this number of calculations represents a high effort. That is why two additional considerations follow.

2.2 Error factors are considered to be the same for each torso position (cartesian/angular with subscript B) and its time derivatives, that is,

$$s_1 = s_{x_B} = s_{\dot{x}_B} = s_{\ddot{x}_B} \qquad s_2 = s_{y_B} = s_{\dot{y}_B} = s_{\ddot{y}_B}$$
$$s_3 = s_{z_B} = s_{\dot{z}_B} = s_{\ddot{z}_B} \qquad s_4 = s_{\theta_B} = s_{\dot{\theta}_B} = s_{\ddot{\theta}_B}$$
$$s_5 = s_{\phi_B} = s_{\dot{\phi}_B} = s_{\ddot{\phi}_B} \qquad s_6 = s_{\psi_B} = s_{\dot{\psi}_B} = s_{\ddot{\psi}_B}$$

2.3 From Fig. 3 it can be seen that some parameter effects act in the same way, for example ($\theta_{31}, \theta_{41}, \theta_{51}$), ($\theta_{32}, \theta_{42}, \theta_{52}$) and ($\theta_{61}, \theta_{62}$), while some others act in opposite way as (θ_{21}, θ_{22}). That is, the last error factors are

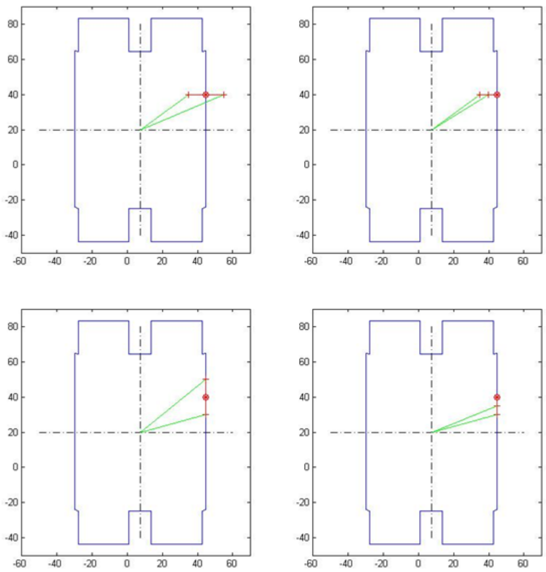

Fig. 5. Effect of error factors on the computation of the ZMP.

$$s_7 = s_{\theta_{31}} = s_{\theta_{41}} = s_{\theta_{51}} = s_{\dot{\theta}_{31}} = s_{\dot{\theta}_{41}} = s_{\dot{\theta}_{51}} = \\ = s_{\ddot{\theta}_{31}} = s_{\ddot{\theta}_{41}} = s_{\ddot{\theta}_{51}}$$
$$s_8 = s_{\theta_{32}} = s_{\theta_{42}} = s_{\theta_{52}} = s_{\dot{\theta}_{32}} = s_{\dot{\theta}_{42}} = s_{\dot{\theta}_{52}} = \\ = s_{\ddot{\theta}_{32}} = s_{\ddot{\theta}_{42}} = s_{\ddot{\theta}_{52}}$$
$$s_9 = s_{\theta_{21}} = s_{\dot{\theta}_{21}} = s_{\ddot{\theta}_{21}} = 2 - s_{\theta_{22}} = 2 - s_{\dot{\theta}_{22}} = 2 - s_{\ddot{\theta}_{22}}$$
$$s_{10} = s_{\theta_{11}} = s_{\dot{\theta}_{11}} = s_{\ddot{\theta}_{11}}$$
$$s_{11} = s_{\theta_{61}} = s_{\theta_{62}} = s_{\dot{\theta}_{61}} = s_{\dot{\theta}_{62}} = s_{\ddot{\theta}_{61}} = s_{\ddot{\theta}_{62}}$$
$$s_{12} = s_{\theta_{12}} = s_{\dot{\theta}_{12}} = s_{\ddot{\theta}_{12}}$$

The number of error factors finally is 12, which reduces the combination number to 2^{12}, i.e. 4,096.

These two last considerations may not have strict theoretical support, but they are shown to greatly reduce the simulation times without loss of consistency in the results. Future results on trajectory planning would allow to tell if they should be modified.

4. RESULTS

Results are shown in Fig. 6 for $NP = 102$. As expected, it can be seen that the transformation between the foot perimeter and the ZMP safety zone is not linear, since the rectangle turns into an irregular shape. These results were obtained by starting the Mathematica® program with a 10% error, on the left and a 5% error, on the right. It is clear that an increase in the error leads to a smaller zone that has to be considered for placing the ZMP during trajectory planning.

These results have an additional practical use: one can propose new positions for the pressure sensors on the feet of the robot based on the estimated error of its kinematic model. This way, one can more precisely verify if the measured ZMP is located within the safety zone.

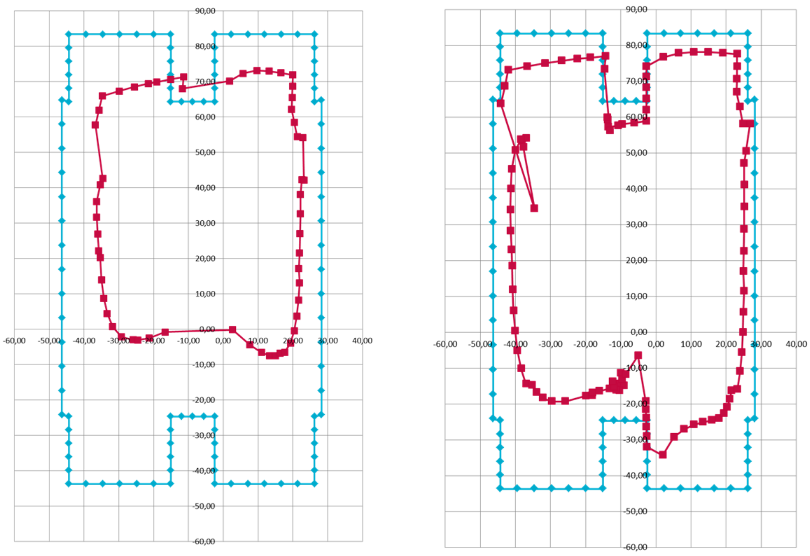

Fig. 6. ZMP safety zone in red obtained with a 10% error on the left and 5% error on the right.

5. CONCLUSIONS AND FUTURE WORK

A safety zone is a subset of the support polygon and it is of interest since some trajectory planning algorithms require it to assure stability according to the ZMP criterion. Sometimes this safety zone is arbitrarily defined, so the aim of this work was to have a better definition of it by using the kinematic model of the robot and the formal computation of the ZMP.

ZMP was fixed at each point of the perimeter of the support polygon and the corresponding configurations of the system in terms of position, velocity and acceleration of each of the robot links were calculated. According to minor modifications on these configurations, due to different error factors disturbing the parameters, new values for ZMP have been recalculated and located within the support polygon to define a safety zone. Results confirm the intuition that the transformation of the foot perimeter into the ZMP safety zone is not linear.

We expect that the proposed changes on the definition of the safety zone will allow us to better define the constraints used in the algorithm previously published in López-García (2012) and thus improve the robustness of the gait cycles that are generated for the Scout biped.

The present work represents the beginning of the study of this alternative ZMP safety zone computation methodology. In the future we will explore changes in the number of evaluated perimeter points and error factors definitions/constraints, depending mainly on the results obtained from trajectory planning.

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