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Fuzzy Fault Tolerant Control based on Unmeasurable Premise Variables: Quadratic Stability and LMIs

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Abstract: In this paper, a fault tolerant Fuzzy-Model-Predictive Control (FMPC) with integral action method for a class of nonlinear systems is proposed. At each sampling time, MPC solves an optimization to achieve desired set points and control objectives. The feasibility of optimization problem provides the guarantee of the nominal asymptotic stability. However the optimization can be infeasible due to faults. This motivates the development of the proposed approach to recover feasibility without violating constraints imposed on control inputs and system states. Nonlinear systems subject to actuators and/or sensors faults are described by Takagi-Sugeno (T-S) fuzzy model. The objective of this approach is to design a Fault Tolerant Controller (FTC). State vector and faults are estimated by a T-S fuzzy observer. The gains of the fuzzy observer and the pre-stabilized control law are obtained by solving a Linear Matrix Inequality (LMI) derived from the Lyapunov theory. The validity of the proposed FTC strategy with Unmeasurable Premise Variables (UPV) and its application to faults tolerance is illustrated by an academic example.

Keywords: Takagi-Sugeno models, FTC, MPC, Integral action, T-S fuzzy observer, LMI.

1. INTRODUCTION

For nonlinear systems, there exist some form of constraints due to physical, economic, safety or performance requirements on control inputs, control rates and/or system states. The ability to handle input and states constraints systematically in the control algorithm is one of the primary advantages of MPC. The MPC structure allows FTC to be embedded: constraints can be redefined, internal model and the control objectives can be changed. As the MPC is implemented in regulator form, the desired state of the reference model must be subtracted from the measured plant state. A large class of nonlinear systems can be well approximated by T-S fuzzy model. The T-S fuzzy modeling is based on the decomposition of the nonlinear system dynamic behavior around several operational areas (Takagi and Sugeno, 1985). According to the area where the system is, each sub-model contributes more or less to approximate the overall system behavior (Nagy, 2010). The Stability and stabilization of T-S fuzzy models are studied in (Tanaka et al., 1996), (Tanaka et al., 2001) and (Aouaouda et al., 2013). Among all the proposed approaches, Lyapunov theory and formulation of the stability conditions in terms of LMI are used. (Tanaka et al., 1998) studied quadratic stability but they found that it is difficult or impossible to calculate a common Lyapunov matrix satisfying a set of LMIs, as well as the number of submodels increases. (Tanaka et al., 2003) developed polyquadratic and non-quadratic approaches. These approaches are extended in (Bergsten and Palm, 2002) for observer design applied to state and unknown input estimation. Direct measurement of main characteristics of nonlinear systems is not available or very expensive. Estimation and observation techniques of these dynamic parameters are required for a control. In this paper, fuzzy model based observer is proposed for T-S fuzzy systems. The objective is to ensure the convergence of the state estimation errors to zero. When fault occurs, the main objective is to conserve the stability and the performances of the system. This paper relies on the idea of accommodating and tolerating actuators and/or sensors faults to maintain current performances closed to desired performances. The layout of this paper is as follows: in second section, a T-S fuzzy modelling methods are given. Then, FMPC with integral action is proposed. The section 4 shows the simulation results. Conclusion is finally presented in the last section.

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2. T-S FUZZY MODELLING

The T-S fuzzy model is based on rules such as (Takagi and Sugeno, 1985): IF PREMISE THEN CONSEQUENCE. Premises are obtained from linguistic propositions allowing the evaluation of weighting functions. The consequences correspond to sub-models. The considered system is based on an interpolation between the local linear models. Assume a set of N local models describing the dynamic behavior of the nonlinear system in different operation areas. Let h and g two nonlinear functions and $y \in \Re^p$ the output such as, the system state space representation is:

$$\begin{cases} \dot{x}(t) = h(x(t), u(t)) \\ y(t) = g(x(t)) \end{cases} \tag{1}$$

where $x \in \Re^n$ stands for the state and $u \in \Re^m$ denotes the control input. Activation function (weighting) $\mu_j(x(t), u(t))$ is normalized; it determines the activation degree of the j^{th} associated local model, by providing a gradual transition from this model to local model neighbors. These functions are generally triangular shaped, sigmoidal or Gaussian and satisfy the following properties:

$$\sum_{j=1}^{N} \mu_j(x(t), u(t)) = 1 \tag{2}$$

and $0 \leq \mu_j(x(t), u(t)) \leq 1,\ \forall\ j \geq 0$.

