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Cyclic pursuit formation control without collisions in multi-agent systems using discontinuous vector fields ⋆

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Abstract: This paper addresses the formation control problem without collisions for multi-agent systems. A general solution is proposed for the case of any number of agents moving on a plane. The solution is based on two modifications of the well known cyclic pursuit algorithm. First, a normalized version of the cyclic pursuit algorithm is employed in order to ensure finite time converge to the desired formation. Second, when two or more agents are in danger of collision, a repulsive vector field is added to the previous control law. The repulsive vector fields display an unstable focus structure suitably scaled. The proposed control law ensures that the agents reach the desired geometric pattern in finite time and that they stay at a distance greater than some prescribed lower bound for all times. Moreover, the closed-loop system does not exhibit undesired equilibria. Numerical simulations illustrate the good performance of the proposed solution.

Keywords: Multi-agent systems; Formation control; Collision avoidance; Discontinuous control.

1. INTRODUCTION

Formation control in multi-agent systems has received much attention in last years because of the wide range of applications as exploration, rescue tasks, toxic residues cleaning, etc. (Lin et al. (2004), Yan et al. (2010)). A very important issue in formation control is the collision avoidance problem, either with other agents or obstacles (Mousavi et al. (2012)). One difficulty arises when there exist constrains in the communication among agents, otherwise, the computational charge increases for completely centralized systems. In order to decentralize the scheme, it is assumed that every robot knows, for all time, the position of a specific subset of robots and, eventually, the position of any robot that violates a minimal distance specification (Bibuli et al. (2012)).

Initially, the non collision strategies were developed based on attractive and repulsive vector fields obtained as the negative gradient of potential functions. Usually, the attractive potential functions are centred, for each agent, at its desired position while the repulsive functions are centred at the position of the other agents or obstacles (Do (2006), Hernandez-Martinez (2011)). The repulsive potential functions depend on the distance of every pair of robots and appear smoothly when the distance becomes smaller and tend to infinity when the distance tends to zero. Then, the first drawback we find in this kind of algorithm is that we can only ensure that there will be no theoretical collisions, but the distance between two agents could become too small, and this could lead to possible actual collisions if the physical dimensions of agents are taken into account. Moreover, the combination of repulsive and attractive vector fields based on potential functions results in the appearance of undesired equilibrium points that can provoke the agents to get stuck at an undesired formation. In this same context, a solution to this problem was proposed with the requirement of having a totally centralized scheme (Dimarogonas (2006)).

In recent work, a new strategy for designing the repulsive vector fields has been proposed (Hernandez-Martinez (2013)). This approach is based on the use of scaled unstable focus structures centred at the position of others agents. These functions cannot be obtained as the gradient of a scalar function of the distance between agents. Although this technique can also lead to undesired equilibria, these can be easily removed. The key point is to use an unstable focus scaled by a function depending on the distance among agents. This scaling function vanishes when the agents are far enough and tend to infinity as distance tends to zero. The analysis in Hernandez-Martinez (2013) is presented for the case of two agents only.

In this paper we study the non collision problem in formation control using discontinuous vector fields for an arbitrary number of agents. In one hand we undertake the design of attractive vector fields based on the well known cyclic pursuit algorithm but, unlike the results reported in the literature (Marshal et al. (2004)), we focus our analysis to normalized vector fields. That is, regarding only the attractive field, the agents move at constant known velocity. On the other hand, the repulsive vector fields have the unstable focus structure scaled by a suitable constant. As mentioned before, the general problem of an arbitrary number of robots is treated and the designed controllers

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are proven to be effective from the case where no collision risk exist to the one when a robot is rounded by a set of robots and there could be collision with any of them.

This paper is organized as follows. In section II we state the problem establishing a couple of standing assumptions. The main contribution is given in section III, where we take into account all the possible scenarios of collisions. We start with the case where no collision risk exists. After this, the case of possible collision between two agents is analysed. Based on this simple case we extend the study to risk of multiple collisions. Some simulation results are presented in section IV. Finally, in section V, we list the conclusions and outlooks of this research.

