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PD controller for the stabilization of Third-order Unstable Linear Delayed Systems with Possible Complex Conjugate Poles

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Abstract: This work considers the stabilization problem of third-order unstable linear systems with time-delay, specifically with one unstable pole and a couple of complex conjugate poles. Necessary and sufficient conditions are stated to guarantee the stability of the closed-loop delayed system by a PD controller. Finally, the performance of the proposed control strategy is evaluated by numerical simulation in the application to an academic example.

Keywords: Time-delay, unstable system, complex conjugate poles, PD control.

1. INTRODUCTION

Control systems with time-delay play an important role in the modeling of real-life phenomena, for example, they are used for the analysis of energy or mass transport. Time delays can also appear due to the time associated with the transmission of information to remote locations and in digital control systems due to the time involved in computing control signals, data acquisition, etc. Niculescu (2001). In general, control system performance is very sensitive to all these delays. In fact, a closed-loop control system may become unstable as a consequence of delays Zhong (2006).

The effect of time delay can be compensated by removing the exponential term from the characteristic equation of the process as introduced by Smith (1957). This technique consists in counteracting the time delay effects by predicting the effects of current inputs by the analysis of future outputs. The main limitation of the original Smith Predictor (SP) is the fact that the prediction scheme lacks of a stabilization step. This limitation restricts the usefulness of the Smith Predictor just to open-loop stable plants. To solve this problem, some modifications of the SP original structure have been proposed. For example, in Rao and Chidambaram (2006) it is presented an efficient modification of the Smith predictor in order to control unstable first order system with time delay by using the direct synthesis method. Moreover, in Normey-Rico and Camacho (2009) it is proposed a modification to the original Smith structure to deal with unstable first and second order delayed systems.

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On the other hand, some recent works have been focused to the stability analysis of time-delay systems. Most of those approaches are based on Lyapunov-Krasovskii and Lyapunov-Razumikhin. These results are expressed in terms of algebraic Riccati equations Zhong (2006), linear matrix inequalities Fridman and Shaked (2002); Lee et al. (2004), etc. In Michiels et al. (2002), the control of time delay is tackled using a numerical method to shift the unstable eigenvalues to the left half plane by static state feedback, applying small changes to the feedback gain.

Another solution in order to deal with the time-delay systems is to use Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) controllers. PID controller is widely used in the control of industrial processes due to its simple structure and control stability in many practical processes Silva et al. (2005). Recent works inspired on PID controller have yielded the characterization of gains to stabilize time-delay systems. For instance, in Silva et al. (2002) they have been solved the stabilization of first-order systems with time-delay using a version of Hermite-Biehler Theorem derived by Pontryagin (1995) applicable to quasi-polynomial. Moreover, the method used in Silva et al. (2002) was generalized to the second-order integrating processes with time-delay Ou et al. (2006). For a first-order unstable system with time-delay, the D-partition technique was applied to characterize the stability domain in the space of system and controller parameters as shown in Hwang and J.Hwang (2004).

Otherwise, using the Nyquist stability criterion, in Muro et al. (2009) and Lee et al. (2010), they are presented the conditions to stabilize linear systems with one unstable pole, n stable poles and time delay. However in Muro et al.

(2009) the problem is solved by static output feedback, while in Lee et al. (2010) the result is extended to the use PID control.

In order to generalize this class of systems is interesting to consider not only the existence of real poles, but also the possible existence of complex conjugate poles. On this approach, in Hernandez et al. (2013) a forward step is given, where is consider the stabilization of a system with an unstable pole and one pair of complex conjugate poles by static output feedback. In same topic, the proposal of this work is present the necessary and sufficient conditions to ensure the stability of the closed-loop third-order delayed system with one unstable pole, and a couple of possible complex conjugate poles. The problem is solved using PD controller. It is important to note that the PD control provides a less restrictive condition than the simple P Control, for that reason in some cases is better apply a PD controller. Moreover, the necessary and sufficient condition in terms of the maximal delay admissible for the stabilizability is established and the range of the stabilizing control parameters (k_p, k_d) is also derived. Finally, in order to evaluate the proposed strategy, the PD controller are applied to an academic example to give an idea of the performance of the control strategy.

