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Observer-based stabilization of a class of unstable delayed systems with a couple of complex conjugate stable poles ⋆

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Abstract: This work deals with the problem of the stabilization and control of a class of linear unstable system plus time-delay. An observer is designed in order to obtain delay-free dynamics of the system, allowing to stabilize the delayed unstable plant. Stability conditions are given in terms of the parameters of the system and the time delay size. The regions of stability for the gains of the proposed observer based controller are computed and the results are tested in numerical simulations.

Keywords: Linear systems, Time-delay, Unstable systems, Observer-based control.

1. INTRODUCTION

Time-delay systems appear commonly in different industrial processes. Delay phenomena is associated with systems where transport of material and/or information occurs. Systems with delays arise in engineering, biology, physics, traffic flow modeling, etc. Sipahi et al. (2011). Time delays are often a source of complex behaviors such as oscillations, bad performance or even instability in dynamical systems, thus considerable attention had been paid on the stability analysis and the control design for time-delay processes, Niculescu (2001), Richard (2003). Hence, the study of systems with delays has been a subject of great interest during the last decades.

Different strategies have been developed to design controllers for delayed processes. A common approach is to approximate the time-delay operator by means of a Taylor or Padé series which could lead to a non minimum-phase system with rational transfer function representation. On the other hand, some recent works have been devoted to the analysis of stability and stabilization of systems with delay based on Lyapunov-Krasovskii and Lyapunov-Razumikhin approaches, Kolmanovskii and Richard (1999), Fridman and Shaked (2002), Gu et al. (2003).

A common approach to deal with time delay systems is the Smith Predictor, (Smith (1957), Palmor (1996)) which consists in counteracting the time delay effects by mean of strategies intended to estimate the effects of current inputs over future outputs. Unfortunately the Smith Predictor is restricted to stable plants. In order to handle with unstable plants, some modifications of the SP original structure have been proposed (see for instance Seshagiri et al. (2007) and Normey-Rico and Camacho (2008)). In Rao and Chidambaram (2006), it was presented an efficient modification to the Smith predictor to control unstable second order systems with time delay, using the direct synthesis method. With a different perspective, Normey-Rico and Camacho (2009), propose a modification to the Smith predictor to deal with high-order unstable delayed systems. However, it is necessary a discrete implementation to cancel unstable roots, yielding high order discrete controllers which need precise parameters, Normey-Rico et al. (2012).

Classic controllers P, PI and PID are also studied to delayed processes. For example Silva et al. (2004) design PID controllers for first order unstable systems. An step forward is given by Xiang et al. (2007), where conditions for the stabilization of second order unstable systems with PID controllers are proposed.

Many chemical and biological systems exist whose dynamics present second or higher order behavior. Continuous stirred tank reactors, polymerization reactors and bioreactors are inherently unstable by design; these types of systems can be modeled as open-loop unstable systems plus time delay. The stabilization of linear systems with one unstable pole, n real stable poles and time delay, are tackled in Lee et al. (2010) by static output feedback and PI-PID controllers. In Hernandez et al. (2013), necessary and sufficient conditions for the stabilization of linear

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systems by static output feedback for SISO systems with one unstable pole, a couple of complex conjugate stable poles and time delay, are provided. Using an observer based control scheme, the stabilization of high order systems with real poles, two of them unstable, is dealt in Novella Rodríguez et al. (2014).

This paper is concerned with the stabilization problem of systems with two unstable poles and a pair of complex conjugate stable poles plus time delay. The control scheme relies on an observer-based structure with a memory observer and a memoryless state feedback, so only two gains are enough to stabilize the observer scheme, and two other to stabilize the open loop unstable plant. Necessary and sufficient conditions are stated to guarantee the existence of the proposed scheme in terms of an algebraic relation between the size of the delayed term and the system time constants. On the contrary of modified Smith predictors, the scheme only contains discrete time-delays (and not distributed ones), which makes easy its practical implementation (see Zhong (2006) for details on numerical implementation of modified Smith predictor scheme).

This work is organized as follows; section 2 introduces the problem statement. In section 3 some preliminary results are briefly presented. Then, in section 4 the proposed control strategy is presented, furthermore necessary and sufficient conditions for the existence of the stabilizing observer-based control structure are given. In section 5 some numerical simulations of an academic example are presented. Finally, some conclusions are given in the last section.

