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Port Hamiltonian approach for inducing limit cycles ⋆

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Abstract: The induction of limit cycles by means of the **IDA-PBC** methodology, for an affine nonlinear system, was introduced. To this we expressed the closed-loop system as a Port Hamiltonian system, with the property of almost all their trajectories asymptotically converge to a convenient limit set, except for a set of measure zero. The control strategy was tested using the pendulum actuated by a DC-motor, having obtained satisfactory results.

Keywords: Port Hamiltonian system, Stabilization, Lyapunov method, Nonlinear control, Limit cycles

1. INTRODUCTION

The compact set stabilization is used to produce periodical stable orbits. There are several oscillatory systems that can be controlled by means of set stabilization. For instance, the stabilization of the well-known inverted pendulum cart system to its up-right position, starting from the hanging position, can be accomplished in two steps: first the system is rendered into a homoclinic orbit, which, in fact, can be seen as a limit set; finally, the second step consists of switching from a nonlinear to a linear controller in order to stop the pendulum in its unstable position and the cart in some desired position. Other useful applications are found in digital and analogical electronics (clock generator circuits, function generators, ramp function applications, PWM converter, motor drivers, etc.) and the mechanisms field (controlled oscillating to rotational movement converters as in cam or quick return mechanisms, development of mechanism for probe vehicles design again induced oscillations, etc.) Boylestad et al. (1999); Tocci (1988); Rao and Yap (1995); Shigley et al. (1989).

The first line of searching about set stabilization problem was originally introduced in Fradkov (1994); Fradkov et al. (1995); Fradkov (1996); Andrievsky et al. (1996); Fradkov et al. (1997).All these works were devoted to showing that the well-known speed-gradient method allows the desired energy hypersurface with an arbitrary small control effort. In Shiriaev and Fradkov (2000, 2001); Shiriaev (2000), a control strategy for the invariant set stabilization was developed. This strategy is also based on the Chetaev's method and the speed-gradient method, in conjunction with the V-detectability property. In other very important works, developed by Pagano et al. (2005); Aracil et al. (2005); Gómez-Estern et al. (2005); Freidovich et al. (2009); Albea et al. (2007); Fantoni and Lozano (2002); Shiriaev and Canudas-de Wit (2005).

In this document the Interconnection and Damping Assignment Passivity-Based Control (**IDA-PBC**) is applied to obtain asymptotic stability towards a limit set. The strategy consists of shaping the closed-loop system as a target port Hamiltonian system, which assures that almost all the trajectories converge to some limit set. The corresponding stability analysis was carried out applying the theorem of LaSalle in conjunction with the abovementioned properties of the Hamiltonian systems Ortega and Garcia-Canseco (2004a,b); Garcia-Canseco et al. (2005).

Due the fact a review of the works related to the **IDA-PBC** is beyond the scope of this work; the interested reader could be referred to Acosta et al. (2005); Blankenstein et al. (2002); D Mahindrakar et al. (2006); Ortega et al. (2002); Gómez-Estern and Van der Schaft (2004); Rodriguez and Ortega (2003); Viola et al. (2007); Fujimoto et al. (2003); Duindam et al. (2009); Batlle et al. (2008, 2009).

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2. COMPACT SET STABILIZATION BY USING THE IDA-PBC METHODOLOGY

Problem statement: Consider the single control system described by:

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

where $x \in R^n$ and $u \in R$; with $n \geq 2$. The maps $f(x)$ and $g(x)$ are at least locally smooth from N into R^n; where N is an open and connect set. For simplicity, it is assumed that $x = 0$ is the equilibrium point of the system (1), that is, $f(0) = 0$. The control objective is to propose a control law $u(x)$, which, when in closed-loop with the system (1), makes it follow some admissible limit cycle, E, which is an isolated closed curve if the system is of second order or an energy level for higher orders, where any trajectory started inside of E remains inside it forever Marquez (2003); Ghaffari et al. (2009). In the present case, the limit cycle E is defined, as:

$$E = \{x(t) \in R^n : S(x(t)) = \mu\}; t \geq 0, \tag{2}$$

where the set formed by $S(x) \leq \mu$ is compact. To achieve this, $S(x)$ is selected as strictly positive, with a local minimum at the origin. That is, $S(x)$ could be selected in such a way, that:

$$\nabla S(x)|_{x=0} = 0 \ ; \ \nabla^2 S(x)\big|_{x=0} > 0, \tag{3}$$

where $\nabla = \partial/\partial x$ and $\nabla^2 S(x)$ is the Hessian of $S(x)$.[1] In the sequel, this condition over the set E is referred to as assumption **A1**. For example, in a second-order system, the curve:

$$S(x) = (1 - \cos x_1) + \frac{1}{2}x_2^2;$$

satisfies the condition of **A1**, if $\mu < \overline{\mu} < 2$.

Remark 1: *The approach consists of a guarantee that the state variable x of (1) in a closed-loop, converges to some bounded energy level set $S(x) = \mu$, where the set $\{x \in R^n, n \geq 2 : S(x) \leq \mu\}$ is compact. In short, we try to make the trajectories of the system (1) remain inside of some bounded admissible surface or some level energy set, where its structure is shaped by applying the **IDA-PBC** methodology. We use this methodology because it allows maintenance and exploiting the system's original structure, without the need to introduce dynamic nonlinear cancelations or nonlinearly domination Ortega (2003), as is done with traditional control techniques. For instance, in Fradkov (1994); Fradkov et al. (1995, 1997), the problem of controlling oscillations is accomplished using the descendant gradient method, while in Shiriaev and Fradkov (2000, 2001); Shiriaev (2000) the traditional Lyapunov method provides sufficient conditions to guarantee stability in invariant sets.*

2.1 Review of the IDA-PBC methodology

This methodology consists of finding a state feedback $u(x)$, which transforms the original system (1) into a desired port Hamiltonian system Garcia-Canseco et al. (2005), with a particular wanted stability property. In the present case, is is necessary to force the closed-loop system to converge asymptotically to the surface $S(x) = \mu$. That is, the closed-loop system must be written as if it were an explicit Hamiltonian system of the form:

$$\dot{x} = (J(x) - D(x))\, \nabla H_d(x), \tag{4}$$

where $D(x) = D^T(x) > 0$ and $J(x) = -J^T(x)$; $H_d(x)$ is the desired Hamiltonian of the closed-loop system, having the property of:

$$H_d(x) = 0; \forall x \in E.$$

Thus, we say that the systems (1) and (4) match, for some control law $u(x)$, if their solutions are the same. That is, $(x,u(x))$ is a solution of (1), if and only if $x(t)$ is a solution of (4). To make (1) equal (4), the following applies:

$$f(x) + g(x)u = (J(x) - D(x))\, \nabla H_d(x), \tag{5}$$

where $J(x)$, $D(x)$ and $H_d(x)$ are the design control variables. Due to the fact that $u(x)$ only acts in the range space of $g(x)$, we must verify the following matching condition:

$$g^{\perp}(x)\, [f(x) - (J(x) - D(x))\, \nabla H_d(x)] = 0; \tag{6}$$

where $g^{\perp}(x)$ is a full-rank left annihilator of $g(x)$; *i.e.*, $g^{\perp}(x)g(x) = 0$; for all $x \in R^n$. It should be emphasized that in the case where the target system (4) is given, $u(x)$ can be directly computed, by:

$$u = -\frac{g^T(x)}{g^T(x)g(x)}\, [(J(x) - D(x))\, \nabla H_d(x) - f(x)]\,. \tag{7}$$

In general, the system (4) is refereed as the desired closed-loop or the target Hamiltonian system. Notice that the above expression is well defined, as long as $g(x) \neq 0$; for all $x \in \mathbf{D} \subset R^n$, where the set $\mathbf{D}$ is related to the domain of attraction of the system (4).