The rest of the paper is organized as follows. Section 2 presents the problem statement. The Section 3 presents the proposed control strategy, establishing the necessary and sufficient conditions to stabilize the system by a PD controller. Section 4 is devoted to presenting an academic example. Finally, Section 5 presents some conclusions.

2. PROBLEM STATEMENT

Consider the class of single–input single–output (SISO) linear time invariant (LTI) systems with time–delay in the input–output path given by

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} \tag{1}$$

where $U(s)$ is the input signal, $Y(s)$ is the output signal, $\tau > 0$ is a constant time–delay and $G(s)$ is the delay-free transfer function of the system given in the form

$$G(s) = \frac{\alpha}{(s-a)(s^2 + 2\zeta\omega_n s + {\omega_n}^2)} \tag{2}$$

that consist of one unstable pole, this is $a > 0$, and a couple of real poles. In particular, assuming ζ as the damping relation and ω_n as the undamped natural frequency, it is clear that when $0 < \zeta < 1$ the second order subsystems will produce a couple of complex conjugate poles.

This work proposes the stability analysis of the class of time–delay systems (1)-(2) in closed–loop with the following control action:

- PD controller: $C_{pd}(s) = k_p\,(1 + k_d s)$

Notice that for the previous controller, the obtained closed–loop system will have the general form

$$\frac{Y(s)}{R(s)} = \frac{C_{pd}(s)G(s)e^{-\tau s}}{1 + C_{pd}(s)G(s)e^{-\tau s}} \tag{3}$$

where $G(s)$ is given by (2). It is clear that the exponential term $e^{-\tau s}$ located at the denominator of the transfer function (3), leads to a system with an infinite number of poles and where the closed–loop stability properties must be carefully stated.

The objective of this work is to present necessary and sufficient conditions for the stabilization of the class systems given by (1)-(2) in closed–loop with PD control action. Moreover, it is intended to characterize all possible values for the parameters k_p and k_d suc that the closed–loop system is asymptotically stable.

3. PRELIMINARY RESULTS

This section presents a preliminary result that will be useful in order to obtain the main result of this work. Consider the following unstable third-order system with time–delay

$$\frac{Y(s)}{U(s)} = \frac{\alpha}{(s-a)(s^2 + 2\zeta\omega_n s + \omega_n^2)}e^{-\tau s} \tag{4}$$

and the proportional output feedback

$$U(s) = R(s) - kY(s) \tag{5}$$

where $R(s)$ is a new input signal and gain $k > 0$.

Lemma 1. Consider the unstable third-order input delayed system (4) and the output feedback (5). Then there exist $k > 0$ that stabilize the closed–loop system

$$\frac{Y(s)}{R(s)} = \frac{\alpha e^{-\tau s}}{(s-a)(s^2 + 2\zeta\omega_n s + \omega_n^2) + k\alpha e^{-\tau s}} \tag{6}$$

if and only if

$$\tau < \frac{1}{a} - \frac{2\zeta}{\omega_n}.$$

This result can be demonstrated with an analysis in the frequency domain. The proof of this result can be viewed in Hernandez et al. (2013). Based on the well-known Nyquist stability criteria, in what follows it is presented the proof of Lemma 1 following a procedure that will be fundamental latter on the proof of the main results of the work.

Proof. [Proof of Lemma 1] Suppose that $\tau < \frac{1}{a} - \frac{2\zeta}{\omega_n}$ is fulfilled. From the Nyquist stability criteria in order to have a stable system it is required to have a counterclockwise rotation to the point $(-1, 0)$. The open loop transfer function takes the form

$$\frac{Y(s)}{U(s)} = Q(s) = \frac{\alpha}{(s-a)(s^2 + 2\zeta\omega_n s + \omega_n^2)}e^{-\tau s}$$

from where it is possible to obtain its phase expression $\angle Q(j\omega)$ in the frequency domain ω as

$$\angle Q(j\omega) = -\left(\pi - \arctan\frac{\omega}{a}\right) - \omega\tau - \arctan\left(\frac{2\zeta\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right)$$

Notice first that the magnitud expression $M_{Q(j\omega)} = \|Q(j\omega)\|$ is an strictly decreasing function of ω.