2. PROBLEM FORMULATION

Consider the following class of single-input single-output (SISO) linear systems with delay term:

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s}, \tag{1}$$

where $U(s)$ and $Y(s)$ are the input and output signals respectively, $\tau \geq 0$ is the constant time delay and $G(s)$ is the delay-free transfer function. Notice that with respect to the class of systems (1) a traditional control strategy based on an static output feedback of the form:

$$U(s) = k[R(s) - Y(s)], \tag{2}$$

yields a closed-loop system given by:

$$\frac{Y(s)}{R(s)} = \frac{kG(s)e^{-\tau s}}{1 + kG(s)e^{-\tau s}}, \tag{3}$$

where the exponential term $e^{-\tau s}$ located at the denominator of the transfer function (3) leads to a system with an infinite number of poles and where the closed-loop stability properties must be carefully stated.

This work proposes an observer based control scheme in order to stabilize a class of systems characterized by:

$$G(s) = \frac{\alpha e^{-\tau s}}{(s-a)(s-b)(s^2 + 2\zeta\omega_n + \omega_n^2)}, \tag{4}$$

where $\tau \geq 0$, and without loss of generality, $a \geq b > 0$. The parameters ζ and ω_n are the damping relation and the undamped natural frequency respectively. In this paper we are concerned to the problem when $0 < \zeta < 1$, *i.e.* a couple of complex conjugate poles is present in the system.

The proposed control scheme has been designed taking into account the traditional observer theory hence only the plant model and two static gains are enough to get an adequate estimation of an internal delay free variable which will be used in the final stabilizing control scheme.

For sake of simplicity, we denote the stable subsystem $G_{stb}(s)$ as follows:

$$G_{stb}(s) = \frac{\alpha}{s^2 + 2\zeta\omega_n + \omega_n^2}. \tag{5}$$

3. PRELIMINARY RESULTS

To achieve the goal of this work, we have considered some preliminary results which are presented in this section.

Lemma 1. Consider the unstable third-order system with time-delay characterized by:

$$G(s) = \frac{\alpha}{(s-a)(s^2 + 2\zeta\omega_n s + {\omega_n}^2)}. \tag{6}$$

There exists a gain k such that the closed loop system (3) is stable if and only if

$$\tau < \frac{1}{a} - \left(\frac{2\zeta}{\omega_n}\right).$$

This result can be demonstrated with an analysis in the frequency domain. The proof of this result is developed in Hernandez et al. (2013).

Lemma 2. Consider the unstable high-order delayed system (1), with an unstable pole, three stable poles of which a couple are complex conjugates, given by:

$$G(s) = \frac{\alpha e^{-\tau s}}{(s-a)(s+c)(s^2 + 2\zeta\omega_n s + {\omega_n}^2)}. \tag{7}$$

there exist a proportional gain k such that the closed-loop system (3) is stable if and only if

$$\tau < \frac{1}{a} - \frac{1}{c} - \frac{2\zeta}{\omega_n}.$$

Proof 1. The proof of this lemma is done in a similar manner of the one presented in Hernandez et al. (2013). Consequently, this result can be demonstrated with an analysis in the frequency domain. From the Nyquist stability criteria, the system will be stable iff $N + P = 0$, where P is the number of poles in the right half plane "s" and N the number of rotations to the critical point $(-1,\ j0)$ point clockwise (N negative in the counterclockwise direction) in the Nyquist diagram. Therefore the phase expression in the frequency domain ω for (7) is given by:

$$\angle G(j\omega) = -\left(180^\circ - \tan^{-1}\frac{\omega}{a}\right) - \omega\tau \cdots \tag{8}$$
$$\cdots - \tan^{-1}\frac{\omega}{c} - \tan^{-1}\left(\frac{2\zeta\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)$$