Three several methods can be employed to obtain a T-S model, through identification and parameters estimation from experimental data. Wang and Tanaka (Tanaka et al., 1996) obtain this convex polytopic representation by a direct transform of an affine model in the state. This method doesn't generate an approximation error and has an advantage of reducing the local model number. The other way is the linearization around different operating points with $A(\theta) = \nabla_x h(x, u)$, $B(\theta) = \nabla_u h(x, u)$ and $C(\theta) = \nabla_x g(x, u)$. From mathematical viewpoint, this corresponds to approximate the nonlinear function $h(.)$ through its tangent plane in the point (x_i, u_i). In this case, the number n of the local models depends on the desired precision of the modelisation, the nonlinear system complexity and the choice of the activation functions structure. Polytope is obtained with $N = 2^r$ peaks, where r is the number of premise variables. In (Ben Hamouda et al., 2013) and (Ben Hamouda et al., 2014a), the proposed non stationary linearization of a class of nonlinear system is clearly presented in three steps. The fuzzy model obtained is constituted by two sets of sub-Linear Time Invariant (LTI) model representing the lower and upper bounds $(\underline{\theta}, \bar{\theta})$, as described in (Tanaka et al., 2001), (Aouaouda et al., 2013), (Ichalal et al., 2012), (Ben Hamouda et al., 2014b) and (Djemili et al., 2012). The precise knowledge of the upper and lower bounds is not always possible. In fact, premise variable depend on nonlinearities system. For Some systems the lower and upper bounds are unknown. The influence of this problem are discussed in (Nagy, 2010) through an example.

The T-S fuzzy model obtained is given by the following relation:

$$\begin{cases} \dot{x}(t) = \sum_{j=1}^{N} \mu_j(\theta) \left(A_j x(t) + B_j u(t)\right) \\ y(t) = \sum_{j=1}^{N} \mu_j(\theta) C_j x(t) \end{cases} \tag{3}$$

where $A_j \in \Re^{n\times n}$, $B_j \in \Re^{n\times m}$ and $C_j \in \Re^{p\times n}$ are constant matrices and θ represents the premise variables vector depending on system states and input (Orjuela et al., 2009), (Nagy, 2010) and (Rodrigues et al., 2008). $\{A_j, B_j\}$ are the sub-models asymptotically stable matrices. The choice of the premises variables is based on a set of criteria. These criteria are constructed in accordance with stability analysis and/or observability objectives. The choice of this set is important, as it affects the number of sub-models and the global model structure. This degree of freedom is used to facilitate the study of controllability, observability and stability analysis (Tanaka et al., 2001) and (Bergsten and Palm, 2002). A multimodel compound of minimal number of sub-models is preferred. Premise variables should depend on a minimal number of state variables. The structure fuzzy model is described by the weighting functions $\mu_j(\theta)$. The j^{th} rule of the T-S fuzzy models is of the following form:

IF $\theta_1(x, u)$ IS M_{j1} AND ... AND $\theta_r(x, u)$ IS M_{jr}

THEN $\begin{cases} \dot{x}(t) = A_j\ x(t) + B_j\ u(t) \\ y(t) = C_j\ x(t), j = 1, 2, ..., N \end{cases}$

In the next section, a T-S fuzzy controller for nonlinear systems subject to faults is proposed. The main objective is to tolerate faults while achieving tracking desired trajectory.

3. PROPOSED FAULT TOLERANT FUZZY-BASED MODEL PREDICTIVE CONTROL STRATEGY

3.1 Studied MPC-based strategy

The aim of MPC is to minimize a cost function J:

$$J_k = \sum_{l=1}^{H_p} \parallel y_{k+l} - y^d_{k+l|k} \parallel^2_Q + \sum_{l=0}^{H_u-1} \parallel \Delta u_{k+l|k} \parallel^2_R \tag{4}$$

to compute the optimal control for $x_{k+1} = \bar{A}^j x_k + \bar{B}^j u_k$, subject to the following constraints:

$$\begin{aligned} &x_{\min} \leqslant x_l \leqslant x_{\max},\ \text{where } k+1 \leqslant l \leqslant k + H_p. \\ &u_{\min} \leqslant u_l \leqslant u_{\max},\ \Delta u = u_k - u_{k-1}, \\ &\Delta u_{\min} \leqslant \Delta u_l \leqslant \Delta u_{\max},\ \text{where } k \leqslant l \leqslant k + H_u - 1 \\ &\Delta U = \left[\, \Delta u_k\ \ \Delta u_{k+1}\ \cdots\ \Delta u_{k+H_u-1} \,\right]^T \end{aligned}$$