2. PROBLEM STATEMENT

Consider a group of n mobile agents denoted by $N = \{R_1, \ldots, R_n\}$ moving in a plane, where the position of agent R_i is specified by $z_i(t) = [x_i(t), y_i(t)]^T \in \mathbb{R}^2$, $i = 1, 2, \ldots, n$. Every robot is described by the kinematic model

$$\dot{z}_i = u_i, i = 1, 2, \ldots, n, \tag{1}$$

where $u_i = [u_{i1}, u_{i2}]^T \in \mathbb{R}^2$ is the velocity along the X and Y axes. In this paper we adopt the cyclic pursuit formation graph. Therefore, the desired position of robot R_i, say z_i^*, with respect to robot R_{i+1} is defined by a constant vector $c_{i+1,i} = [h_{i+1,i}, v_{i+1,i}]^T$, that is,

$$\begin{aligned} z_i^* &= z_{i+1} + c_{i+1,i}, i = 1, 2, \ldots, (n-1) \\ z_n^* &= z_1 + c_{1n}. \end{aligned} \tag{2}$$

Control goal

The goal is to design decentralized control laws $u_i = \alpha(z_i, z_i^*)$, $i = 1, 2, \ldots, n$ such that:

i) The agents reach a desired formation, that is, $\lim_{t\to\infty}(z_i(t) - z_i(t)^*) = 0$, and
ii) There are no collisions among agents. Moreover, at all times robots remain at some distance greater than a predefined distance d from each other , i.e., $\|z_i(t) - z_j(t)\| \geq d$, $\forall t \geq 0$, $i \neq j$.

Throughout the paper, the following Assumptions are supposed to hold:

Assumption 1. The desired relative position of the mobile agents is assumed to satisfy the closed-formation condition (Hernandez-Martinez (2010)), that is,

$$c_{21} + c_{32} + \cdots + c_{1n} = 0. \tag{3}$$

Assumption 2. Agent R_i knows the position of agent R_{i+1} for all time and, eventually, can detect the presence of any other agent within a circle of radius d.

Assumption 3. The initial conditions of all agents satisfy $\|z_i(0) - z_j(0)\| > d$, $\forall i \neq j$.

3. CONTROL DESIGN

The strategy to reach the desired formation pattern is designed based on attractive vector fields proportional to the error,

$$\gamma_i = -k\tilde{z}_i, \quad i = 1, 2, \ldots, n, \tag{4}$$

where $\tilde{z}_i = z_i - z_i^*$ and $k > 0$. In this paper, we consider a normalized version of (4) to treat a suitable system where all the agents move at the same known velocity; namely,

$$\gamma_i = -\mu\frac{\tilde{z}_i}{\|\tilde{z}_i\|}, \quad i = 1, 2, \ldots, n, \tag{5}$$

where, μ is the constant velocity of all agents.

In order to avoid collision between any pair of robots, repulsive vector fields have to be designed. These vector fields are proposed in such a way that, for robot R_i there exists an unstable counterclockwise focus, centred at the position of the other potentially colliding robots. The general expression of the repulsive vector fields is

$$\beta_i = \varepsilon \sum_{j=1, j\neq i}^{n} \delta_{ij} \begin{bmatrix} (x_i - x_j) - (y_i - y_j) \\ (x_i - x_j) + (y_i - y_j) \end{bmatrix}, \tag{6}$$

where $\varepsilon > 0$ and the functions δ_{ij} depend on the distance between R_i and R_j, in the following way

$$\delta_{ij} = \begin{cases} 1, & \text{if} \quad \|z_i - z_j\| \leq d, \\ 0, & \text{if} \quad \|z_i - z_j\| > d, \end{cases} \tag{7}$$

where d is the minimum allowed distance between any pair of agents. It is clear that $\delta_{ij} = \delta_{ji}$, $i \neq j$ if the sensed area is the same for all agents Finally, the proposed control laws take the form

$$u_i = \gamma_i + \beta_i, \quad i = 1, 2, \ldots, n. \tag{8}$$

Our first result states the finite time convergence property when there is no risk of collision.

Proposition 1. Consider system (1) and the control law (5). Then, in the closed-loop system (1)-(5) the mobile robots converge to the desired formation in finite time.

Before proving Proposition 1, we present a preliminary Lemma.

Lemma 1. Consider the dynamical system $\dot{x} = Ax$, where the matrix A is Hurwitz. Then the *normalized* system $\dot{x} = AD(x)x$ with $D(x) = \text{diag}\left\{\frac{1}{\|x_1\|}, \ldots, \frac{1}{\|x_n\|}\right\}$ and $x = [x_1, \ldots, x_n]^T$, is stable with finite time convergence.