We can note in this case, as $P = 1$, there exist a gain k that stabilizes the system iff $N = -1$, namely, iff there exists a counterclockwise rotation to the critical point $(-1, 0)$ in the Nyquist diagram. To ensure the Nyquist diagram starts counterclockwise, $\angle G(j\omega) > -180^\circ$ for small frequencies ($\omega \approx 0$). Hence, the phase should increase from -180° for frequencies near to 0, and then decrease, and there is one and only one intersection with the negative real axis in the anticlockwise for some positive frequency. In order to find the necessary and sufficient condition for the existence to obtain a counterclockwise rotation in the Nyquist diagram, it is possible to derive the expression of phase such that:

$$\frac{d}{dw}\angle G(j\omega) = -\tau + \frac{a}{a^2+\omega^2} - \frac{c}{c^2+\omega^2} \cdots \tag{9}$$
$$-\frac{2\omega_n\zeta(\omega+\omega_n^2)}{\omega^4+2\omega\omega_n^2(2\zeta^2-1)+\omega^4},$$

then, for small frequencies (namely $\omega \approx 0$), the phase equation must satisfy $\angle G(j\omega) > -180^\circ$, solving for τ, the stability condition for the system (7) is:

$$\tau < \frac{1}{a} - \frac{1}{c} - \frac{2\zeta}{\omega_n}.$$

4. MAIN RESULT

Taking into account the class of systems studied in this work and characterized by the transfer function (4) with a, $b > 0$, $0 < \zeta < 1$, and the time delay $\tau \geq 0$, and assuming without loss of generality $a \geq b$. An observer based control is designed in order to obtain an estimation of the internal states of the system to be used as control signals for the original process.

As a first step, the stability conditions for the controller and the observer system are stated separately. This conditions will be used later in order to state the closed loop stability conditions for the proposed observer based controller.

4.1 Controller

First, taking into consideration the controller structure shown in Fig. 1, with the control law $u(t) = r(t) - k_1 w(t) - k_2 y(t)$, let us introduce the following result.

Lemma 3. Consider the delayed system given by (1) and (4), and the control law mentioned above. There exist gains k_1 and k_2 such that the closed-loop system is stable if and only if

$$\tau < \frac{1}{b} - \frac{2\zeta}{\omega_n}.$$

The aim of the foregoing proof is to apply the stability conditions given in Lemma 2 to the state feedback strategy shown in the Fig. 1.

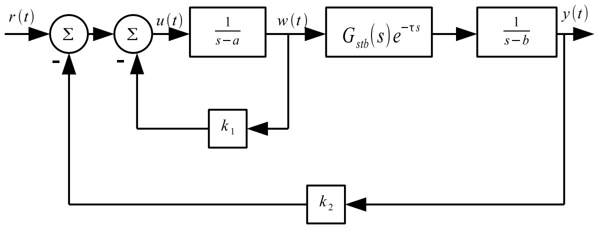

Fig. 1. Controller Scheme

Proof 2. Sufficiency. Let us consider a time delay such that $\tau < \frac{1}{b} - (2\zeta/\omega_n)$. Then, $\tau = \frac{1}{b} - (2\zeta/\omega_n) - \beta$, for some $\beta > 0$. Therefore, it is possible to select a k_1 such that $\beta > \frac{1}{k_1 - a} > 0$. Then, it is easy to determine

$$\tau < \frac{1}{b} - \frac{2\zeta}{\omega_n} - \frac{1}{k_1 - a}. \tag{10}$$

Finally, we can conclude of Lemma 2, assuming $c = k_1 - a$, there exists a gain k_2 such that the closed loop plant is stable.

Necessity. Considering the delayed system 4, and the state feedback controller shown in Fig. 1, with constant gains k_1 and k_2 such that the process is stable. The closed loop transfer function of the plant can be written as follows:

$$\frac{Y(s)}{R(s)} = \frac{\alpha e^{-\tau s}}{(s-b)(s^2+2\zeta\omega_n s+\omega_n^2)(s+\phi)+\alpha k_2 e^{-\tau s}}, \tag{11}$$

with $\phi = k_1 - a$. It is well known that a k_2 that stabilizes the delayed system (11) must also stabilize the delay free system (see for instance Niculescu (2001) or Malakhovski and Mirkin (2006)), which implies that $\phi > 0$. Indeed, from the preliminary results, it is possible to conclude $\tau < \frac{1}{b} - (2\zeta/\omega_n) - \frac{1}{\phi}$, with $\phi = k_1 - a$, (note that $\phi > 0$ is a free parameter function of k_1). Let us consider $\beta > \frac{1}{\phi} > 0$, and denoting $\beta = \frac{1}{b} - (2\zeta/\omega_n) - \tau$, therefore:

$$\tau = \frac{1}{b} - \frac{2\zeta}{\omega_n} - \beta < \frac{1}{b} - \frac{2\zeta}{\omega_n}.$$