where y is the predicted response and y_d is the output desired trajectory. The matrices Q and R are used to weight the corresponding control errors and control actions. The R matrix helps to keep the control inputs within bounds, making sure that smooth control actions result. H_p and H_u are output and control prediction horizons, respectively. In general, a short control horizon makes the system more robust to uncertainties such as parameter variations. It is also assumed that the dynamic system defined by the model (A_j, B_j) is controllable. The controllability condition is required to ensure that the MPC optimization solved at each step is feasible. This optimization can be formulated as a quadratic programming (QP) problem (Maciejowski, 2002). Only the first control increment $\Delta u(k)$ is implemented and the optimization is re-solved at each step. The feasibility with the MPC provides the guarantee of the nominal asymptotic stability. When fault occurs, the optimization may become infeasible.In the next subsection, an FTC strategy is proposed to recover feasibility without violating constraints imposed on control inputs and system states.

3.2 The structure of the proposed FTC strategy

The FTC strategy scheme is given by Fig. 1. The method uses a T-S fuzzy observer to estimate system states and to detect faults (green part). The proposed FMPC approach should maintain system output closed to the desired trajectory obtained by the reference model and preserve stability conditions even when the faults occurs (black part).

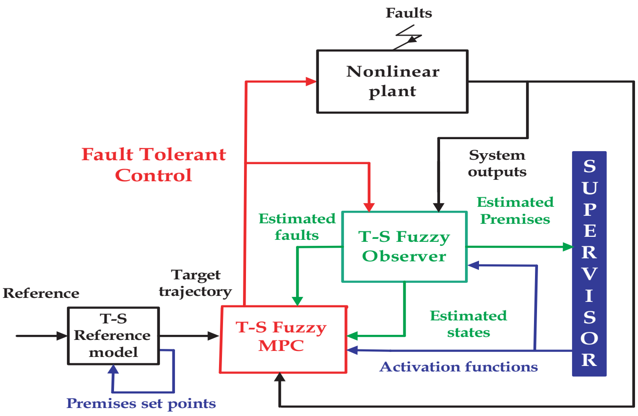


Fig. 1. The Fault Tolerant Control strategy based on T-S fuzzy diagnosis with UPV

The proposed j^{th} control law signal generated (red part) in the nominal operating is given by the following:

$$\begin{cases} u^j_{k+i|k} = -k^j_x\,(\hat{x}_{k+i|k} - x_{k+i|k}) - k^j_I\, x^I_{k+i|k} + q_j, \\ i = 0, \ldots, H_u - 1 \text{ and } j = 1, \ldots, N \end{cases} \tag{5}$$

where $\dot{x}_I = y_d - y$, $k^1_I = k^2_I = 1$ are the integral action gain, $\hat{x}$ represents the estimated state, $k_{u1}, k_{u2}, \ldots, k_{uN}$ are the N state feedback gains and q_j the j^{th} predicted control input calculated by the FTC. After the end of the control horizon, the MPC control q_j is set to zero and the control law becomes:

$$u^j_{k+i|k} = -k^j_x\,(\hat{x}_{k+i|k} - x_{k+i|k}) - k^j_I\, x^I_{k+i|k},\ i \geqslant H_u \tag{6}$$

Kale and Chipperfield (Kale and Chipperfield, 2005) introduce a straightforward strategy by assuming state feedback as a baseline controller to which MPC control signals are added. The pre-stabilization provides an effective tool to guarantee nominal closed-loop stability using the MPC controller, (Afonso and Galvao, 2010). Stability proofs of such formulation are given in (Maciejowski, 2002). Providing further robustness and accuracy is the feedback aim (Ben Hamouda et al., 2013). The integral action helps to drive the tracking error to zero. In the faulty case, the nonlinear system described by (1) becomes:

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{N} \mu_i(\theta_f)\left(A_i x_f(t) + B_i u_f(t) + E^i_a f(t)\right) \\ y_f(t) = \sum_{i=1}^{N} \mu_i(\theta_f)\left(C_i x_f(t) + E^i_s f(t)\right) \end{cases} \tag{7}$$

where $f \in \Re^f$ is the fault signal and E_a and E_s represent the fault matrices with appropriate dimensions.
The strategy given by Fig. 1 is proposed to determine the control inputs $u_f(t)$ such that:

- the closed-loop system (7) is stable,
- $x_f(t)$ converges asymptotically to the reference state vector even in the presence of faults.