Proof. Since A is Hurwitz, then, for every matrix $Q = Q^T > 0$ there exists a matrix $P = P^T > 0$ such that the Lyapunov equation $A^T P + PA = -Q$ holds and $V = x^T Px$ is a Lyapunov function for the system $\dot{x} = Ax$. Taking $V = x^T Px$ as a Lyapunov function candidate and evaluating the time derivative along the trajectories of the normalized system we have

$$\dot{V} = x^T \left(D(x)A^T P + PAD(x)\right) x.$$

By construction, $D(x) > 0$, moreover, if $x_{min} = \min(x_1, \ldots, x_n)$ and bounding by the largest eigenvalue

$$\|D(x)\| \leq \frac{1}{\|x_{min}\|},$$

therefore

$$\dot{V} \leq x^T \left(A^T \frac{1}{\|x_{min}\|} P + P\frac{1}{\|x_{min}\|} A\right) x.$$

Since $\frac{1}{\|x_{min}\|}$ is a scalar and the equation $A^T P + PA = -Q$ holds. Then, taking $Q = I$ the time derivative is bounded from above by

$$\dot{V} \leq -\frac{1}{\|x_{min}\|}\|x\|^2.$$

Taking the euclidean norm, we can bound $\|x\| \leq \sqrt{n}\|x_{max}\|$, where $x_{max} = \max(x_1, \ldots, x_n)$, then

$$\dot{V} \leq -\sqrt{n}\frac{\|x_{max}\|}{\|x_{min}\|}\|x\|$$

If we regard the quadratic form $x^T Px$ as a norm for vector x, we can write

$$\dot{V} \leq -\sqrt{n}\ell\frac{\|x_{max}\|}{\|x_{min}\|}\left(x^T Px\right)^{\frac{1}{2}},$$

where ℓ is a proportionality constant. This leads to finally write the last expression as

$$\dot{V} \leq -\sqrt{n}\ell\frac{\|x_{max}\|}{\|x_{min}\|}V^{\frac{1}{2}},$$

that according to Bhat and Bernstein (2000), ensures convergence in finite time.

Now, keeping in mind Lemma 1, we carry on with the proof of Proposition 1 by showing the closed-loop system (1)-(5) has the same form of the normalized system above.

Proof. Consider the error coordinates $\tilde{z}_i = z_i - z_i^*$, $i = 1, \ldots, n$. For the closed-loop system (1)-(5) the dynamics of the system is given by

$$\begin{aligned} \dot{\tilde{z}}_1 &= -\mu\frac{\tilde{z}_1}{\|\tilde{z}_1\|} + \mu\frac{\tilde{z}_2}{\|\tilde{z}_2\|}, \\ &\vdots \\ \dot{\tilde{z}}_n &= -\mu\frac{\tilde{z}_n}{\|\tilde{z}_n\|} + \mu\frac{\tilde{z}_1}{\|\tilde{z}_1\|}. \end{aligned} \tag{9}$$

Or, in matrix form, by

$$\dot{\tilde{z}} = -\mu\left(\mathcal{L}(G) \otimes I_2\right)\tilde{z}^*, \tag{10}$$

where $\tilde{z} = [\tilde{z}_1, \ldots, \tilde{z}_n]^T$ is the error vector, $\mathcal{L}(G)$ is the Laplacian matrix of the desired formation, $\mu > 0$ is the constant velocity of agents, $\otimes$ denotes the Kronocker product, $I_2 \in \mathbb{R}^{2\times 2}$ is the identity matrix and $\tilde{z}^*$ is the modified error vector given by

$$\tilde{z}^* = \left[\frac{\tilde{z}_1}{\|\tilde{z}_1\|}, \ldots, \frac{\tilde{z}_n}{\|\tilde{z}_n\|}\right]^T, \tag{11}$$

which yields to write

$$\dot{\tilde{z}} = -\mu\left(\mathcal{L}(G) \otimes I_2\right)\left(D(\tilde{z}) \otimes I_2\right)\tilde{z} \tag{12}$$

or

$$\dot{\tilde{z}} = -\mu\left(\mathcal{L}(G)D(\tilde{z}) \otimes I_2\right)\tilde{z}, \tag{13}$$

where

$$D(\tilde{z}) = \text{diag}\left\{\frac{1}{\|\tilde{z}_1\|}, \ldots, \frac{1}{\|\tilde{z}_n\|}\right\}. \tag{14}$$

Since, the closed-loop system has the same form as the normalized system in the Lemma 1, then it is enough to show that the system $\dot{\tilde{z}} = -\mu\left(\mathcal{L}(G) \otimes I_2\right)\tilde{z}$ is asymptotically stable what was shown in Hernandez-Martinez (2010). Then, this implies convergence in finite time for the system (1)-(5). This concludes the proof. □

In order to analyse the relative distance between any pair of agents, it is useful to define the variables $p_{ij} = x_j - x_i$ and $q_{ij} = y_j - y_i$.