4.2 Observer

In most of the practical applications, the internal variables are not measured. Thus, an observer based on an output injection strategy is proposed. Let us take into consideration the static output injection scheme shown in Fig. 2. The stability of the observer can be tackled as follows.

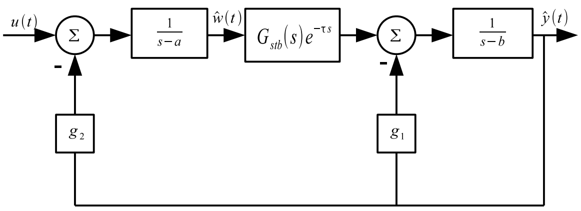

Fig. 2. Observer Scheme

Lemma 4. Considering the delayed system given by (1) and (4), and the static output injection scheme shown in

Fig. 2, there exist constants g_1 and g_2 such that the closed-loop system is stable if and only if

$$\tau < \frac{1}{a} - \frac{2\zeta}{\omega_n}.$$

Proof 3. The proof can easily be derived from a dual procedure of the previous result.

4.3 Observer Based Controller

Finally, the main result of this work is presented; we propose an observed based controller as in Fig. 3, where the observer allows to estimate the state variables, to be used in state feedback controller. It is important to note that, in the scheme, only four proportional gains are enough to get a stable closed loop behavior. As a consequence of the previous results, the following lemma can be stated.

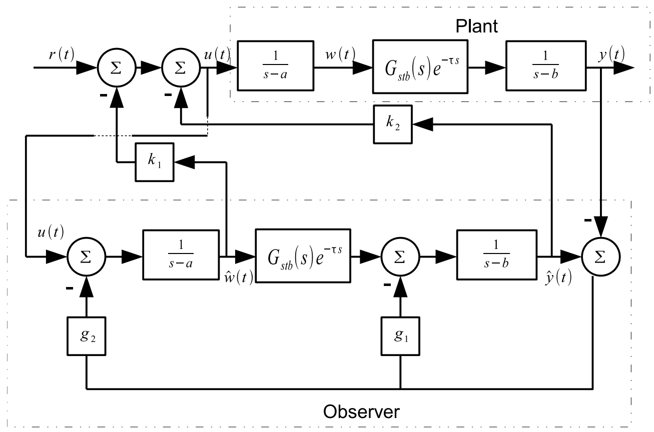

Fig. 3. Observer-Controller Scheme

Theorem 1. Consider the observer based controller scheme shown in Fig. 3. There exist proportional gains k_1, k_2, g_1 and g_2 such that the closed-loop system is stable if and only if

$$\tau < \frac{1}{a} - \frac{2\zeta}{\omega_n}.$$

Proof 4. Consider a possible state space representation of the system (4) characterized by the following equation:

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + A_1x(t-\tau) + Bu(t) \\ y(t) &= Cx(t), \end{aligned} \tag{12}$$

where the state id defined by $[w(t)x_1(t)x_2(t)z(t)]^T$, being $[x_1(t)x_2(t)]^T$ the internal states of $G_{stb}(s)$, and the following matrices,

$$A_0 = \begin{bmatrix} a & 0 & 0 & 0 \\ 1 & -2\,\zeta\,\omega_n & -\omega_n^2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & b \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = [\,\alpha\ 0\ 0\ 0\,]^T$$

$$C = [\,0\ 0\ 0\ 1\,]$$

Note that the state state representation characterized by (12) can be returned to its transfer function representation by means of:

$$\frac{Y(s)}{U(s)} = C(sI - (A_0 + A_1e^{-\tau s}))^{-1}B, \tag{13}$$

which brings us back to the delayed transfer function (4). The dynamics of the estimated states and the control law can be described as follows.