Since $\angle Q(j\omega)$ has an initial phase angle of $-\pi$, then there exists an adequate $k > 0$ such that the Nyquist diagram will start on the left of the point $(-1, 0)$. In order to get the required rotation to the point $(-1, 0)$ the function $\angle Q(j\omega)$ should be increasing around $\omega = 0$, this is $\angle Q_p(j\omega) > -\pi$ for $\omega \approx 0$, since

$$\frac{d}{d\omega}\left(\angle Q(j\omega)\right)\bigg|_{\omega=0} = \left[\frac{a}{\omega^2 + a^2} - \frac{2\zeta\omega_n(\omega^2 + \omega_n^2)}{\omega^4 + 2\omega_n^2\omega^2\left(2\zeta^2 - 1\right) + \omega_n^4} - \tau\right]_{\omega=0}$$

$$\frac{d}{d\omega}\left(\angle Q(j\omega)\right)\bigg|_{\omega=0} = \frac{1}{a} - \frac{2\zeta}{\omega_n} - \tau > 0$$

producing the desired condition

$$\tau < \frac{1}{a} - \frac{2\zeta}{\omega_n}.$$

4. MAIN RESULTS

This Section presents necessary and sufficient conditions to stabilize the class of third-order linear input delayed system of the form (1)-(2) by means of PD controller.

Notice from (3) that the control strategy proposed in this work produces a direct open–loop transfer function which can be expressed in the form $Q(s)$

$$Q_{pd}(s) = C_{pd}(s)G(s)e^{-\tau s} \tag{7}$$

for $G(s)$ given by (2).

Theorem 1. Consider the class of third-order delayed systems with one unstable pole given by (1)-(2) and possible including complex conjugate poles. There exists a PD controller such that the corresponding closed–loop system is stable if and only if

$$\tau < \frac{1}{a} + \sqrt{\frac{1}{a^2} + \frac{2\left(2\zeta^2 - 1\right)}{\omega_n^2}} - \frac{2\zeta}{\omega_n}.$$

Proof. From the Nyquist stability criteria the system is stable iff $N + P = 0$, being P the number of poles in the right half plane "s" and N the number of rotations to the -1 point clockwise in the Nyquist diagram. In this case, as $P = 1$, from the Nyquist stability criteria, in order to have a stable system it is required to have a counterclockwise rotation to the point $(-1, 0)$.

Consider the system given by equations (1)-(2) and using a PD controller $C_{pd}(s) = k_p(1 + k_d s)$, the open-loop response in the frequency domain is represented by

$$Q_{pd}(j\omega) = k_p\alpha\frac{1 + k_d j\omega}{(j\omega - a)\left((j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2\right)}e^{-j\omega\tau}. \tag{8}$$

The phase expression in the frequency domain ω for (8) is obtained as

$$\angle Q_{pd}(j\omega) = -\left(\pi - \arctan\left(\frac{\omega}{a}\right)\right) - \omega\tau + \arctan\left(k_d\omega\right) - \arctan\left(\frac{2\zeta\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right) \tag{9}$$

and the magnitude $M_{Q_{pd}} = \|Q_{pd}(j\omega)\|$ is expressed as

$$M_{Q_{pd}(j\omega)} = k_p\alpha\sqrt{\frac{1 + k_d^2\omega^2}{(\omega^2 + a^2)\left(\omega^4 + 2\omega_n^2\omega^2\left(2\zeta^2 - 1\right) + \omega_n^4\right)}}. \tag{10}$$

As in the proof of Lemma 1, to assure the counterclockwise rotation to the point $(-1, 0)$ in the corresponding Nyquist diagram, it is sufficient to assure that $M_{Q_{pd}(j\omega)}$ is a monotonically decreasing function of ω and that the angle $\angle Q_{pd}(j\omega)$ should be an increasing function around $\omega = 0$ since $\angle Q_{pd}(0) = -\pi$.