Remark 1. On the plane $p_{ij} - q_{ij}$, we identify the origin as the point where a collision between agents i-th and j-th occurs and a circle of radius d, centered at the origin. Outside this circle, only attractive vector fields prevail while inside the circle the discontinuous repulsive vector fields appear. This is shown in Figure 1.

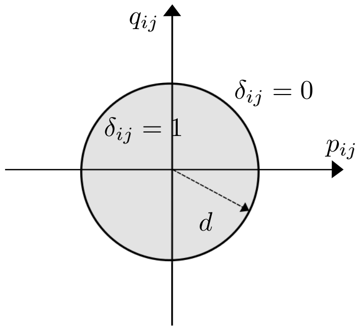

Fig. 1. Relative distance between agents i-th and j-th on the plane $p_{ij} - q_{ij}$.

Now consider the case when there exist collision risk between agents R_r and R_s only while the rest of agents are far enough each one from the others. Then $\delta_{sr} = 1$, and the dynamics of R_r and R_s are

$$\begin{aligned} \dot{z}_r &= -\mu\frac{\tilde{z}_r}{\|\tilde{z}_r\|} + \varepsilon\begin{bmatrix}(x_r - x_s) - (y_r - y_s)\\(x_r - x_s) + (y_r - y_s)\end{bmatrix}, \\ \dot{z}_s &= -\mu\frac{\tilde{z}_s}{\|\tilde{z}_s\|} + \varepsilon\begin{bmatrix}(x_s - x_r) - (y_s - y_r)\\(x_s - x_r) + (y_s - y_r)\end{bmatrix}. \end{aligned} \tag{15}$$

Proposition 2. Consider the system (1) and the control law (8) along with definitions (5)-(7). Suppose there is risk of collision between two agents only at time instant t and $\varepsilon > \frac{\mu}{d}$. Then, in the closed-loop system (1)-(8) the mobile robots reach their desired position and they stay for all $t \geq 0$ at a distance greater than or equal to d.

Proof. Assume first that there are $n-2$ agents without danger of a collision, then it is necessary to show R_r and R_s will avoid collision between them and keep at some minimum distance from each other. As mentioned in Remark 1, there exists a surface, given by

$$\sigma_{rs} = p_{rs}^2 + q_{rs}^2 - d^2 = 0 \tag{16}$$

where the composite control law become discontinuous. Under the mentioned scenario, the trajectories defined by p_{rs} and q_{rs} lie inside the circle $\sigma_{rs} \leq 0$. To determine the behaviour of trajectories under the action of repulsive vector fields, the time derivative of (16) is computed and evaluated along the closed-loop system.

The dynamics of relative position variables is

$$\begin{bmatrix}\dot{p}_{rs}\\\dot{q}_{rs}\end{bmatrix} = -\mu\left(\frac{\tilde{z}_s}{\|\tilde{z}_s\|} - \frac{\tilde{z}_r}{\|\tilde{z}_r\|}\right) + 2\varepsilon\begin{bmatrix}p_{rs} - q_{rs}\\p_{rs} + q_{rs}\end{bmatrix}. \tag{17}$$

Then,

$$\begin{aligned} \dot{\sigma}_{rs} &= 2\left[\,p_{rs}\;\; q_{rs}\,\right]\begin{bmatrix}\dot{p}_{rs}\\\dot{q}_{rs}\end{bmatrix}, \\ &= 2\left(-\mu\left[p_{rs}, q_{rs}\right]\left(\frac{\tilde{z}_s}{\|\tilde{z}_s\|} - \frac{\tilde{z}_r}{\|\tilde{z}_r\|}\right)\right) \\ &\quad + 4\varepsilon\left(p_{rs}^2 + q_{rs}^2\right) \end{aligned} \tag{18}$$

At this point, it is necessary to show that $\dot{\sigma}_{rs}$, in the internal region, is positive. That means that, the resulting vector field inside the discontinuous surface points outwards. To this sake, first consider the case when the attractive vector field points to inside the surface. Then, the constant ε should be selected in such a way that $\dot{\sigma}_{rs} > 0$. Taking the case where attractive fields for both agents are negative and restricting our analysis to the limit of the discontinuous surface, where $p_{rs}^2 + q_{rs}^2 = d^2$, we can easily check that the derivative of σ_{rs} is bounded from below by

$$\dot{\sigma}_{rs} > 2\left(-2\mu d + 2\varepsilon d^2\right), \tag{19}$$

therefore if $\varepsilon > \frac{\mu}{d}$ then $\dot{\sigma}_{rs} > 0$.