It is important to mention that there are many strategies to solve the above matching conditions, because (6) involves the solution of a nonlinear **PDE**, subject to the sign constraint $H_d(x) > 0$. Since the proposed approach produces stable oscillations of the closed-loop system, **Lemma 1** is introduced, which allows shaping of the target system, such that all the trajectories converge to a compact set, except for a set of Lebesgue measure zero.

Now, in order to shape the dynamics of the corresponding target system, the energy function $H_d(x)$ is proposed as:

$$H_d(x) = \frac{1}{2}\left(S(x) - \mu\right)^2, \tag{8}$$

with $\mu > 0$ and $S(x)$ selected according the assumption **A1**, and $D(x) \in R^{n \times n}$ selected as:

$$D(x) = \phi(x)g(x)g^T(x); \tag{9}$$

where $\phi(x) \geq \varepsilon > 0$. Then, the corresponding target system (4) is:

$$\dot{x} = (S(x) - \mu)\left(J(x) - \phi(x)g(x)g^T(x)\right) \nabla S(x). \tag{10}$$

So, based on the system (10) we propose the following lemma, which will help us to carry out the corresponding stability analysis.

Lemma 1: *Under assumption **A1** the equilibrium point $x = 0$ of the system (10), is a repeller point* (see the Appendix for details).

Now we proceed to give the sufficient condition to assure the stability towards the compact set E. To this end, we present the main proposition of this work:

[1] Since $S(x)$ is in most cases only locally strictly positive, the constant μ should be selected, such that it be always inside of the range $\overline{\mu} > \mu > 0$; where the set defined by $\{x \in R^n : S(x) \leq \overline{\mu}\}$ is bounded and convex.

Proposition 1: *Consider the nonlinear controlled system (1). If $S(x)$ and $J(x) = -J^T(x)$, solve the following* ***PDE****:*

$$g^{\perp}(x)f(x) = (S(x) - \mu)\, g^{\perp}(x)J(x)\nabla S(x), \tag{11}$$

where $S(x)$ and μ are selected according to assumption **A1***, then, the system (1), in closed-loop with:*

$$\begin{aligned} u &= -G\left[Z\left(J(x) - \phi(x)g(x)g^T(x)\right)\nabla S(x) - f(x)\right]; \\ G &= \frac{g^T(x)}{g^T(x)g(x)}; Z = (S(x) - \mu). \end{aligned} \tag{12}$$

which asymptotically converges to E except for a set of measure zero. An estimation of its domain of attraction is given by the largest bounded set $\mathbf{D} = \{x \in R^n : \frac{1}{2}\left(S(x) - \mu\right)^2 \leq c\}$.

Proof: From relations (5), (11), in conjunction with Lemma 1, the closed-loop system is equivalent to the target system (10). Hence, the stability analysis can be carried out over the latter system. Thus, computing the time derivative of H_d, defined in (8), around the trajectories of (10), leads to:

$$\dot{H}_d(x) = (S(x) - \mu)\left(\nabla H_d\right)^T\left(J(x) - \phi(x)g(x)g^T(x)\right)\nabla S(x). \tag{13}$$

So, after using the identity $\nabla H_d = (S(x) - \mu)\nabla S(x)$ and defining y, as:

$$y(x) = g^T(x)\nabla S(x), \tag{14}$$

it is quite easy to see that the equation (13) can be simplified, as:

$$\dot{H}_d(x) = -\phi(x)y^2(x)\left(S(x) - \mu\right)^2 \leq 0. \tag{15}$$

That is, $H_d(x) \leq H_0(0)$. Now, from **A1** it can be assured that, by construction, $H_d(x)$ is radially bounded, implying that $x(t) \in L_\infty$, as long as $x(0) \in \mathbf{D}$. Hence, $\dot{H}_d$ is uniformly continuous. On the other hand, $H_d(x)$ has a limit because is a non-increasing function and bounded from below. Therefore, from the Barbalat lemma, the following is obtained:

$$\lim_{t\to\infty} \dot{H}_d(x) = \phi(x)y^2(x)\left(S(x) - \mu\right)^2 = 0. \tag{16}$$