$$\dot{\hat{x}}(t) = A_0\hat{x}(t) + A_1\hat{x}(t-\tau) + Bu(t) - G(C\hat{x}(t) - y(t)), \tag{14}$$

where $\hat{x}(t)$ is the estimated state of $x(t)$, and the gain vectors K and G are defined by

$$K = [\,k_1\ 0\ 0\ \cdots\ 0\ k_2\,],$$

$$G = [\,g_2\ 0\ 0\ \cdots\ 0\ g_1\,]^T.$$

Let $e(t) := x(t) - \hat{x}(t)$, then we have:

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t) = (A_0 - GC)e(t) + A_1e(t-\tau), \tag{15}$$

and the controlled system:

$$\dot{x}(t) = A_0x(t) + A_1x(t-\tau) - BK\hat{x}(t). \tag{16}$$

Noting $x_e = [x(t)\ e(t)]^T$ and after a simple manipulation of variables we have the following closed loop system with the observer and the controller proposed in the Fig. 3

$$\dot{x_e}(t) = \begin{bmatrix} A_0 - BK & BK \\ 0 & A_0 - GC \end{bmatrix} x_e(t) + \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix} x_e(t-\tau) \tag{17}$$

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} x_e(t).$$

It is easy to see that the observer based controller proposed satisfies the separation principle. Hence, the stability of the observer scheme is enough to assure an adequate error convergence, i.e. there exist proportional gains g_1 and g_2 such that $\lim_{t\to\infty}[\hat{w}(t) - w(t)] = 0$ if and only if

$$\tau < \frac{1}{a} - \frac{2\zeta}{\omega_n},$$

then, considering the fact of the observer and controller can be designed separately and reminding the stability conditions stated previously in Lemmas 3 and 4, is clear that the observer stability condition is more restrictive than the controller one, namely,

$$\frac{1}{a} - \frac{2\zeta}{\omega_n} < \frac{1}{b} - \frac{2\zeta}{\omega_n},$$

therefore, there exist k_1, k_2, g_1 and g_2 such that the closed-loop system is stable if and only if

$$\tau < \frac{1}{a} - \frac{2\zeta}{\omega_n}.$$

Remark 1. It is important to note that even if the main interest of the result is when dealing with linear systems containing a couple of complex stable poles, the same result allows to deal with a system with real poles. Assuming that the parameter $\zeta > 1$, then we have a overdamped system with two real poles in the left half part of the plane s. For this case the result holds.

5. EXAMPLE

The proposed approach in this work will be evaluated by means an academic example in order to illustrate the performance in the observer-controller scheme.

Consider the delayed high-order systems with two unstable poles, and a pair of complex conjugate poles characterized by the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{1}{(s-1.17)(s-0.5)(s^2+10.50s+29.26)}e^{-0.2s}, \tag{18}$$

where $\tau = 0.2$, $a = 1.17$, $b = 0.5$, $\zeta = 0.97$ and $\omega_n = 5.40$. It is clear from the system parameters, since:

$$\tau < \frac{1}{1.17} - \left(\left(\frac{2 * 0.97}{5.40}\right)\right) = 0.48,$$

that the stability condition given in Theorem 1 is satisfied, so this system can be stabilized by means of the observer based controller proposed in this work.

Since the control scheme holds the separation property the observer and controller can be designed independently. Then the following propositions are given in order to compute the stabilizing gains.

Controller gains

Proposition 1. A simple methodology for choosing the values for the gains of the controller will be given below.

Step 1. In order to ensure the existence of a proportional gain k_2 such that the closed loop system is stable, from the proof of Lemma 3, eq.(10) we obtain

$$k_1 > \frac{1}{\frac{1}{b} - \frac{2\zeta}{\omega_n} - \tau} + a. \tag{19}$$

It is worth stress that if k_1 is selected near to the bound stated in (19), the gain margin for the gain k_2 will be reduced.

Step 2. Once a gain k_1 is selected, we can compute the gain k_2 by means of a frequency domain analysis, Nyquist stability criterion for instance, for the *auxiliary system* (11) such that the controller scheme shown in the Fig. 1 is stable.