This implies that there will exist a repulsive resulting vector field between R_r and R_s such that they will reject each other at least until they reach a distance d. Moreover, since $\|z_r(0) - z_s(0)\| \geq d$, then the agents not only avoid the collision but also $\|z_r(t) - z_s(t)\| \geq d$ for all time. □

Remark 2. It is important to note that in case that the attractive vector fields outside the surface point to $\sigma_{ij} = 0$, then a sliding behaviour shall exist on the surface such that the agents keep at a distance d, until there exist conditions for the trajectories to leave this surface. Even though the control laws proposed in this paper are not intended to produce a sliding mode motion.This situation might occur depending on the initial conditions.

To generalize the problem we are discussing, it is insightful to consider now the situation of having three different robots R_r, R_s and R_t possible collision risks between R_r and R_s and R_r and R_t as shown in Figure 2.

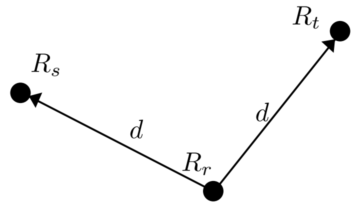

Fig. 2. Agent R_r in risk of collision with both R_s and R_t.

Proposition 3. Consider the system (1) and the control law (8) along with definitions (5)-(7). Suppose that there exists a risk of collision among three agents at time instant t, as shown in Fig. 2, and $\varepsilon > \frac{\mu}{d}$. Then, in the closed-loop system (1)-(8) the mobile robots reach their desired position and they stay at a distance greater than or equal to d for all $t \geq 0$.

Proof. In this case, the discontinuous surface consists of two different components. Each one is related with a pair of agents, that is

$$\sigma = \begin{bmatrix} \sigma_{rs} \\ \sigma_{rt} \end{bmatrix} = \begin{bmatrix} p_{rs}^2 + q_{rs}^2 - d^2 \\ p_{rt}^2 + q_{rt}^2 - d^2 \end{bmatrix}. \tag{20}$$

On the other hand, based in Fig. 2, since $\delta_{rs} = \delta_{rt} = 1$ we obtain the dynamics

$$\begin{aligned} \begin{bmatrix} \dot{p}_{rs} \\ \dot{q}_{rs} \end{bmatrix} &= -\mu\left(\frac{\tilde{z}_s}{\|\tilde{z}_s\|} - \frac{\tilde{z}_r}{\|\tilde{z}_r\|}\right) \\ &\quad + 2\varepsilon \begin{bmatrix} p_{rs} - q_{rs} \\ p_{rs} + q_{rs} \end{bmatrix} + \varepsilon \begin{bmatrix} p_{rt} - q_{rt} \\ p_{rt} + q_{rt} \end{bmatrix}, \end{aligned} \tag{21}$$

$$\begin{aligned} \begin{bmatrix} \dot{p}_{rt} \\ \dot{q}_{rt} \end{bmatrix} &= -\mu\left(\frac{\tilde{z}_t}{\|\tilde{z}_t\|} - \frac{\tilde{z}_r}{\|\tilde{z}_r\|}\right) \\ &\quad + 2\varepsilon \begin{bmatrix} p_{rt} - q_{rt} \\ p_{rt} + q_{rt} \end{bmatrix} + \varepsilon \begin{bmatrix} p_{rs} - q_{rs} \\ p_{rs} + q_{rs} \end{bmatrix}. \end{aligned} \tag{22}$$

To analyse the behaviour on the discontinuity surface consider the positive definite function

$$V = \frac{1}{4}\sigma^T\sigma. \tag{23}$$

Evaluating the derivative of V we have

$$\begin{aligned} \dot{V} &= \frac{1}{2}\sigma^T\dot{\sigma} = \frac{1}{2}\left(\sigma_{rs}\dot{\sigma}_{rs} + \sigma_{rt}\dot{\sigma}_{rt}\right) \\ &= \sigma_{rs}\left([p_{rs}, q_{rs}]\begin{bmatrix} \dot{p}_{rs} \\ \dot{q}_{rs} \end{bmatrix}\right) \\ &\quad + \sigma_{rt}\left([p_{rt}, q_{rt}]\begin{bmatrix} \dot{p}_{rt} \\ \dot{q}_{rt} \end{bmatrix}\right). \end{aligned} \tag{24}$$