The decreasing property of $M_{Q_{pd}}$ can be assured by considering the equivalent condition

$$\frac{d}{d\omega}\left(\frac{M_{Q_{pd}}^2}{k_p^2\alpha^2}\right) < 0$$

that after some computations produces

$$\left(\frac{\omega_n^2\left(4a^2\zeta^2 - a^2\left(k_d^2\omega_n^2 + 2\right) + \omega_n^2\right)}{a^4\omega_n^8}\right) > 0$$

or equivalently, it should be satisfied

$$k_d < \sqrt{\frac{1}{a^2} + \left(\frac{2\left(2\zeta^2 - 1\right)}{\omega_n^2}\right)}. \tag{11}$$

As in the case of the Lemma 1, since

$$\frac{d}{d\omega}\left(\angle Q_{pd}(j\omega)\right)\bigg|_{\omega=0} = \frac{a}{\omega^2 + a^2} + \frac{k_d}{k_d^2\omega^2 + a^2} - \frac{2\zeta\omega_n(\omega^2 + \omega_n^2)}{\omega^4 + 2\omega_n^2\omega^2\left(2\zeta^2 - 1\right) + \omega_n^4} - \tau$$

evaluating at $\omega = 0$, it is obtained

$$\frac{d}{d\omega}\left(\angle Q_{pd}(0)\right) = -\tau + \frac{1}{a} + k_d - \frac{2\zeta}{\omega_n} > 0$$

and therefore, the angle function will be increasing if

$$\tau < \frac{1}{a} + k_d - \frac{2\zeta}{\omega_n}. \tag{12}$$

From (11) and (12) it is possible to choose k_d within the range

$$\tau - \frac{1}{a} + \left(\frac{2\zeta}{\omega_n}\right) < k_d < \sqrt{\frac{1}{a^2} + \left(\frac{2\left(2\zeta^2 - 1\right)}{{\omega_n}^2}\right)}. \quad (13)$$

Finally, (12) can be rewritten by using (13) as the condition

$$\tau < \frac{1}{a} + \sqrt{\frac{1}{a^2} + \left(\frac{2\left(2\zeta^2 - 1\right)}{\omega_n^2}\right)} - \left(\frac{2\zeta}{\omega_n}\right). \quad (14)$$

The following corollary provide useful procedures to compute the range of k_p values that stabilize system (1)-(2) by a PD controller.

Corollary 1. Assume that conditions of the Theorem 1 hold. The range of k_p values that stabilize system (1)-(2) by means of a PD controller is given by

$$k_p(\omega_{c_1}) < k_p < k_p(\omega_{c_2})$$

where ω_{c_i} for $i = 1, 2$ are the crossover frequencies and $k_p(\omega_{c_i})$ are given by

$$k_p(\omega_{c_i}) = \frac{1}{\alpha}\sqrt{\frac{\left(\omega_{c_i}^2 + a^2\right)\left(\omega_{c_i}^4 + 2\omega_n^2\omega_{c_i}^2\left(2\zeta^2 - 1\right) + \omega_n^4\right)}{1 + k_d^2\omega_{c_i}^2}} \quad (15)$$

with $\omega_{c_1} = 0$ and ω_{c_2} being the smaller, non null, positive solution of

$$\arctan\left(\frac{\omega_{c_2}}{a}\right) - \omega_{c_2}\tau + \arctan\left(k_d\omega_{c_2}\right) - \arctan\left(\frac{2\zeta\left(\frac{\omega_{c_2}}{\omega_n}\right)}{1 - \left(\frac{\omega_{c_2}}{\omega_n}\right)^2}\right) = 0. \quad (16)$$

Proof. In order to assure the required counterclockwise rotation to the critical point $(-1, 0)$ when the condition $\tau < \frac{1}{a} + \sqrt{\frac{1}{a^2} + \left(\frac{2(2\zeta^2-1)}{\omega_n^2}\right)} - \left(\frac{2\zeta}{\omega_n}\right)$ holds, consider the magnitude expression $M_{Q_{pd}(j\omega)} = \|Q_{pd}(j\omega)\|$ given in (10) which decrease monotonically.