From the above, x approaches to M as long $t \to \infty$, where M is defined as:

$$M = \{y(x) = 0\} \cup \{E\}. \tag{17}$$

Clearly, there are two possibilities: a) $y(x) = 0$ in M; or b) $S(x) = \mu$ in M. From **A1**, $x(t) = 0$, if $y(x) = 0$. However, it must be remembered that, according to Lemma 1, $x(t) = 0$ is a repeller point, which is a case that its probability is almost zero. On the other hand, $x \in E$ assures that x converges to the defined limit set. These arguments allow us to claim that the closed-loop system converges asymptotically to E, except for a set of measure zero.

Notice that, if $S(x)$ is a global strictly positive and proper function, and $g(x)$ has a constant rank for all $x \in R^n$, then global asymptotic stability is assured; however, if the above properties are solely locally fulfilled, then only local asymptotic stability of the closed-loop system can be assured. Finally, some important properties must be mentioned:

P1: *Under assumption A1 if the initial conditions verify the following inequality:*

$$H_d(x(0)) \leq \bar{c} < \frac{\mu^2}{2},$$

then, $x(t) \to E$ as long as $t \to \infty$, because $H_d(x) = 0$ whenever $S(x) = \mu$ and $H_d(x) = \frac{\mu^2}{2}$. Therefore, selecting 0¡ε $< \bar{c} < \mu^2/2$, it can always be assured that $\mathbf{D}_{\bar{c}}$*=*$\{x \in R^n : \frac{1}{2}\left(S(x) - \mu\right)^2 \leq \bar{c}\}$ *includes the limit set E but not the origin. According to LaSalle's Theorem, any motions starting in $\mathbf{D}_{\bar{c}}$ converge to set E. That is, the limit cycle is said to be convergent. In a similar way, it can be shown that the origin is unstable, since any motion starting arbitrarily near the origin converges to the limit set E. That is, these motions diverge from the origin.*

P2: *Under conditions in P1: If, $x(0) = x_0$, is initialized inside of, $\Omega_0 = \{x \in \mathbf{D} : S(x) \leq c\}$, then $x(t) \in \Omega_0$; for all $t \geq 0$ (see the Appendix).*

Comments: To clarify some limitations of the proposed control approach, it should be noted that, as is well known, the main obstacle with **iDA-PBC** is the so-called matching equation for making the closed-loop a port Hamiltonian system, because the matching condition consists of a set of **PDE**'s, which generally are very difficult to solve as the system order increases; however, novel methods to solve these equations numerically are constantly being developed. The linear case is easy to solve by means of numerical or algebraic methods. On the other hand, function $\phi(x) > \epsilon > 0$ can be seen as a modulation factor, which allows the increasing or decreasing of the magnitude of the stabilizer controller.

3. NUMERICAL EXAMPLE

The pendulum actuated by a DC-motor:

Now, it is required to induce a limit set for the well-known pendulum actuated by a **DC**-motor Mahindrakar and Sankaranarayanan (2008), shown in Fig. 3 and described by:

$$\begin{aligned} \dot{\theta} &= \omega, \\ \dot{\omega} &= \frac{1}{J}\left(k_m I - B\omega - mgl\sin\theta\right), \\ \dot{I} &= \frac{1}{L}\left(-RI - k_e\omega + u\right), \end{aligned} \tag{18}$$