To evaluate $\dot{V}$ on the discontinuous surface $p_{rs}^2 + q_{rs}^2 = p_{rt}^2 + q_{rt}^2 = d^2$ we consider the case when the trajectories lie in the internal region of $\sigma_{ij} = 0$, which implies $\sigma_{rs}, \sigma_{rt} < 0$, then

$$\begin{aligned} \dot{V} &= \sigma_{rs}\left(-\mu[p_{rs}, q_{rs}]\left(\frac{\tilde{z}_s}{\|\tilde{z}_s\|} - \frac{\tilde{z}_r}{\|\tilde{z}_r\|}\right) + 2\varepsilon d^2\right) \\ &\quad + \sigma_{rt}\left(-\mu[p_{rt}, q_{rt}]\left(\frac{\tilde{z}_t}{\|\tilde{z}_t\|} - \frac{\tilde{z}_r}{\|\tilde{z}_r\|}\right) + 2\varepsilon d^2\right) \\ &\quad + \varepsilon\left(\sigma_{rs} + \sigma_{rt}\right)[p_{rs}, q_{rs}]\begin{bmatrix} p_{rt} \\ q_{rt} \end{bmatrix} \\ &\quad + \varepsilon\left(\sigma_{rt} - \sigma_{rs}\right)\left(p_{rs}q_{rt} - p_{rt}q_{rs}\right). \end{aligned} \tag{25}$$

Without loss of generality, assume that $\sigma_{rs}, \sigma_{rt} < \sigma^*$, with $\sigma^* < 0$. Then consider the terms which depend on the relative positions of agents. Geometrically, the most negative case occur when

$$\begin{bmatrix} p_{rs} \\ q_{rs} \end{bmatrix} = -\begin{bmatrix} p_{rt} \\ q_{rt} \end{bmatrix} \tag{26}$$

then, the time derivative $\dot{V}$ is bounded from above by

$$\dot{V} \leq \sigma^*\left(-2\mu d + 4\varepsilon d^2 - 2\varepsilon d^2\right). \tag{27}$$

Since we want to ensure $\dot{V} < 0$ it is necessary to select ε such that

$$-2\mu d + 2\varepsilon d^2 > 0, \tag{28}$$

finally, if

$$\varepsilon > \frac{\mu}{d}, \tag{29}$$

then the trajectories of the closed-loop system point outwards of the surfaces $\sigma_{rs} = \sigma_{rt} = 0$. This concludes the proof. □

From now on, we add more agents to the collision problem. We have shown that 2 or 3 agents can not get closer than a predefined distance d. Therefore, the case when robot R_s is surrounded by 6 other agents is the limit of the above situation. Figure 3 shows this case. There, R_s is in danger of collision with 6 other robots, and any of these 6 robots is in danger of collision with other 2 robots only. This occurs when all of them are at distance d from each other.

Now we are in position to state our main result.

Proposition 4. Consider the system (1) and the control law (8) along with definitions (5)-(7). Suppose that there exists a risk of collision among $p+1$ agents at time instant t, as shown in Figure 3 and $\varepsilon > \frac{\mu}{d}$. Then, in the closed-loop system (1)-(8) the mobile robots reach their desired position and they stay at a distance greater than or equal to d for $p = 2, \ldots, 6$.

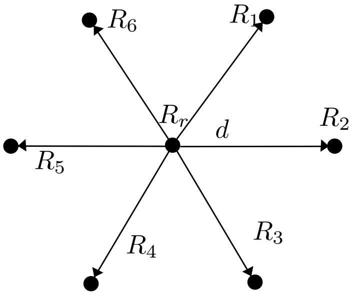

Fig. 3. Agent R_r in risk of collision with 6 different robots.