In order to illustrate the proposition given above, it is possible to compute the stability gains region for the controller of the system (18) for a time delay in the range $0 < \tau < 0.48$, (Fig. 4).

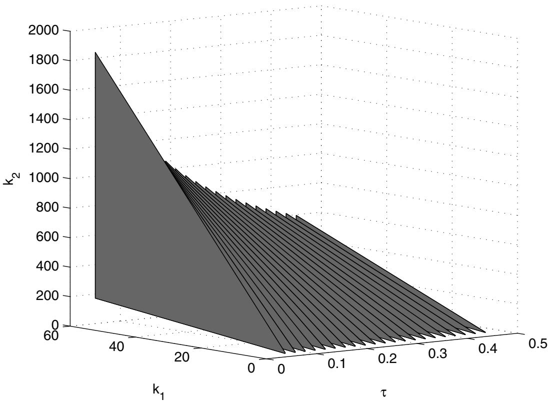

Fig. 4. Stabilizing gains k_1 and k_2 for different delays

Observer gains

Proposition 2. As in the controller design, the computation of the gains g_1 and g_2 can be obtained in a dual way to Remark 1, i.e.,

Step 1. In order to ensure the existence of a proportional gain g_2 such that the closed loop system is stable, from Lemma 1 we obtain

$$g_1 > \frac{1}{(\frac{1}{a} - \frac{2\zeta}{\omega_n} - \tau)} + b.$$

Step 2. Once the gain g_1 is selected, we can compute the gain g_2 by means of a frequency domain analysis, Nyquist stability criterion for instance, such that the controller scheme shown in Fig. 2 is stable.

It is possible to compute the stability gains region for the observer of the system (18) for a time delay in the range $0 < \tau < 0.48$, (Fig. 5).

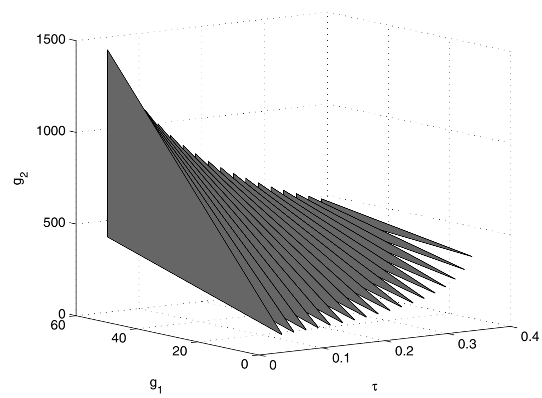

Fig. 5. Stabilizing gains g_1 and g_2 for different delays

Strategy performance

Taking into account the propositions 1 and 2, the stabilizing gains for this example are chosen as follows: $k_1 = 21.177$, $k_2 = 35$, $g_1 = 20.5$ and $g_2 = 80$. The Fig. 6 illustrate the output performance of the observer-controller in numerical simulations whit different initial conditions

$\hat{y}(0) = 0.3$. The Fig. 7 show the error between $y(t)$ and the error $e(t) = \hat{y}(t) - y(t)$ with $\hat{y}(0) = 0.3$.

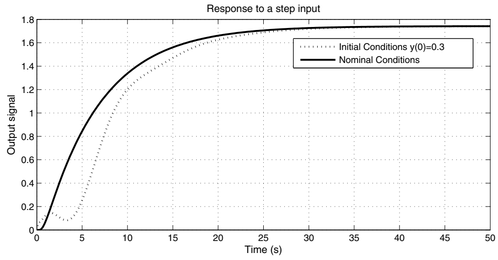

Fig. 6. Performance of the closed loop system

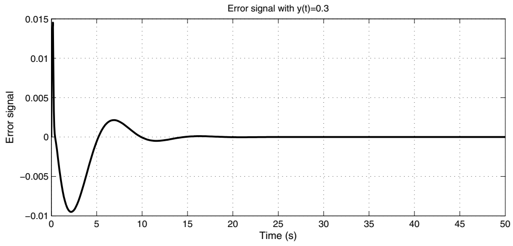

Fig. 7. Error Signal

6. CONCLUSIONS

Unstable systems with time delay usually add complications for its study, which produces a challenge in the stabilization of the system. An observer-controller is proposed in this paper in order to stabilize high-order system with two unstable poles, a couple of complex conjugates poles and time-delay. The necessary and sufficient conditions are stated in order to ensure stability of the closed loop system and the existence of the control scheme. The procedure for the computation of the constant gains is easy and can be realized by mean a frequency domain analysis. The system behavior in closed-loop is illustrated by a numerical example, where is possible show the performance of the proposed control strategy. In addition, is illustrates the behavior of stability regions of the proportional gains, when the time delay increases.