From the proof of Theorem 1, the properties of the function $\angle Q_{pd}(j\omega)$ states that starting with an adequate magnitude and the initial value $\angle Q_{pd}(0) = -\pi$ there exists a counterclockwise rotation to the point $(-1, 0)$. This implies that the phase diagram intersects the negative real axis for some positive frequency different from zero. From this fact, the range of k_p values that stabilize system (1)-(2) is given by $k_p(\omega_{c_1}) < k_p < k_p(\omega_{c_2})$, such that the frequencies ω_{c_1} y ω_{c_2} are computed from (16). Then, substituing that values in (15), we obtain the corresponding values for k_p such that the controlled system is stable. Consequently, there exists exactly one counterclockwise rotation to the point $(-1, 0)$.

5. EXAMPLE

The performance of the control strategy is evaluated by considering the linear approximation of an unstable continuously stirred tank reactor (CSTR). The following example has been taken from Bequette (2003). This model is particularly applied to the chemical process of a Propylene Glycol reactor system. A general description of the CSTR is shows in the Fig. 1 and a mathematical model is presented below.

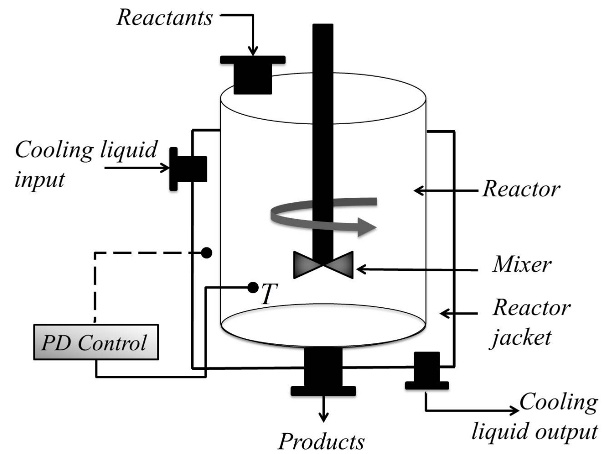

Fig. 1. Unstable Continuously Stirred Tank Reactor (CSTR).

Differential equations of the process shown in Fig. 1 are characterized as follows:

(1) Balance of the mass on component A is
$$\frac{dC_A}{dt} = \frac{F}{V}\left(C_{Af} - C_A\right) - k_0 e^{\frac{-\Delta E}{RT}} C_A.$$
(2) Energy balance in the reactor,
$$\frac{dT}{dt} = \frac{F}{V}\left(T_f - T\right) + \frac{-\Delta H}{RT} k_0 e^{\left(\frac{-\Delta E}{RT}\right)} C_A - \frac{UA}{VpC_p}\left(T - T_j\right),$$
(3) Energy balance in the cooling liquid,
$$\frac{dT_j}{dt} = \frac{F_{jf}}{V_j}\left(T_{jf} - T_j\right) + \frac{UA}{V_j p_j C_{pj}}\left(T - T_j\right),$$

where k_0 is thefrequency factor, ΔE is the activation energy and R is the ideal gas constant

The variables and parameters are now defined such as independent variables (input variables): Product flow A (F), flow cooling liquid (reactor jacket) F_{jf}. Dependent variables (output variables): Concentration of the product A (C_A), temperature in the Reactor (T), temperature in the cooling liquid reactor jacket (T_j). Measurable disturbances: A product concentration A in the reactor input (C_{Af}). Input temperature product A (T_f).

The parameters of the systems are characterized as follows: $V = 85ft^3$, $-\Delta E = 32.400Btu/lbmol$, $U = 75btu/hr°F$, $-\Delta H = 39.000Btu/lbmolPO$, $A = 88ft^2$, $k_0 = 16.96x10^{12}hr^{-1}$, $R = 1.987Btu/lbmol$, $V_j = 21.25ft^3\ °F$, $pc_p = 53.25Btu/ft^3\ °F$, $p_j c_{pj} = 55.6Btu/ft^3\ °F$.