Proof. The Proof is lengthy and rather technical. Because of lack of space we present here a sketch of Proof only. For simplicity, and without loss of generality, let's call R_r the robot at the center of the configuration shown in Fig. 3 and assume that there are p robots $R_1, R_2, \ldots, R_p$ around R_r. The cases $p = 1$ and $p = 2$ have already been analysed, then $p = 3, 4, 5, 6$. In this case, the discontinuous surface σ is composed of p components given by

$$\sigma = \begin{bmatrix} \sigma_{r1} \\ \sigma_{r2} \\ \vdots \\ \sigma_{rp} \end{bmatrix} = \begin{bmatrix} p_{r1}^2 + q_{r1}^2 - d^2 \\ p_{r2}^2 + q_{r2}^2 - d^2 \\ \vdots \\ p_{rp}^2 + q_{rp}^2 - d^2 \end{bmatrix}. \tag{30}$$

According to Fig. 3, $\delta_{r1} = \delta_{r2} = \cdots = \delta_{rp} = 1$, therefore the dynamics of relative position variables are

$$\begin{aligned} \begin{bmatrix} \dot{p}_{rk} \\ \dot{q}_{rk} \end{bmatrix} &= -\mu \left(\frac{\tilde{z}_k}{\|\tilde{z}_k\|} - \frac{\tilde{z}_r}{\|\tilde{z}_r\|} \right) + 2\varepsilon \begin{bmatrix} p_{rk} - q_{rk} \\ p_{rk} + q_{rk} \end{bmatrix} \\ &\quad + \sum_{j=1, j\neq k}^{p} \varepsilon \begin{bmatrix} p_{rj} - q_{rj} \\ p_{rj} + q_{rj} \end{bmatrix}. \end{aligned} \tag{31}$$

Consider the positive definite function

$$V = \frac{1}{4}\sigma^T \sigma. \tag{32}$$

The derivative in terms of the functions σ_{ij} is

$$\dot{V} = \frac{1}{2}\sum_{k=1}^{p} \sigma_{rk}\dot{\sigma_{rk}} \tag{33}$$

and, rewriting the last expression in terms of relative position variables we have

$$\dot{V} = \sum_{k=1}^{p} \sigma_{rk}[p_{rk}, q_{rk}] \begin{bmatrix} \dot{p}_{rk} \\ \dot{q}_{rk} \end{bmatrix}. \tag{34}$$

If $\dot{V}$ is evaluated along the dynamics of these variables it becomes

$$\begin{aligned} \dot{V} = \sum_{k=1}^{p} \sigma_{rk} \Bigg[& -\mu[p_{rk}, q_{rk}] \left(\frac{\tilde{z}_k}{\|\tilde{z}_k\|} - \frac{\tilde{z}_r}{\|\tilde{z}_r\|} \right) \\ & + 2\varepsilon(p_{rk}^2 + q_{rk}^2) \\ & + \varepsilon[p_{rk}, q_{rk}] \sum_{j=1, j\neq k}^{p} \begin{bmatrix} p_{rj} - q_{rj} \\ p_{rj} + q_{rj} \end{bmatrix} \Bigg]. \end{aligned} \tag{35}$$

The main idea is to show that, again, under the condition $\varepsilon > \frac{\mu}{d}$, it follows that $\dot{V} < 0$ in the inner region of the hyper surface σ.

Since there does not exist any condition where a given agent can be in a risk of collision with more than 6 robots, this concludes de proof. □

4. SIMULATION RESULTS

In order to illustrate the performance of the proposed algorithm, a numerical simulation was carried out. The simulated system consist of nine mobile robots $R_1, \ldots, R_9$ and the goal is to reach the formation shown in Figure 4, where the relative position vectors are defined as $c_{21} = c_{32} = [0, 1.5]^T$, $c_{43} = c_{54} = [-1.5, 0]^T$, $c_{65} = c_{76} = [0, -1.5]^T$, $c_{87} = [1.5, 0]^T$, $c_{98} = [0, 1.5]^T$ and $c_{19} = [1.5, -1.5]^T$. The constant velocity when no collision risk exists is $\mu = 1$ and the minimum allowed distance is $d = 1$. According to the condition found above, the parameter ε was set to $\varepsilon = 2 \geq \frac{\mu}{d} = 1$ to ensure the minimum distance condition will not be violated.

Results are shown in Fig. 5 and Fig. 6. In Fig. 5 all the possible distances among agents have been drawn. Despite the apparent complexity of this graph, two aspects are to be emphasized. First, note that the distance between any pair of agents is always greater than or equal to the predefined distance $d = 1$. Second, and perhaps more interesting, note that some agents converge to their desired positions without sliding, while some others reach the discontinuity surfaces $\sigma_{rs} = 0$, and slide for some time interval until they eventually escape and reach the desired configuration. On the other hand, in Figure 6 the positions of agents corresponding to selected time instants are shown. The convergence to the desired formation becomes clear.