When the actuator and/or sensors are faulty, UPV depend on the estimated faulty state vector, hence the fuzzy control law is based on the estimated premise variables. The following control strategy is then used:

$$u_f(t) = \sum_{j=1}^{N} \mu_j(\hat{\theta}_f)\left(-\hat{f}(t) - k^j_x\left(\hat{x}_f(t) - x(t)\right) + u(t)\right) \tag{8}$$

where $\hat{f}$ is the fault estimate vector and $u(t)$ is the nominal control input given by (5) and (6), with the interpolation mechanism (blue part): $u(t) = \sum_{j=1}^{N} \mu_j(\hat{\theta}(t)) u_j(t)$. The activation functions μ_1 and μ_2 are defined by:

$$\mu_1(\hat{\theta}_f) = \frac{\hat{\theta}_f(\hat{x}_f) - \underline{\theta}}{\bar{\theta} - \underline{\theta}} \quad \text{and} \quad \mu_2(\hat{\theta}_f) = 1 - \mu_1(\hat{\theta}_f) \tag{9}$$

Generally, it is assumed that state variables are accessible to control the system. Unfortunately in practice, it is rarely that state variables are directly measurable. It is the reason why a reliable estimation of unmeasurable variables is necessary. The FMPC design objective is to compute k^j_x in such a way that the closed-loop system including the state and fault estimations is stable. To estimate simultaneously $\hat{x}_f(t)$ and $\hat{f}(t)$, a T-S fuzzy observer is used for system (7):

$$\begin{cases} \dot{\hat{x}}_f(t) = \sum_{i=1}^{N} \mu_i(\hat{\theta}_f)\left(A_i \hat{x}_f(t) + B_i u_f(t) + E^i_a \hat{f}(t) + L_i(y_f - \hat{y}_f)\right) \\ \dot{\hat{f}}(t) = \sum_{i=1}^{N} \mu_i(\hat{\theta}_f)\left(G_i C_i(x_f - \hat{x}_f(t)) + G_i E^i_s(f - \hat{f}(t))\right) \end{cases} \tag{10}$$

The extended error system, containing the two error dynamics $x_f(t) - \hat{x}_f(t)$ and $f(t) - \hat{f}(t)$:

$$\begin{pmatrix} \dot{x}_f(t) - \dot{\hat{x}}_f(t) \\ \dot{f}(t) - \dot{\hat{f}}(t) \end{pmatrix} = \sum_{i=1}^{N} \mu_i(\hat{\theta}_f) \begin{pmatrix} A_i - L_i C_i & E^i_a - L_i E^i_s \\ -G_i C_i & -G_i E^i_s \end{pmatrix} \begin{pmatrix} x_f(t) - \hat{x}_f(t) \\ f(t) - \hat{f}(t) \end{pmatrix} \tag{11}$$

The tracking error $e(t) = x(t) - x_f(t)$ is given by:

$$\dot{e}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \mu_i(\theta)\mu_j(\theta_f) \begin{pmatrix} (A_i - B_i K^j_x)e(t) - E^i_a(f(t) - \hat{f}(t)) \\ -B_i K^j_x(x_f(t) - \hat{x}_f(t)) \end{pmatrix} + I_{n\times n}\Delta_1(t) \tag{12}$$

where $\Delta_1(t) = \sum_{i=1}^{N} (\mu_i(\theta) - \mu_i(\theta_f))\,(A_i x(t) + B_i u(t))$.
An extended error system $\tilde{e}(t)$, containing the tracking error $e(t)$, the state estimation error $x_f(t) - \hat{x}_f(t)$ and the fault estimation error $f(t) - \hat{f}(t)$, can be expressed as:

$$\dot{\tilde{e}}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \mu_i(\hat{\theta}_f)\mu_j(\theta_f)\tilde{A}_{ij}\tilde{e}(t) + \Gamma\Delta(t) \tag{13}$$

where:

$$\tilde{e}(t) = \begin{pmatrix} x(t) - x_f(t) \\ x_f(t) - \hat{x}_f(t) \\ f(t) - \hat{f}(t) \end{pmatrix}, \Gamma = \begin{pmatrix} I_{n\times n} & 0 \\ 0 & I_{n\times n} \\ 0 & 0 \end{pmatrix}, \ \Delta = \begin{pmatrix} \Delta_1(t) \\ \Delta_2(t) \end{pmatrix},$$

$$\tilde{A}_{ij} = \begin{pmatrix} A_i - B_i K^j_x & -B_i K^j_x & -E^i_a \\ 0 & A_i - L_i C_j & E^i_a - L_i E^j_s \\ 0 & -G_i C_j & -G_i E^j_s \end{pmatrix} \text{ and}$$

$$\Delta_2(t) = \sum_{i=1}^{N} \left(\mu_i(\theta_f) - \mu_i(\hat{\theta}_f)\right) \begin{pmatrix} A_i x_f(t) + B_i u_f(t) \\ +E^i_a f(t) \end{pmatrix}$$

Hypothesis 1. It is assumed that the following conditions are checked:

- The term $\Delta(t)$ is bounded.
- The open-loop system is stable.