Let us consider the jacket temperature as the manipulated variable and the concentration of the CSTR as the controlled variable. The next step is to linearize around an operating point the previous differential equations describing the system. The operating point is characterized under the following conditions (Bequette, 2003) : $C_{Af} = 0.132lbmol/ft^3$, $C_A = 0.066lbmol/ft^3$, $F = 340ft^3/hr$, $F_{jf} = 28.75ft^3/hr$, $T = 101.1°F$, $T_j = 55°F$, $T_f = 60°F$,

$T_{jf} = 50°F$. To linearize the differential equations, a Taylor series approximation, or a Jacobian matrix linear transformation around the operating point may be used. From the system linearization, it is possible to obtain a mathematical model using the following transfer function

$$\frac{C_A(s)}{T_{jf}(s)} = \frac{0.0646}{s^3 + 9.332s^2 + 16.89s - 34.45} e^{-.25s}, \qquad (17)$$

which has an unstable pole, a pair of complex conjugate poles. For the current example, the parameters of the system are $a = 1.1772$, $\tau = 0.25$, $\zeta = 0.97$ and $\omega_n = 5.4$.

Since the stability condition given in Theorem 1 is satisfied, so this system can be stabilized by a PD controller. According to (13), a stabilizing gain of k_d is taking from the range $-0.1904 < k_d < 0.8846$. Let us consider $k_d = 0.4$, then $\omega_c = 3.5266$ is solved numerically from (16), and $34.439 < k_p < 87.5176$ is determined from (15). For this example consider $k_p = 40$, such that $C_{pd}(s) = 40(1 + 0.4s)$. The Nyquist diagram is shown in Fig. 2, which indicates that the Nyquist criterion holds.

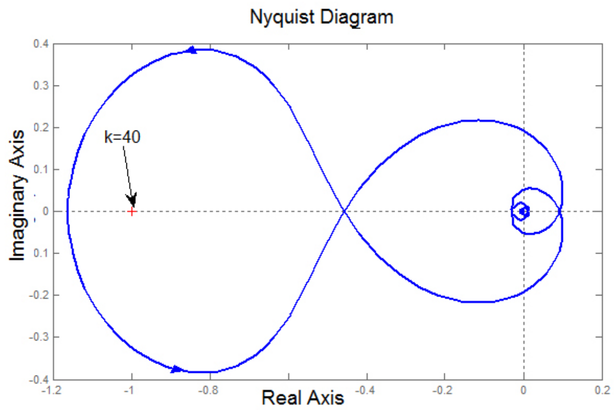

Fig. 2. Nyquist Diagram for PD controller

Finally, the Fig. 3 illustrates the performance of the output response of the system which is stabilized by means of the PD controller.

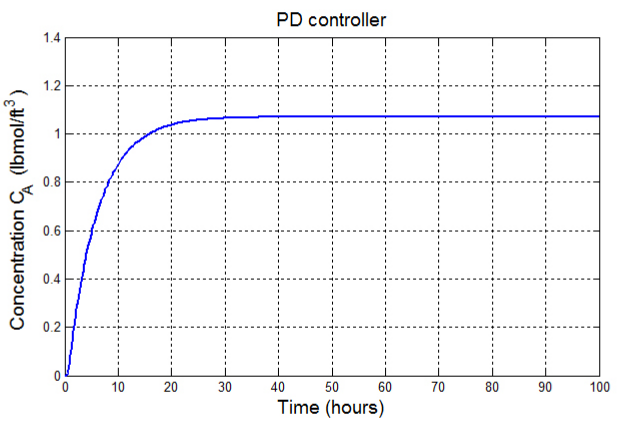

Fig. 3. Output signal for PD controller

6. CONCLUSION

Unstable systems with time delay are commonly found in industrial processes (such as material transport and chemical processes). This time delays add complications for study, besides being a challenge for system stabilization. This paper presents necessary and sufficient conditions for the stabilization by PD controller of third-order systems, with one unstable pole and couple of possible complex conjugate stable poles and time-delay. The comparable result has been reported in the literature in Lee et al. (2010) where only stable real poles of the model are presented. However, notice that the proposal of this work is to give a further step in this topic, addressing the particular problem of time-delayed systems with possible complex conjugate poles. In the same topic, this PD controller, provides a less restrictive condition than the simple P Control, such that, the delay can be larger than in the case of P Control. Besides, it is provide the procedure for determining the ranges of the stabilizing controller parameters k_p and k_d. The adequate performance of the proposed strategy was illustrated by means of an academic example.

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