where θ and ω are the pendulum angular position and the angular velocity; variables, respectively; I and u are the armature circuit electric current and the applied voltage, respectively, where the former is the input. The parameters J, m, l, and g are the system total inertia, the pendulum mass, the pendulum arm length and the gravity constant, respectively; coefficients B and k_m are the pendulum viscous friction and the constant motor torque, respectively. Finally, L, R, and k_e are the armature circuit inductance, the armature circuit resistance and the back electromotive force constant, respectively. In order to obtain the needed $S(x)$, the linear transformation $z = k_m I - B\omega$, is introduces, which transforms system (18) into:

$$\dot{\theta} = \omega;\ \dot{\omega} = \frac{1}{J}\left(z - mgl\sin\theta\right);\ \dot{z} = \alpha u + \beta(x), \tag{19}$$

where,

$$\alpha = \frac{k_m}{L};$$
$$\beta(x) = -\left(\frac{Bk_m}{J} + \frac{Rk_m}{J}\right) I + \left(\frac{B^2}{J} - \frac{k_e k_m}{J}\right)\omega + R;$$
$$R = \frac{Bmgl}{J}\sin\theta.$$

That is, for this particular case,

$$f(x) = \begin{bmatrix} \omega \\ \frac{1}{J}(z - mgl\sin\theta) \\ \beta(x) \end{bmatrix}; \; J = \begin{bmatrix} 0 & w_1 & w_2 \\ -w_1 & 0 & w_3 \\ -w_3 & -w_3 & 0 \end{bmatrix};$$
$$g(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Therefore, the matching condition leads to:

$$\omega = (S-\mu)\left(w_2\frac{\partial S}{\partial z} + w_1\frac{\partial S}{\partial \omega}\right); \qquad (20)$$
$$\frac{1}{J}(z - mgl\sin\theta) = (S-\mu)\left(w_3\frac{\partial S}{\partial z} - w_1\frac{\partial S}{\partial \theta}\right).$$

Now, letting:

$$w_1 = \frac{1}{J(S(x)-\mu)};\; w_2 = 0;\; w_3 = \frac{1}{J(S(x)-\mu)};$$

it is easy to check that a solution that verifies (20), is given by:

$$S(x) = (1-\cos\theta) + \frac{z^2}{2} + \frac{J\omega^2}{2mgl}.$$

Notice that the function $S(x)$ is strictly positive and proper, as long as $S(x) < 2$, implying that $|\theta| < \pi$. Now, if $y = \sin\theta = 0$, then $\theta = 0$ and by equation (19), $\omega = z = 0$. That is the system is also detectable with respect to y. Hence, the assumptions in the **P1** are verified and the closed-loop system asymptotically converges to the limit set $S(x) = 2\lambda$; with $0 < \lambda < 1$.

To illustrate the performance of the proposed controller, a numerical simulation was carried out. The corresponding physical values, taken similarly to the ones chosen in Sira-Ramirez and Agrawal (2004), were:

$J = 45\times 10^{-5} kgm^2$; $B = 6\times 10^{-4} Nms/r$;
$G = 40\times 10^{-3} kgm^2/s^2$; $L = 43mH$;
$R = 30\Omega$; $k_m = 56\times 10^{-3} Nm/A$;
$k_e = 0.08Vs/r$; $l = 0.11m$.

The initial conditions were fixed as ($\theta = \pi$,$\dot{\theta} = 0.5r/s$,$I = 0A$), and the control parameters, as $\phi(x) = 1$ and $\mu = 1$. Figure 4 shows the obtained results. From this figure, it can be seen that the controller effectively makes the system to reach the limit set.

4. CONCLUSIONS

The showed technique solved the stabilization problem of a compact set by means of the **IDA-PBC** methodology. The control strategy consisted of expressing the closed-loop system as if it were a desired Port Hamiltonian system, having the property of being asymptotically stable towards a convenient limit set, whatever the initial conditions were, except for a set of measure zero. The asymptotic stability towards the limit set, E, is assured by using La Salle's Theorem in conjunction with some structural properties of the target Hamiltonian system. To assure the effectiveness of the obtained control strategy,is offered a simulation applied to the pendulum actuated by a DC-motor where convincing results were obtained.