The stability analysis of system (14), guarantying the tracking performance under the L^2-gain, allows to introduce the *Theorem* 2.

Theorem 2. The tracking error $e(t)$, the state estimation error $x_f(t)-\hat{x}_f(t)$ and the fault estimation error $f(t)-\hat{f}(t)$ converge asymptotically to zero, if there exists symmetric positive definite matrices X_1 and P_2, $P_3 = I$, gain matrices K_x^j, $\bar{L}_i$ and G_i and a positive scalar $\bar{\gamma}$ solutions of the following optimization problem:

$$\min_{X1, P_2, K_x^j, \bar{L}_i, G_i} \quad \bar{\gamma},$$

such that the following LMIs are verified:

$$\begin{pmatrix} \Omega_i & -B_i K_x^j & -E_a^i & -B_i K_x^j & X1 & X1 & 0 \\ * & \Xi_{ij} & \Psi_{ij} & 0 & 0 & 0 & P_2 \\ * & * & Z_{ij} & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\bar{\gamma} I & 0 \\ * & * & * & * & * & * & -\bar{\gamma} I \end{pmatrix} < 0, \qquad (14)$$

$$\begin{aligned} &\Omega_i = A_i X1 + X1 A_i^T \\ &\Xi_{ij} = P_2 A_i + A_i^T P_2 - \bar{L}_i C_j - C_j^T \bar{L}_i^T \\ &\Psi_{ij} = P_2 E_a^i - \bar{L}_i E_s^j - C_j^T \bar{G}_i^T \\ &Z_{ij} = -\bar{G}_i E_s^j - {E_s^j}^T G_i^T \\ &i, j = 1, \ldots, N \end{aligned}$$

The gains of the controller are K_x^j and the gains of the observer are given by $L_i = P_2^{-1} \bar{L}_i$ and G_i. The attenuation rate is obtained by $\gamma = \sqrt{\bar{\gamma}}$.

Proof: The proof is given in the appendix.

4. SIMULATION RESULTS

Consider the nonlinear system described by the following differential equations form (Ben Hamouda et al., 2013):

$$\begin{cases} \dot{x}_1(t) = -x_1(t) + u(t) \\ \dot{x}_2(t) = x_1(t) - |x_2(t)|\, x_2(t) - 10 \\ y(t) = x_2(t) \end{cases} \qquad (15)$$

Let us consider the following T-S reference model:

$$\begin{cases} \dot{x}(t) = \sum_{j=1}^{2} \mu_j(\theta)\left(A_j x(t) + B_j u(t)\right) \\ y(t) = \sum_{j=1}^{2} \mu_j(\theta)\left(C_j x(t)\right) \end{cases} \qquad (16)$$

where $A_1 = \begin{bmatrix} -1 & 0 \\ 1 & -2\bar{\theta} \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 0 \\ 1 & -2\underline{\theta} \end{bmatrix}$, $B_1 = B_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$, $C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$ and $\theta = |x_2|$ with $\bar{\theta} = 9.5$ and $\underline{\theta} = 0.5$. The choice of the values was made based on the respect of the activation function propriety, as explained in the second section (part B). The objective is to design a constrained FMPC. Through figures, simulation results demonstrate the effectiveness and the applicability of the proposed FTC strategy. A T-S fuzzy model with premise variable depend on unmeasurable state variables is used to design the observer and the controller. The tuning parameters used in the MPC are given in Table 1.

Table 1. Controller tuning parameters

Sample time Te	$0.5\ s$
Prediction horizon H_p	$8\ Te$
Control horizon H_u	$6\ Te$
Input constraints	$-25 \leqslant u_k \leqslant 25$
Output constraints	$-6 \leqslant y_k \leqslant 6$, $\forall k \geqslant 0$
Input weights R	0.1
Output weights Q	1

Two scenario faults (actuator and sensor) are given by (a) in Fig.2 and (a) in Fig.4, with $\dot{f}(t) = 0$:

$$f(t) = \begin{cases} f_1(t), & 25 \le t < 40 \\ f_1(t) + f_2(t), & t \ge 40 \end{cases}$$

Solutions satisfying stability conditions under LMIs in *Theorem* 2. are found with the attenuation rate value: $\gamma = 0.9417$. Solving the optimization problem results in the following matrices:

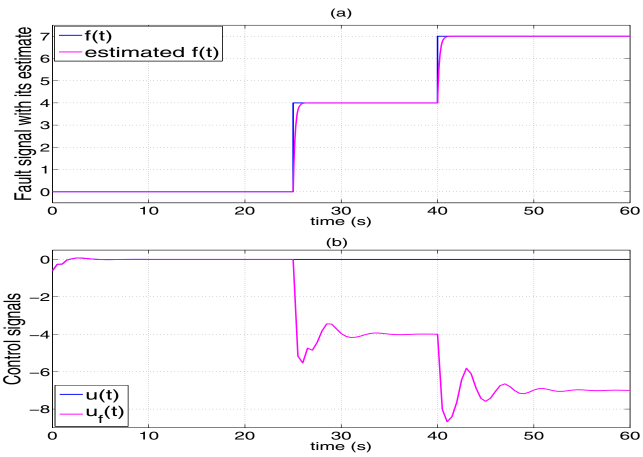
$$X1 = \begin{bmatrix} 0.3073 & 0 \\ 0 & 0.6144 \end{bmatrix}, \; P_2 = \begin{bmatrix} 0.8973 & 0.2107 \\ 0.2107 & 0.9286 \end{bmatrix}$$

The LMIs given by (14) are solved with the YALMIP toolbox (Lofberg, 2004) and the semidefinite programming SeDuMi-Solver. The SeDuMi interface was developed at the Laboratory of Architecture and Analysis of Systems (LAAS) by D. Peaucelle et al. The designed controller and observer gains are:

$$K_x^1 = K_x^2 = \begin{bmatrix} 0.0096 & 0.6146 \end{bmatrix}, \; L_1 = \begin{bmatrix} 1.8528 \\ -10.5144 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 0.6584 \\ 1.9390 \end{bmatrix}, \; G_1 = \begin{bmatrix} 5.3346 \\ 0 \end{bmatrix} \; \textit{and } G_2 = \begin{bmatrix} 4.9879 \\ 0 \end{bmatrix}$$

The top of Fig.2 and Fig.4 show that actuator and sensor faults are estimated with a high accuracy. From (b) in Fig.2 and (b) in Fig.4, it is observed that the nominal control law obtained using the FTC strategy is equal to $u_f(t)$ before the occurrence of faults. Figures 3 and 5 illustrate the state estimation errors together with the state tracking errors. From (f) in Fig.5, it is shown how $\mu_1(\hat{\theta})$ increases when the fault occurs. From the simulation results, it is concluded that the performances of the FMPC strategy are very satisfactory and allow normal functioning of the system even in the occurrence of actuator sensor faults.



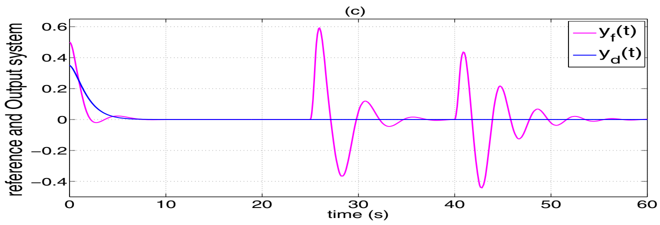


Fig. 2. Fault with its estimate (a), FMPC (b) and output system (c) signals vs.time in the actuator faulty case

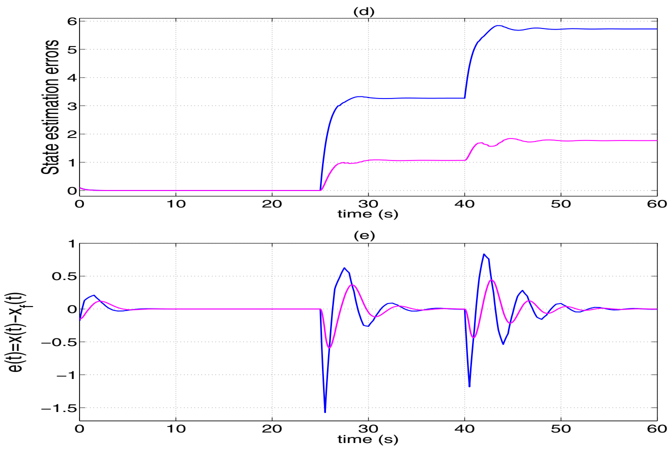


Fig. 3. State estimation errors (d) and state tracking errors (e) signals vs.time in the actuator faulty case