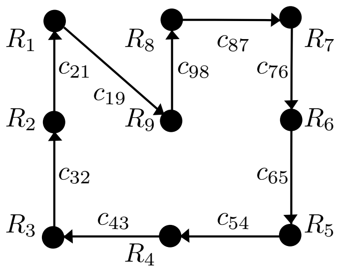

Fig. 4. Desired formation.

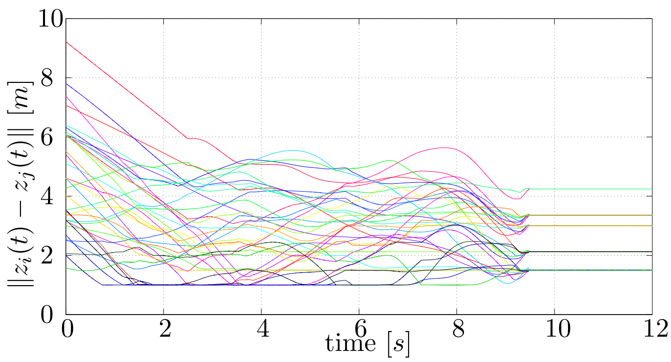

Fig. 5. Distances among the nine agents.

5. CONCLUSIONS

A solution to the general non collision problem in formation control has been proposed. This solution is based on

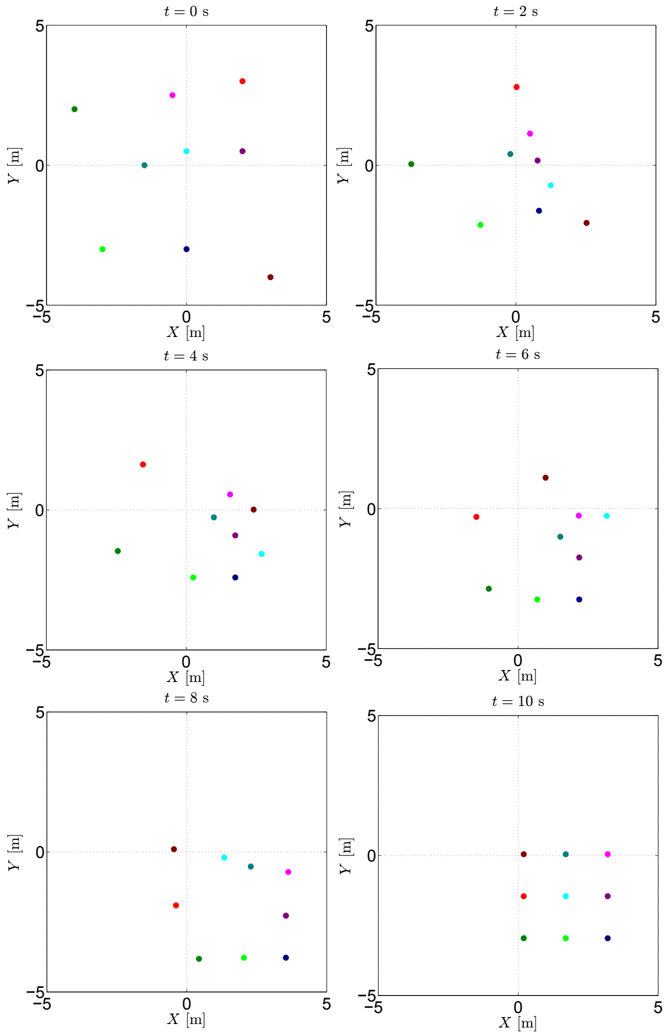

Fig. 6. Agent distribution in the plane at different times.

the combination of attractive and repulsive vector fields. The attractive forces are designed proportional to the error of each robot. The repulsive vector fields are designed as unstable focus centred at the position of the other robots. Besides, the attractive field was normalized to ensure the agents move at constant velocity when no danger of collision exists. As a by-product, finite time convergence is ensured. We analysed geometrically all the possible cases of multiple collisions and we proved the proposed control law is suitable in all situations, ensuring that the agents reach the desired geometric pattern in finite time and that they stay at a distance greater than a predefined bound. As a further research, the analysis can be extended not only to a more general class of formation graph but also to non-holomonic robots.

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