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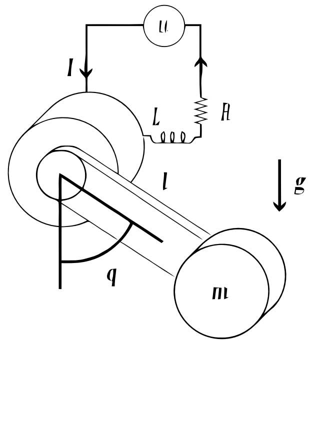

Fig. 1. The DC-motor actuated pendulum system.

Appendix A. RELEVANT PROOFS

Proof of lemma 1: After substituting (8) and (9) into (4), the resulting target system (10)can be obtained. Now, to check that $x = 0$ is a repelling point, it must be pointed out that the time derivative of $S(x)$, around the trajectories of (10), leads to the following equality:

$$\dot{S}(x) = -(S(x) - \mu)\phi(x)\left(g^T(x)\nabla S(x)\right)^2,$$

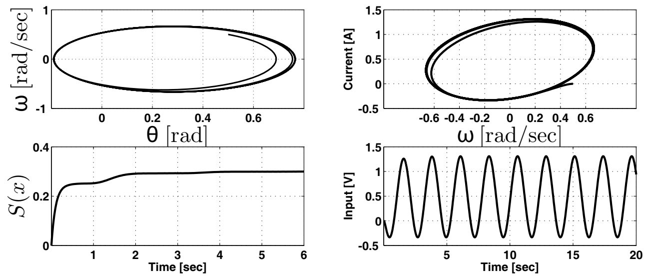

Fig. 2. Closed-loop response of the DC-motor actuated pendulum system.

where $\phi(x) \geq \varepsilon > 0$; for all $x \in R^n$. Then, $\dot{S} > 0$ as long as $S(x) < \mu$. This means that, if $x(t_*)$ belongs to the set $S(x) \leq \mu_*$; where $0 < \mu_* < \mu$, then necessarily $x(t)$ tends to leave out of this set. Notice that the values of $|x(t)|$, increase continuously until $S(x) = \mu$. This is justified because by assumption $S(x) \leq \mu$ is bounded and proper in its arguments. The other case, when $x(t_*) \in R^n$, such that, $S(x) > \mu$. Then, $\dot{S}(x(t)) < 0$ for $t \geq t_*$. That is, the values of the $|x(t)|$ decrease continuously until $S(x) = \mu$. Therefore, the single point $x = 0$ is a repelling point of the target system. It should be noted that the case $g^T(x)\nabla S(x) = 0$ is neglected it because leads to $x = 0$, by the assumption **A1**.

Proof of P2: First of all, it must be noted that the set $S(x) \leq c$ is compact by assumption. Next, it is shown that $S(x_0) \leq \delta$, then $S(x) \leq \delta$. By contradiction, suppose that, for some $t_1 > 0$, we have that, $S(x(t_1)) > \delta$. So, there are two numbers, $\varepsilon_0 \geq 0$ and $\varepsilon_1 > 0$, that the following are obtained: $S(x_0) = \delta - \varepsilon_0$ and $S(x(t_1)) = \delta + \varepsilon_1$. However, $H_d(x)$ is a non-increasing function and bounded from below, implying that:

$$\begin{aligned} & H_d(x(t_*)) \leq H_d(x_0) \\ & \frac{1}{2}\left(\varepsilon_1 + (1-\lambda)\delta\right)^2 = H_d(x(t_*)) \\ & H_d(x_0) = \frac{1}{2}\left(-\varepsilon_0 + (1-\lambda)\delta\right)^2 . \end{aligned} \tag{A.1}$$

This leads to a contradiction, because $\varepsilon_1 \geq 0$ and $\varepsilon_0 > 0$. Consequently, $S(x(t)) < \delta$ for all $t \geq 0$.