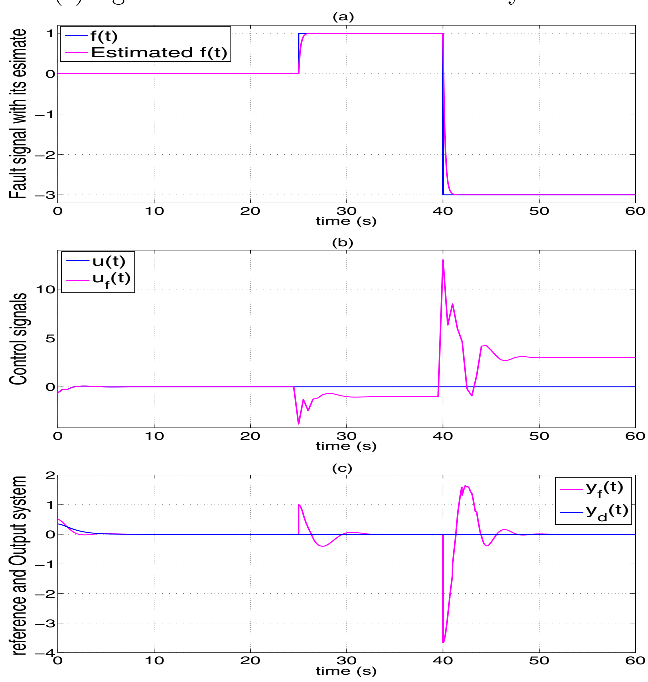
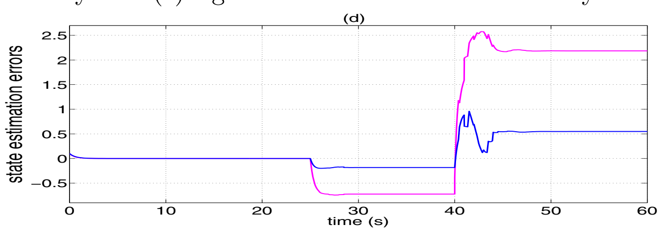


Fig. 4. Fault with its estimate (a), FMPC (b) and output system (c) signals vs.time in the sensor faulty case



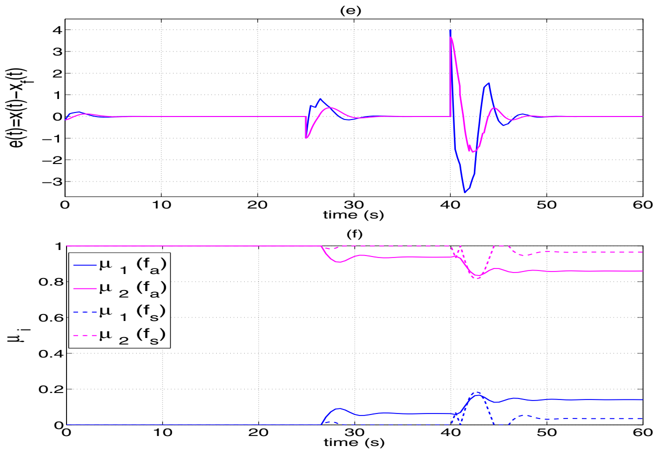


Fig. 5. state estimation errors (d), state tracking errors (e) signals in the sensor faulty case and variation of the activation functions according to the UPV vs.time

5. CONCLUSION

The main contribution of this paper is the development of a new FTC strategy for a class of nonlinear systems. A T-S fuzzy observer is designed for the proposed strategy, to estimate unmeasurable states and faults. The proposed controller accommodates faults properly and ensures the stability convergence of the closed-loop system. Thus the FMPC maintained good tracking performances. The occurrence of faults did not cause infeasibility problems, instability and constraints dissatisfaction. New sufficient conditions for the existence of the robust FTC are developed in terms of LMI constraints. The proposed FMPC with integral action is shown to be able of providing an optimal control law in the nominal and faulty operation.

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Appendix A. PROOF

Lemma 3. Let us consider two matrices X and Y of appropriate dimensions. The following inequality is verified for each matrix Q:

$$X^TY + XY^T \leq X^TQ^{-1}X + YQY^T$$

Lemma 4. (Schur complement) The following two inequalities are equivalent:

1. $\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0$ where $Q = Q^T$ and $R = R^T$
2. $R > 0$, $Q - SR^{-1}S^T > 0$.