REFERENCES

Fridman, E. and Shaked, U. (2002). An improved stabilization method for linear time-delay systems. *Automatic Control, IEEE Transactions on*, 47(11), 1931 – 1937. doi:10.1109/TAC.2002.804462.

Gu, K., Kharitonov, V.L., and Chen, J. (2003). *Stability of Time-Delay Systems.* Birkhäuser.

Hernandez, M., del Muro, B., Cortes, D., and Araujo, I. (2013). Stabilization of third-order systems with possible complex conjugate poles and time delay. *Revista Mexicana de Ingeniería Química*, 12(2), 351–360.

Kolmanovskii, V. and Richard, J.P. (1999). Stability of some linear systems with delays. *Automatic Control, IEEE Transactions on*, 44(5), 984 –989. doi: 10.1109/9.763213.

Lee, S.C., Wang, Q.G., and Xiang, C. (2010). Stabilization of all-pole unstable delay processes by simple controllers. *Journal of Process Control*, 20(2), 235 – 239.

Malakhovski, E. and Mirkin, L. (2006). On Stability of Second-order Quasi-polynomials with a Single Delay. *Automatica*, 42, 1041–1047.

Niculescu, S.I. (2001). *Delay Effects on Stability: A Robust Control Approach.* Springer.

Normey-Rico, J.E. and Camacho, E.F. (2009). Unified approach for robust dead-time compesator design. *Journal of Process Control*, 19, 38–47.

Normey-Rico, J.E. and Camacho, E.F. (2008). Dead-time compensators: A survey. *Control Engineering Practice*, 16(4), 407–428.

Normey-Rico, J.E., Garcia, P., and Gonzalez, A. (2012). Robust stability analysis of filtered smith predictor for time-varying delay processes. *Journal of Process Control*, 22(10), 1975 – 1984.

Novella Rodríguez, D.F., del Muro-Cuéllar, B., and Sename, O. (2014). Observer-based scheme for the control of high order systems with two unstable poles plus time delay. *Asia-Pacific Journal of Chemical Engineering*, 9(2), 167–180.

Palmor, Z.J. (1996). Time delay compensation smith predictor and its modifications. *The Control Handbook*, 224– 237.

Rao, A.S. and Chidambaram, M. (2006). Control of unstable processes with two rhp poles, a zero and time delay. *Asia-Pacific Journal of Chemical Engineering*, 1(1-2), 63–69.

Richard, J.P. (2003). Time-Delay Systems:An Overview of Some Recent Advances and Open Problems. *Automatica*, 39, 1667–1694.

Seshagiri, R.A., Rao, V.S.R., and Chidambaram, M. (2007). Simple analytical design of modified smith predictor with improved performance for unstable first-order plus time delay (fodtp) processes. *Ind. Eng. Chem. Res*, 46(13), 4561–4571.

Silva, G.J., Datta, A., and Bhattacharyya., S. (2004). *PID Controllers for Time-Delay Systems.* Control Engineering. Birkhäuser.

Sipahi, R., Niculescu, S.I., Abdallah, C., Michiels, W., and Gu, K. (2011). Stability and stabilization of systems with time delay. *Control Systems, IEEE*, 31(1), 38–65.

Smith, O.J.M. (1957). Close Control of Loops with Dead Time. *Chem. Eng. Prog.*, 53, 217–219.

Xiang, C., Wang, Q., Lu, X., Nguyen, L., and Lee, T. (2007). Stabilization of second-order unstable delay processes by simple controllers. *Journal of Process Control*, 17(8), 675 – 682.

Zhong, Q.C. (2006). *Robust Control of Time-Delay Systems.* Springer.