The proof of the *Theorem* 2. is established using the following Lyapunov's function:

$$V(\tilde{e}(t)) = \tilde{e}(t)^T P \tilde{e}(t),\ P = P^T > 0 \tag{A.1}$$

where the matrix P is defined as follows:

$$\begin{pmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{pmatrix}$$

The derivative of $V(\tilde{e}(t))$ is written as:

$$\dot{V}(\tilde{e}(t)) = \sum_{i=1}^{N}\sum_{j=1}^{N} \mu_i(\hat{\theta}_f)\mu_j(\theta_f)\left(\tilde{e}(t)^T\ \Upsilon_{ij}\ \tilde{e}(t)\right) \tag{A.2}$$

with

$$\Upsilon_{ij} = \Lambda\left(\begin{pmatrix} P_1A_i - P_1B_iK_x^j & -P_1B_iK_x^j & -P_1E_a^i \\ 0 & P_2A_i - P_2L_iC_j & P_2E_a^i - P_2L_iE_s^j \\ 0 & -P_3G_iC_j & -P_3G_iE_s^j \end{pmatrix}\right)$$

where $\Lambda(X)$ denote the Hermitian of the matrix X:

$$\Lambda(X) = X^T + X$$

The derivative of the Lyapunov function is negative if the following inequality is satisfied

$$\Upsilon_{ij} < 0,\ i,j = 1,\ldots,N \tag{A.3}$$

using the lemma of congruence as follows:

$$\Upsilon_{ij} < 0 \Leftrightarrow \begin{pmatrix} P_1^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix} \Upsilon_{ij} \begin{pmatrix} P_1^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix} \tag{A.4}$$

The following inequality is obtained:

$$\begin{pmatrix} \xi_{ij}^1 & -B_iK_x^j & -E_a^i \\ * & \xi_{ij}^2 & P_2E_a^i - P_2L_iE_s^j - C_j^TG_i^TP_3 \\ * & * & -P_3G_iE_s^j - {E_s^j}^TG_i^TP_3 \end{pmatrix} < 0 \tag{A.5}$$

where:

$$\xi_{ij}^1 = A_iX1 + X1A_i^T - B_iK_x^jX1 - X1{K_x^j}^TB_i^T$$
$$\xi_{ij}^2 = P_2A_i + A_i^TP_2 - \bar{L}_iC_j - C_j^T\bar{L}_i^T$$

with $X1 = P_1^{-1}$. The inequality (A.5) can be written as:

$$\begin{pmatrix} A_iX1 + X1A_i^T & -B_iK_x^j & -E_a^i \\ * & \xi_{ij}^2 & P_2E_a^i - P_2L_iE_s^j - C_j^TG_i^TP_3 \\ * & * & -P_3G_iE_s^j - {E_s^j}^TG_i^TP_3 \end{pmatrix}$$
$$+ \begin{pmatrix} -B_iK_x^j \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} X1 \\ 0 \\ 0 \end{pmatrix}^T + \begin{pmatrix} X1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} -B_iK_x^j \\ 0 \\ 0 \end{pmatrix}^T < 0 \tag{A.6}$$

Using the *Lemma* 3., the inequality (A.6) becomes:

$$\begin{pmatrix} A_iX1 + X1A_i^T & -B_iK_x^j & -E_a^i \\ * & \xi_{ij}^2 & P_2E_a^i - P_2L_iE_s^j - C_j^TG_i^TP_3 \\ * & * & -P_3G_iE_s^j - {E_s^j}^TG_i^TP_3 \end{pmatrix} +$$
$$\begin{pmatrix} -B_iK_x^j \\ 0 \\ 0 \end{pmatrix}\Theta^{-1}\begin{pmatrix} -B_iK_x^j \\ 0 \\ 0 \end{pmatrix}^T + \begin{pmatrix} X1 \\ 0 \\ 0 \end{pmatrix}\Theta\begin{pmatrix} X1 \\ 0 \\ 0 \end{pmatrix}^T < 0 \tag{A.7}$$

where Θ is a symmetric definite positive matrix. Using the *Lemma* 4., we obtain the LMIs of *Theorem* 2., with $\bar{L}_i = P_2L_i$, $\bar{G}_i = P_3G_i$ and $\Theta = I$.

The objective is to minimize the L^2-gain of the perturbation transfer from $\Delta(t)$ to the errors $\tilde{e}(t)$, this is formulated by:

$$\frac{\|\tilde{e}(t)\|_2}{\|\Delta(t)\|_2} < \gamma, \quad \|\Delta(t)\|_2 \neq 0 \tag{A.8}$$

Then, the problem can be formulated as follows:

$$\dot{V}(\tilde{e}(t)) + \tilde{e}(t)^T\tilde{e}(t) - \gamma\Delta(t)^T\Delta(t) < 0 \tag{A.9}$$